



Standard Test Method Practice for (Analytical Procedure) Determining Transmissivity of Confined Nonleaky Aquifers by Critically Damped Well Response to Instantaneous Change in Head (Slug)¹

This standard is issued under the fixed designation D5881; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This test method covers determination of transmissivity from the measurement of water-level response to a sudden change of water level in a well-aquifer system characterized as being critically damped or in the transition range from underdamped to overdamped. Underdamped response is characterized by oscillatory changes in water level; overdamped response is characterized by return of the water level to the initial static level in an approximately exponential manner. Overdamped response is covered in Guide D4043; underdamped response is covered in D5785, D4043.

1.2 The analytical procedure in this test method is used in conjunction with Guide D4043 and the field procedure in Test Method D4044 for collection of test data.

1.3 *Limitations*—Slug tests are considered to provide an estimate of the transmissivity of an aquifer near the well screen. The method is applicable for systems in which the damping parameter, ζ , is within the range from 0.2 through 5.0. The assumptions of the method prescribe a fully penetrating well (a well open through the full thickness of the aquifer) in a confined, nonleaky aquifer.

1.4 All observed and calculated values shall conform to the guidelines for significant digits and rounding established in Practice D6026.

1.5 *Units*—The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.5.1 The procedures used to specify how data are collected/recorded or calculated, in this standard are regarded as the industry standard. In addition, they are representative of the significant digits that generally should be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any considerations for the user's objectives and it is common practice to increase or reduce significant digits of reported data to be commensurate with these considerations. It is beyond the scope of this standard to consider significant digits used in analytical methods for design.

1.6 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 ASTM Standards.²

D653 Terminology Relating to Soil, Rock, and Contained Fluids

D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction

D4043 Guide for Selection of Aquifer Test Method in Determining Hydraulic Properties by Well Techniques

D4044 Test Method for (Field Procedure) for Instantaneous Change in Head (Slug) Tests for Determining Hydraulic Properties of Aquifers

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² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For Annual Book of ASTM Standards volume information, refer to the standard's Document Summary page on the ASTM website.

*A Summary of Changes section appears at the end of this standard

~~D5785 Test Method for (Analytical Procedure) for Determining Transmissivity of Confined Nonleaky Aquifers by Underdamped Well Response to Instantaneous Change in Head (Slug Test)~~
~~D6026 Practice for Using Significant Digits in Geotechnical Data~~

3. Terminology

~~3.1 Definitions~~ For definitions of common technical terms in this standard, refer to Terminology [D653](#).

~~3.2 Definitions of Terms Specific to This Standard:~~

~~3.2.1 aquifer, confined~~—an aquifer bounded above and below by confining beds and in which the static head is above the top of the aquifer.

~~3.2.2 critically damped well response~~—characterized by the water level responding in a transitional range between underdamped and overdamped following a sudden change in water level.

~~3.2.3 observation well~~—a well open to all or part of an aquifer.

~~3.3 Symbols and Dimensions:~~

~~3.3.1 T~~ —transmissivity [$L^2 T^{-1}$].

~~3.3.2 S~~ —storage coefficient [nd].

~~3.3.3 L~~ —static water column length above top of aquifer [L].

~~3.3.4 L_e~~ —effective length of water column in a well, equal to $L_c + (r_c^2/r_s^2)(b/2)$ [L].

~~3.3.5 L_c~~ —length of water column within casing [L].

~~3.3.6 L_s~~ —length of water column within well screen [L].

~~3.3.7 g~~ —acceleration of gravity [LT^{-2}].

~~3.3.8 h~~ —hydraulic head in the aquifer [L].

~~3.3.9 h_o~~ —initial hydraulic head in the aquifer [L].

~~3.3.10 h_s~~ —hydraulic head in the well screen [L].

~~3.3.11 r_c~~ —radius of well casing [L].

~~3.3.12 r_s~~ —radius of well screen [L].

~~3.3.13 t~~ —time [T].

~~3.3.14 t'~~ —dimensionless time [nd].

~~3.3.15 t^\wedge~~ —dimensionless time [nd].

~~3.3.16 w~~ —water level displacement from the initial static level [L]. [18](#)

~~3.3.17 w_o~~ —initial water level displacement [L]. [sist/aa9b14c7-36be-4831-a963-beeb4066c2a6/astm-d5881-18](#)

~~3.3.18 α~~ —dimensionless storage parameter [nd].

~~3.3.19 β~~ —dimensionless inertial parameter [nd].

~~3.3.20 γ~~ —damping constant [T^{-1}].

~~3.3.21 τ~~ —wavelength [T].

~~3.3.22 ω~~ —angular frequency [T^{-1}].

~~3.3.23 ζ~~ —dimensionless damping factor [nd].

4. Summary of Test Method

~~4.1~~ This test method describes the analytical procedure for analyzing data collected during an instantaneous head (slug) test for well and aquifer response at and near critical damping. Procedures in conducting a slug test are given in Test Method [D4044](#). The analytical procedure consists of analyzing the response of water level in the well following the change in water level induced in the well.

~~4.2 Theory~~—The equations that govern the response of well to an instantaneous change in head are treated at length by Kipp (1).³ The flow in the aquifer is governed by the following equation for cylindrical flow:

$$\frac{S}{T} \frac{dh}{dt} = \frac{1}{r} \frac{d}{dr} \left(r \frac{dh}{dr} \right) \quad (1)$$

where:

h = hydraulic head;

⁴ The boldface numbers in parentheses refer to a list of references at the end of this standard.

T = aquifer transmissivity, and
 S = storage coefficient.

4.2.1 The initial condition is at $t = 0$ and $h = h_o$, and the outer boundary condition is as $r \rightarrow \infty$ and $h \rightarrow h_o$.

4.2.1.1 An equation is given by Kipp (1) for the skin factor, that is, the effect of aquifer damage during drilling of the well. However, this factor is not treated by Kipp (1) and is not considered in this procedure.

4.2.2 The flow rate balance on the well bore relates the displacement of the water level in the well riser to the flow into the well:

$$\pi r_c^2 \frac{dw}{dt} = 2\pi r_s T \frac{dh}{dr} \Big|_{r=r_s} \quad (2)$$

where:

r_c = radius of the well casing, and

w = displacement of the water level in the well from its initial position.

4.2.3 The fourth equation describing the system relating h_s and w , comes from a momentum balance equation of Bird et al (2) as referenced in Kipp (1):

$$\frac{d}{dt} \int_{-b}^0 \pi r_s^2 p v dz = (-pv_2^2 + p_1 - p_2 - \rho g b) \pi r_s^2 \quad (3)$$

where:

v = velocity in the well screen interval;

b = aquifer thickness;

p = pressure;

ρ = fluid density;

g = gravitational acceleration, and

r_s = well screen radius.

The numerical subscripts refer to the planes described above and shown in Fig. 1. Atmospheric pressure is taken as zero.

5. Solution

5.1 Kipp (1) derives the following differential equation to represent for the response of the displacement of water level in the well:

$$\frac{d^2w}{dt^2} + \left(\frac{g}{L_e} \right) w = \frac{g}{(h_s - h_o)} / L_e \quad (4)$$

where:

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L_e = effective water column length, defined as:

$$L_e = L + (r_c^2 h_s^2) (b/2) \quad (5)$$

where:

b = aquifer thickness with initial conditions:

$$\text{at } t = 0, w = w_o \quad (6)$$

$$dw/dt = w_o^* \quad (7)$$

$$h_s = L = h_o \quad (8)$$

5.2 Kipp (1) introduces dimensionless variables and parameters in converting these equations to dimensionless form, solves the equations by Laplace transforms, and inverts the solution by a Laplace-transform-inversion algorithm.

5.2.1 The following dimensionless parameters are among those given by Kipp (1):

dimensionless water-level displacement:

$$w' = w/w_o \quad (9)$$

dimensionless time:

$$t' = (tT)/(r_s^2 S) \quad (10)$$

and:

$$\hat{t} = t'/\beta^{1/2} \quad (11)$$

dimensionless storage:

$$a = (r_c^2)(2r_s^2 S) \quad (12)$$

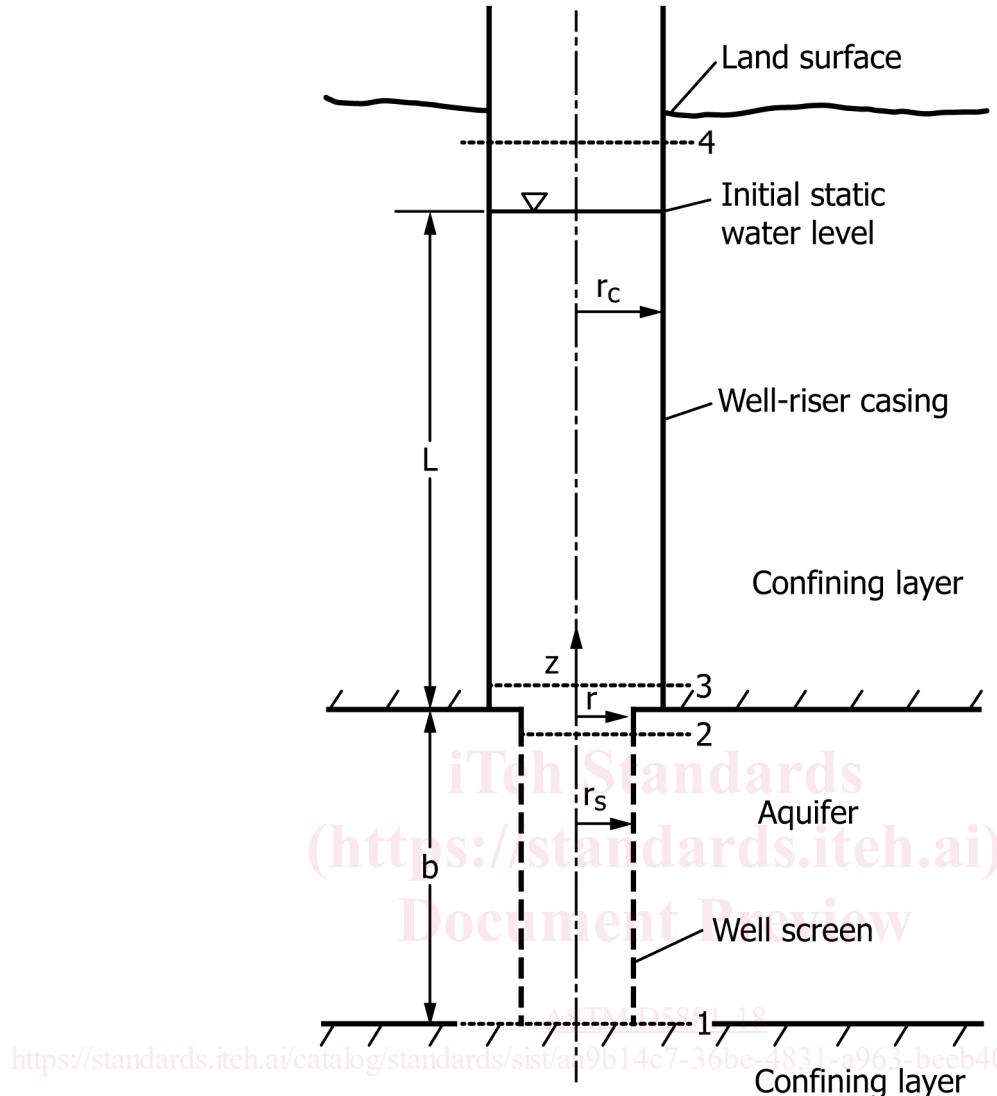


FIG. 1 Well and Aquifer Geometry from Kipp (1)

dimensionless inertial parameter:

$$\beta = (L \epsilon g) (T / (r_s^2 S))^2 \quad (13)$$

dimensionless skin factor:

$$\alpha = f / r_s \quad (14)$$

dimensionless frequency parameter:

$$\omega = \frac{[-d^2(\sigma + \frac{1}{4} \ln \beta) + 4\beta]^{\frac{1}{2}}}{2\beta} \quad (15)$$

dimensionless decay parameter:

$$\gamma = \frac{\alpha(\sigma + \frac{1}{4} \ln \beta)}{2\beta^{\frac{1}{2}}} \quad (16)$$

and dimensionless damping factor:

$$\zeta = \frac{\alpha(\sigma + \frac{1}{4} \ln \beta)}{2\beta^{\frac{1}{2}}} \quad (17)$$

5.3 For ζ less than one, the system is underdamped; for ζ greater than one, the system is overdamped. For ζ equal to one, the system is critically damped, yet the inertial effects are quite important (1). For ζ greater than about five, the system responds as if the inertial effects can be neglected and the solution of Cooper et al. (3) (given in Guide D4043) is applicable. For ζ about 0.2

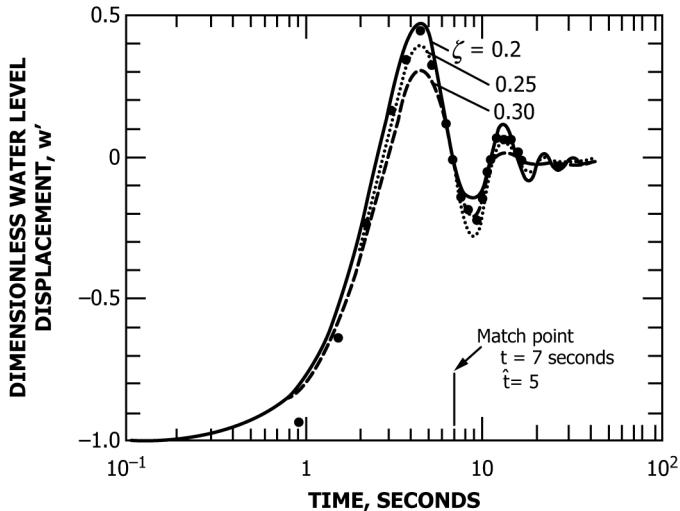


FIG. 2 Slug-Test Data Overlay on Type Curves for Three Different Damping Factors, Modified from Kipp (1)

or less, the approximate solution of vander Kamp (4) is valid (given in Test Method D5785). The solution of Kipp (1), the subject of this test method, is applicable for the transition zone between systems that are underdamped and overdamped. Solutions are given here for ζ ranging from 0.2 to 5.0.

6. Significance and Use

- 6.1 The assumptions of the physical system are given as follows:
 - 6.1.1 The aquifer is of uniform thickness, with impermeable upper and lower confining boundaries.
 - 6.1.2 The aquifer is of constant homogeneous porosity and matrix compressibility and constant homogeneous and isotropic hydraulic conductivity.
 - 6.1.3 The origin of the cylindrical coordinate system is taken to be on the well-bore axis at the top of the aquifer.
 - 6.1.4 The aquifer is fully screened.
 - 6.1.5 The well is 100 % efficient, that is, the skin factor, f , and dimensionless skin factor, σ , are zero.
- 6.2 The assumptions made in defining the momentum balance are as follows:
 - 6.2.1 The average water velocity in the well is approximately constant over the well-bore section.
 - 6.2.2 Frictional head losses from flow in the well are negligible.
 - 6.2.3 Flow through the well screen is uniformly distributed over the entire aquifer thickness.
 - 6.2.4 Change in momentum from the water velocity changing from radial flow through the screen to vertical flow in the well are negligible.

NOTE 1—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself assure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.

7. Procedure

- 7.1 The overall procedure consists of conducting the slug test field procedure (see Test Method D4044) and analysis of the field data using this test method.

NOTE 2—The initial displacement of water level should not exceed 0.1 or 0.2 of the static water column in the well, the measurement of displacement should be within 1 % of the initial water-level displacement and the water-level displacement needs to be calculated independently.

8. Calculation and Interpretation of Results

- 8.1 Plot the normalized water-level displacement in the well versus the logarithm of time.
- 8.2 Prepare a set of type curves from Tables 1–10 by plotting dimensionless water level displacement, w' , versus dimensionless time, t' , using the same scale as in plotting the observed water-level displacement.
- 8.3 Match the semilog plot of water-level displacement to the type curves by translation of the time axis.
- 8.4 From the type curve, record the value of ζ ; from the match point, record the values of t' , and w' from the type curve. From the data plot, record the values of time, t , and water-level displacement, w .
- 8.5 Calculate the effective static water column length, L_e , from the following:

**TABLE 1 Values of the Dimensionless Water Level Displacement,
 w' , Versus Dimensionless Time, t , for Construction of Type
 Curves, $\zeta = 0.1$ and $\alpha = 9988.1$**

| t | w' | t | w' |
|----------------|---------------|-------------------|---------------|
| 3.162278E-02 | -9.994887E-01 | 3.162278E + 00 | 7.100277E-01 |
| 3.636619E-02 | -9.993281E-01 | 3.636619E + 00 | 6.204110E-01 |
| 3.952847E-02 | -9.992086E-01 | 3.952847E + 00 | 4.871206E-01 |
| 4.269075E-02 | -9.990793E-01 | 4.269075E + 00 | 3.138511E-01 |
| 4.743416E-02 | -9.988666E-01 | 4.743416E + 00 | 2.218683E-02 |
| 5.375872E-02 | -9.985483E-01 | 5.375872E + 00 | -3.226809E-01 |
| 6.324555E-02 | -9.979965E-01 | 6.324555E + 00 | -5.191564E-01 |
| 7.115125E-02 | -9.974688E-01 | 7.115125E + 00 | -3.413663E-01 |
| 7.905694E-02 | -9.968794E-01 | 7.905694E + 00 | 3.445623E-05 |
| 8.696264E-02 | -9.962284E-01 | 8.696264E + 00 | 2.889492E-01 |
| 9.486833E-02 | -9.955161E-01 | 9.486833E + 00 | 3.712172E-01 |
| 1.106797E-01 | -9.939077E-01 | 1.106797E + 01 | -1.758246E-02 |
| 1.264911E-01 | -9.920552E-01 | 1.264911E + 01 | -2.697976E-01 |
| 1.423025E-01 | -9.899599E-01 | 1.423025E + 01 | 2.109260E-02 |
| 1.581139E-01 | -9.876230E-01 | 1.581139E + 01 | 1.919487E-01 |
| 1.739253E-01 | -9.850456E-01 | 1.739253E + 01 | -2.455328E-02 |
| 1.897367E-01 | -9.822293E-01 | 1.897367E + 00 | -1.392019E-01 |
| 2.213594E-01 | -9.758851E-01 | 2.213594E + 01 | 9.826209E-02 |
| 2.529822E-01 | -9.686026E-01 | 2.529822E + 01 | -7.129166E-02 |
| 2.846050E-01 | -9.603946E-01 | 2.846050E + 01 | 4.976069E-02 |
| 3.162278E-01 | -9.512748E-01 | 3.162278E + 01 | -3.626029E-02 |
| 3.636619E-01 | -9.359183E-01 | 3.636619E + 01 | -9.997386E-03 |
| 3.952847E-01 | -9.259452E-01 | 3.952847E + 01 | 7.200932E-03 |
| 4.269075E-01 | -9.084819E-01 | 4.743416E + 01 | 5.892951E-03 |
| 4.743416E-01 | -8.947298E-01 | 5.375872E + 01 | 2.737128E-03 |
| 5.375872E-01 | -8.632514E-01 | 6.324555E + 01 | -1.254582E-03 |
| 6.324555E-01 | -8.135785E-01 | 7.115125E + 01 | 2.961127E-04 |
| 7.115125E-01 | -7.673017E-01 | 7.905694E + 01 | -5.757717E-05 |
| 7.905694E-01 | -7.169702E-01 | 8.696264E + 01 | -2.991356E-04 |
| 8.696264E-01 | -6.629659E-01 | 9.486833E + 01 | -1.835296E-04 |
| 9.486833E-01 | -6.056883E-01 | 1.106797E + 02 | -1.426791E-04 |
| 1.106797E + 00 | -4.829810E-01 | 1.264911E + 02 | -1.249977E-04 |
| 1.264911E + 00 | -3.522848E-01 | 1.423025E + 02 | -1.115579E-04 |
| 1.423025E + 00 | -2.171309E-01 | 1.581139E + 02 | -1.001696E-04 |
| 1.581139E + 00 | -8.105198E-02 | 1.739253E + 02 | -9.109389E-05 |
| 1.739253E + 00 | 5.974766E-02 | 1.897367E + 02 | -8.347056E-05 |
| 1.897367E + 00 | 1.802728E-01 | 2.213594E + 02 | -7.152232E-05 |
| 2.213594E + 00 | 4.066508E-01 | 2.529822E + 02 | -6.256450E-05 |
| 2.529822E + 00 | 5.647406E-01 | 2.846050E + 02 | -5.560200E-05 |
| 2.846050E + 00 | 6.811030E-01 | | ... |

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$$\hat{t} = \frac{t}{(L_e/g)^{1/2}} \quad (18)$$

$$L_e = (t/\hat{t})^2 g \quad (19)$$

The effective static water column length should agree, within 20 %, with the effective length calculated from the system geometry (Eq 5).

8.6 Calculate the dimensionless inertial parameter, β , iteratively from the following expression:

$$\beta = [(\alpha \ln \beta)/8\xi^2]^{1/2} \quad (20)$$

where:

ξ = damping parameter;

α = dimensionless storage parameter as given in Eq 12.

8.7 Calculate transmissivity from the following:

$$T = [(\beta g)/L_e]^{1/2} t_s^{-2} \quad (21)$$

8.7.1 Kipp (1) gives an example application of the method, using data from vander Kamp (4) for York Point well 6-2. This well has casing, screen, and well-bore radii of 0.051 m, a water column above the aquifer of 6.5 m, an aquifer thickness of 15 m, and an independently estimated storage coefficient of 8×10^{-5} .

8.7.2 A type curve of dimensionless water-level displacement, w' , plotted against the log of dimensionless time, \hat{t} , for three values of the dimensionless damping factor, ξ , was prepared. Water-level displacement was calculated using an estimated initial displacement of 3.45 cm, and plotted against the log of elapsed time since maximum initial water-level displacement of paper of the same scale as the type curve.

TABLE 2 Values of the Dimensionless Water Level Displacement, w' , Versus Dimensionless Time, t , for Construction of Type Curves, $\zeta = 0.2$ and $\alpha = 19976$

| t | w' | t | w' |
|----------------|---------------|----------------|--------------|
| 3.162278E-02 | -9.994902E-01 | 3.162278E + 00 | 4.939368E-• |
| 3.636619E-02 | -9.993263E-01 | 3.636619E + 00 | 4.349310E-• |
| 3.952847E-02 | -9.992107E-01 | 3.952847E + 00 | 3.465758E-• |
| 4.269075E-02 | -9.990815E-01 | 4.269075E + 00 | 2.343067E-• |
| 4.743416E-02 | -9.988695E-01 | 4.743416E + 00 | 5.160353E-• |
| 5.375872E-02 | -9.985520E-01 | 5.375872E + 00 | -1.543438E-• |
| 6.324555E-02 | -9.980024E-01 | 6.324555E + 00 | -2.671865E-• |
| 7.115125E-02 | -9.974810E-01 | 7.115125E + 00 | -1.818502E-• |
| 7.905694E-02 | -9.968908E-01 | 7.905694E + 00 | -2.600650E-• |
| 8.696264E-02 | -9.962437E-01 | 8.696264E + 00 | 9.764360E-• |
| 9.486833E-02 | -9.955360E-01 | 9.486833E + 00 | 1.324266E-• |
| 1.106797E-01 | -9.939399E-01 | 1.106797E + 01 | 3.871680E-• |
| 1.264911E-01 | -9.921040E-01 | 1.264911E + 01 | -7.304361E-• |
| 1.423025E-01 | -9.900304E-01 | 1.423025E + 01 | -3.623751E-• |
| 1.581139E-01 | -9.877207E-01 | 1.581139E + 01 | 3.430765E-• |
| 1.739253E-01 | -9.851770E-01 | 1.739253E + 01 | -2.397516E-• |
| 1.897367E-01 | -9.824014E-01 | 1.897367E + 01 | -2.051297E-• |
| 2.213594E-01 | -9.761622E-01 | 2.213594E + 01 | 8.187383E-• |
| 2.529822E-01 | -9.690205E-01 | 2.529822E + 01 | -6.259136E-• |
| 2.846050E-01 | -9.609942E-01 | 2.846050E + 01 | 1.402892E-• |
| 3.162278E-01 | -9.521021E-01 | 3.162278E + 01 | -2.331164E-• |
| 3.636619E-01 | -9.371834E-01 | 3.636619E + 01 | -1.031248E-• |
| 3.952847E-01 | -9.262139E-01 | 3.952847E + 01 | -7.347959E-• |
| 4.269075E-01 | -9.105352E-01 | 4.269075E + 01 | -8.050596E-• |
| 4.743416E-01 | -8.975464E-01 | 4.743416E + 01 | -6.352422E-• |
| 5.375872E-01 | -8.673412E-01 | 5.375872E + 01 | -5.870822E-• |
| 6.324555E-01 | -8.201831E-01 | 6.324555E + 01 | -5.087767E-• |
| 7.115125E-01 | -7.766091E-01 | 7.115125E + 01 | -4.500425E-• |
| 7.905694E-01 | -7.295735E-01 | 7.905694E + 01 | -4.046973E-• |
| 8.696264E-01 | -6.794859E-01 | 8.696264E + 01 | -3.675505E-• |
| 9.486833E-01 | -6.267637E-01 | 9.486833E + 01 | -3.366208E-• |
| 1.106797E + 00 | -5.151022E-01 | 1.106797E + 02 | -2.881191E-• |
| 1.264911E + 00 | -3.979593E-01 | 1.264911E + 02 | -2.518280E-• |
| 1.423025E + 00 | -2.786373E-01 | 1.423025E + 02 | -2.236385E-• |
| 1.581139E + 00 | -1.602887E-01 | 1.581139E + 02 | -2.011471E-• |
| 1.739253E + 00 | -3.860371E-02 | 1.739253E + 02 | -1.827551E-• |
| 1.897367E + 00 | 6.204784E-02 | 1.897367E + 02 | -1.674534E-• |
| 2.213594E + 00 | 2.492937E-01 | 2.213594E + 02 | -1.434090E-• |
| 2.529822E + 00 | 3.742380E-01 | 2.529822E + 02 | -1.254123E-• |
| 2.846050E + 00 | 4.694111E-01 | 2.846050E + 02 | -1.113734E-• |

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8.7.3 The data curve was overlaid on the type curve, and shifted horizontally, with the water-level displacement axes coincident, until the best match with the type curve was found. The best fit was for a dimensionless damping factor of 0.25. A match point of $t = 7s$ for $t = 5$ was selected. The resulting graph is shown in Fig. 2.

8.7.4 The effective water column length can be calculated from Eq 19 as follows:

$$L_e = (t/\hat{t})^2 g = (7s/5)^2 (9.80 \text{ m/s}^2) = 19.2 \text{ m} \quad (22)$$

and from the system geometry (see 3.3.4) as:

$$L_e = L_c + (r_c^{-2}/r_s^{-2})(b/2) = 6.5 \text{ m} + (15 \text{ m}/2) = 14 \text{ m} \quad (23)$$

8.7.4.1 The lack of agreement between the estimated and measured effective water column length may be due to factors such as skin effect that may be significant, or assumptions such as “system linearity, high inertial parameter, negligible frictional flowing head loss, radial flow, uniform well-screen flux, and so forth are significantly violated” (1).

8.7.5 Using the value of the dimensionless damping parameter, ζ , Eq 20 can be solved iteratively for β . An initial estimate of the value of α is first made using Eq 12:

$$\alpha = (r_c^{-2})/(2r_s^{-2}S) = (0.051 \text{ m})^2/[2(0.051 \text{ m})^2 (8 \times 10^{-5})] = 6250 \quad (24)$$

so that Eq 20 becomes:

$$\beta = [(adn\beta)/8\zeta]^2 - [(6250 \ln \beta)/8(0.25)]^2 \quad (25)$$

Beginning with any estimate of β , such as the minimum value of 10^6 , a few iterations will produce a value of $\beta = 4.9 \times 10^{-9}$.

8.7.6 The transmissivity, T , is calculated from Eq 21 as follows:

**TABLE 3 Values of the Dimensionless Water Level Displacement,
 w' , Versus Dimensionless Time, t , for Construction of Type
 Curves, $\zeta = 0.5$ and $\alpha = 49940$**

| t | w' | t | w' |
|----------------|---------------|----------------|---------------|
| 3.162278E-02 | -9.994990E-01 | 3.162278E + 00 | 9.492086E-02 |
| 3.636619E-02 | -9.993397E-01 | 3.636619E + 00 | 1.012577E-01 |
| 3.952847E-02 | -9.992213E-01 | 3.952847E + 00 | 8.820339E-02 |
| 4.269075E-02 | -9.990932E-01 | 4.269075E + 00 | 6.762111E-02 |
| 4.743416E-02 | -9.988829E-01 | 4.743416E + 00 | 3.217532E-02 |
| 5.375872E-02 | -9.985688E-01 | 5.375872E + 00 | -8.337546E-03 |
| 6.324555E-02 | -9.980257E-01 | 6.324555E + 00 | -3.647544E-02 |
| 7.115125E-02 | -9.975079E-01 | 7.115125E + 00 | -3.476092E-02 |
| 7.905694E-02 | -9.969310E-01 | 7.905694E + 00 | -2.373581E-02 |
| 8.696264E-02 | -9.962956E-01 | 8.696264E + 00 | -1.338713E-02 |
| 9.486833E-02 | -9.956020E-01 | 9.486833E + 00 | -7.681039E-03 |
| 1.106797E-01 | -9.940425E-01 | 1.106797E + 01 | -6.737283E-03 |
| 1.264911E-01 | -9.922559E-01 | 1.264911E + 01 | -7.879678E-03 |
| 1.423025E-01 | -9.902461E-01 | 1.423025E + 01 | -6.928157E-03 |
| 1.581139E-01 | -9.880166E-01 | 1.581139E + 01 | -5.770595E-03 |
| 1.739253E-01 | -9.855713E-01 | 1.739253E + 01 | -5.154381E-03 |
| 1.897367E-01 | -9.829139E-01 | 1.897367E + 01 | -4.740291E-03 |
| 2.213594E-01 | -9.769780E-01 | 2.213594E + 01 | -3.991538E-03 |
| 2.529822E-01 | -9.702398E-01 | 2.529822E + 01 | -3.447316E-03 |
| 2.846050E-01 | -9.627300E-01 | 2.846050E + 01 | -3.033006E-03 |
| 3.162278E-01 | -9.544800E-01 | 3.162278E + 01 | -2.706963E-03 |
| 3.636619E-01 | -9.407848E-01 | 3.636619E + 01 | -2.330656E-03 |
| 3.952847E-01 | -9.321798E-01 | 3.952847E + 01 | -2.132780E-03 |
| 4.743416E-01 | -9.053980E-01 | 4.269075E + 01 | -1.966362E-03 |
| 5.375872E-01 | -8.786102E-01 | 4.743416E + 01 | -1.759041E-03 |
| 6.324555E-01 | -8.380771E-01 | 5.375872E + 01 | -1.542575E-03 |
| 7.115125E-01 | -8.014756E-01 | 6.324555E + 01 | -1.302071E-03 |
| 7.905694E-01 | -7.627801E-01 | 7.115125E + 01 | -1.152281E-03 |
| 8.696264E-01 | -7.224138E-01 | 7.905694E + 01 | -1.033361E-03 |
| 9.486833E-01 | -6.807796E-01 | 8.696264E + 01 | -9.366315E-04 |
| 1.106797E + 00 | -5.952065E-01 | 9.486833E + 01 | -8.565071E-04 |
| 1.264911E + 00 | -5.088214E-01 | 1.106797E + 02 | -7.312991E-04 |
| 1.423025E + 00 | -4.239899E-01 | 1.264911E + 02 | -6.380141E-04 |
| 1.581139E + 00 | -3.426759E-01 | 1.423025E + 02 | -5.658156E-04 |
| 1.739253E + 00 | -2.592066E-01 | 1.581139E + 02 | -5.082956E-04 |
| 1.897367E + 00 | -1.964942E-01 | 1.739253E + 02 | -4.613954E-04 |
| 2.213594E + 00 | -7.843895E-02 | 1.897367E + 02 | -4.198187E-04 |
| 2.529822E + 00 | -4.874063E-03 | 2.213594E + 02 | -3.613054E-04 |
| 2.846050E + 00 | 6.501678E-02 | 2.529822E + 02 | -3.156807E-04 |
| ... | ... | 2.846050E + 02 | -2.802644E-04 |

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$$T = [(\beta g) / L_e]^{1/2} r_s^2 S = [(4.9 \times 10^9)(9.8 \text{ m/s}^2) / 19.2 \text{ m}]^{1/2} \quad (26)$$

$$(0.051 \text{ m})^2 (8 \times 10^{-5}) = 0.01 \text{ m}^2/\text{s}$$

8.7.7 This example is included to show the application of the method. Because of the difference between the values of L_e exceeds 20 %, this test would not be considered a successful application of this test method.

9. Report

9.1 Prepare the report including the following information. The final report of the analytical procedure will include information from the report on test method selection, Guide D4043, and the field testing procedure, Test Method D4044.

9.1.1 *Introduction*—The introductory section is intended to present the scope and purpose of the slug test method for determining transmissivity and storativity. Summarize the field hydrogeologic conditions and the field equipment and instrumentation including the construction of the control well, and the method of measurement and of effecting a change in head. Discuss the rationale for selecting this test method.

9.1.2 *Hydrogeologic Setting*—Review information available on the hydrogeology of the site; interpret and describe the hydrogeology of the site as it pertains to the method selected for conducting and analyzing an aquifer test method. Compare hydrogeologic characteristics of the site as it conforms and differs from assumptions made in the solution to the aquifer test method.

9.1.3 *Equipment*—Report the field installation and equipment for the aquifer test method. Include in the report, well construction information, diameter, depth, and open interval to the aquifer, and location of control well and pumping equipment. The construction, diameter, depth, and open interval of observation wells should be recorded.

9.1.3.1 Report the techniques used for observing water levels, pumping rate, barometric changes, and other environmental conditions pertinent to this test method. Include a list of measuring devices used during the test method; the manufacturers name, model number, and basic specifications for each major item; and the name and date of the last calibration, if applicable.

**TABLE 4 Values of the Dimensionless Water Level Displacement,
 w' , Versus Dimensionless Time, t , for Construction of Type
 Curves, $\zeta = 0.7$ and $\alpha = 69917$**

| t | w' | t | w' |
|----------------|---------------|----------------|---------------|
| 3.162278E-02 | -9.995070E-01 | 3.162278E + 00 | -6.213039E-02 |
| 3.636619E-02 | -9.993420E-01 | 3.636619E + 00 | -3.354664E-02 |
| 3.952847E-02 | -9.992401E-01 | 3.952847E + 00 | -2.515924E-02 |
| 4.269075E-02 | -9.991031E-01 | 4.269075E + 00 | -2.198880E-02 |
| 4.743416E-02 | -9.988941E-01 | 4.743416E + 00 | -2.246330E-02 |
| 5.375872E-02 | -9.985822E-01 | 5.375872E + 00 | -2.597889E-02 |
| 6.324555E-02 | -9.980437E-01 | 6.324555E + 00 | -2.841030E-02 |
| 7.115125E-02 | -9.975307E-01 | 7.115125E + 00 | -2.670372E-02 |
| 7.905694E-02 | -9.969601E-01 | 7.905694E + 00 | -2.343491E-02 |
| 8.696264E-02 | -9.963325E-01 | 8.696264E + 00 | -2.012564E-02 |
| 9.486833E-02 | -9.956483E-01 | 9.486833E + 00 | -1.743141E-02 |
| 1.106797E-01 | -9.941127E-01 | 1.106797E + 01 | -1.389694E-02 |
| 1.264911E-01 | -9.923583E-01 | 1.264911E + 01 | -1.171415E-02 |
| 1.423025E-01 | -9.903899E-01 | 1.423025E + 01 | -1.010995E-02 |
| 1.581139E-01 | -9.882121E-01 | 1.581139E + 01 | -8.865170E-03 |
| 1.739253E-01 | -9.858299E-01 | 1.739253E + 01 | -7.886036E-03 |
| 1.897367E-01 | -9.832480E-01 | 1.897367E + 01 | -7.099991E-03 |
| 2.213594E-01 | -9.775043E-01 | 2.213594E + 01 | -5.916365E-03 |
| 2.529822E-01 | -9.710195E-01 | 2.529822E + 01 | -5.068451E-03 |
| 2.846050E-01 | -9.638311E-01 | 2.846050E + 01 | -4.431953E-03 |
| 3.162278E-01 | -9.559768E-01 | 3.162278E + 01 | -3.936862E-03 |
| 3.636619E-01 | -9.430270E-01 | 3.636619E + 01 | -3.371269E-03 |
| 3.952847E-01 | -9.350272E-01 | 3.952847E + 01 | -3.076362E-03 |
| 5.375872E-01 | -8.853626E-01 | 4.269075E + 01 | -2.828780E-03 |
| 6.324555E-01 | -8.485776E-01 | 4.743416E + 01 | -2.523926E-03 |
| 7.115125E-01 | -8.158209E-01 | 5.375872E + 01 | -2.206660E-03 |
| 7.905694E-01 | -7.816147E-01 | 6.324555E + 01 | -1.856412E-03 |
| 8.696264E-01 | -7.463554E-01 | 7.115125E + 01 | -1.639455E-03 |
| 9.486833E-01 | -7.104052E-01 | 7.905694E + 01 | -1.467850E-03 |
| 1.106797E + 00 | -6.377118E-01 | 8.696264E + 01 | -1.328729E-03 |
| 1.264911E + 00 | -5.657711E-01 | 9.486833E + 01 | -1.213675E-03 |
| 1.423025E + 00 | -4.963320E-01 | 1.106797E + 02 | -1.034479E-03 |
| 1.581139E + 00 | -4.307045E-01 | 1.264911E + 02 | -9.013608E-04 |
| 1.739253E + 00 | -3.625714E-01 | 1.423025E + 02 | -7.985732E-04 |
| 1.897367E + 00 | -3.142473E-01 | 1.581139E + 02 | -7.168198E-04 |
| 2.213594E + 00 | -2.201264E-01 | 1.739253E + 02 | -6.502288E-04 |
| 2.529822E + 00 | -1.617035E-01 | 1.897367E + 02 | -5.949757E-04 |
| 2.846050E + 00 | -9.684892E-02 | 2.213594E + 02 | -5.085217E-04 |
| ... | ... | 2.529822E + 02 | -4.439977E-04 |
| ... | ... | 2.846050E + 02 | -3.939470E-04 |

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9.1.4 Test Procedures—Report the steps taken in conducting the pretest and test phases. Include the frequency of head measurements made in the control well, and other environmental data recorded before and during the procedure.

9.1.5 Presentation and Interpretation of Test Results:

9.1.5.1 Data—Present tables of data collected during the test.

9.1.5.2 Data Plots—Present data plots used in analysis of the data. Show overlays of data plots and type curve with match points and corresponding values of parameters at match points.

9.1.5.3—Show calculation of transmissivity and coefficient of storage.

9.1.5.4—Evaluate the overall quality of the test on the basis of the adequacy of instrumentation and observations of stress and response and the conformance of the hydrogeologic conditions and the performance of the test to the assumptions (see 5.1).

10. Precision and Bias

10.1 Precision—Test data on precision is not presented due to the nature of this test method. It is either not feasible or too costly at this time to have ten or more agencies participate in an in situ testing program at a given site.

10.2 Bias—There is no accepted reference value for this test method, therefore, bias cannot be determined.

11. Keywords

11.1—aquifer; aquifer tests; groundwater; hydraulic conductivity; observation wells; slug test; storage coefficient; transmissivity

**TABLE 5 Values of the Dimensionless Water Level Displacement,
 w' , Versus Dimensionless Time, t , for Construction of Type
 Curves, $\zeta = 1.0$ and $\alpha = 99881$**

| t | w' | t | w' |
|----------------|---------------|----------------|---------------|
| 3.162278E-02 | -9.995190E-01 | 3.162278E + 00 | -2.219805E-01 |
| 3.636619E-02 | -9.993614E-01 | 3.636619E + 00 | -1.781301E-01 |
| 3.952847E-02 | -9.992445E-01 | 3.952847E + 00 | -1.556584E-01 |
| 4.269075E-02 | -9.991182E-01 | 4.269075E + 00 | -1.371938E-01 |
| 4.743416E-02 | -9.989111E-01 | 4.743416E + 00 | -1.151268E-01 |
| 5.375872E-02 | -9.986024E-01 | 5.375872E + 00 | -9.311931E-02 |
| 6.324555E-02 | -9.980706E-01 | 6.324555E + 00 | -7.022901E-02 |
| 7.115125E-02 | -9.975651E-01 | 7.115125E + 00 | -5.700982E-02 |
| 7.905694E-02 | -9.970039E-01 | 7.905694E + 00 | -4.724055E-02 |
| 8.696264E-02 | -9.963876E-01 | 8.696264E + 00 | -3.986817E-02 |
| 9.486833E-02 | -9.957171E-01 | 9.486833E + 00 | -3.420016E-02 |
| 1.106797E-01 | -9.942169E-01 | 1.106797E + 01 | -2.623916E-02 |
| 1.264911E-01 | -9.925094E-01 | 1.264911E + 01 | -2.105718E-02 |
| 1.423025E-01 | -9.906011E-01 | 1.423025E + 01 | -1.749216E-02 |
| 1.581139E-01 | -9.884982E-01 | 1.581139E + 01 | -1.492222E-02 |
| 1.739253E-01 | -9.862069E-01 | 1.739253E + 01 | -1.299590E-02 |
| 1.897367E-01 | -9.837333E-01 | 1.897367E + 01 | -1.150439E-02 |
| 2.213594E-01 | -9.782635E-01 | 2.213594E + 01 | -9.352879E-03 |
| 2.529822E-01 | -9.721364E-01 | 2.529822E + 01 | -7.879054E-03 |
| 2.846050E-01 | -9.653980E-01 | 2.846050E + 01 | -6.807075E-03 |
| 3.162278E-01 | -9.580927E-01 | 3.162278E + 01 | -5.992307E-03 |
| 3.636619E-01 | -9.461653E-01 | 3.636619E + 01 | -5.080616E-03 |
| 3.952847E-01 | -9.389866E-01 | 3.952847E + 01 | -4.612882E-03 |
| 4.269075E-01 | -9.247502E-01 | 4.269075E + 01 | -4.224055E-03 |
| 5.375872E-01 | -8.944811E-01 | 4.743416E + 01 | -3.749946E-03 |
| 6.324555E-01 | -8.624921E-01 | 5.375872E + 01 | -3.261801E-03 |
| 7.115125E-01 | -8.345350E-01 | 6.324555E + 01 | -2.728883E-03 |
| 7.905694E-01 | -8.058093E-01 | 7.115125E + 01 | -2.401808E-03 |
| 8.696264E-01 | -7.766480E-01 | 7.905694E + 01 | -2.144707E-03 |
| 9.486833E-01 | -7.473366E-01 | 8.696264E + 01 | -1.937288E-03 |
| 1.106797E + 00 | -6.891964E-01 | 9.486833E + 01 | -1.767517E-03 |
| 1.264911E + 00 | -6.328903E-01 | 1.106797E + 02 | -1.501524E-03 |
| 1.423025E + 00 | -5.794237E-01 | 1.264911E + 02 | -1.305668E-03 |
| 1.581139E + 00 | -5.294147E-01 | 1.423025E + 02 | -1.152882E-03 |
| 1.739253E + 00 | -4.759468E-01 | 1.581139E + 02 | -1.035462E-03 |
| 1.897367E + 00 | -4.408436E-01 | 1.739253E + 02 | -9.383461E-04 |
| 2.213594E + 00 | -3.675417E-01 | 1.897367E + 02 | -8.578843E-04 |
| 2.529822E + 00 | -3.213633E-01 | 2.213594E + 02 | -7.322666E-04 |
| 2.846050E + 00 | -2.602688E-01 | 2.529822E + 02 | -6.387252E-04 |
| ... | ... | 2.846050E + 02 | -5.663467E-04 |

**TABLE 6 Values of the Dimensionless Water Level Displacement,
 w' , Versus Dimensionless Time, t , for Construction of Type
 Curves, $\zeta = 1.5$ and $\alpha = 149821$**

| t | w' | t | w' |
|----------------|---------------|----------------|----------------|
| 3.162278E-02 | -9.995363E-01 | 3.162278E + 00 | -3.890578E-01 |
| 3.636619E-02 | -9.993806E-01 | 3.636619E + 00 | -3.398395E-01 |
| 3.952847E-02 | -9.992652E-01 | 3.952847E + 00 | -3.113242E-01 |
| 4.269075E-02 | -9.991407E-01 | 4.269075E + 00 | -2.857245E-01 |
| 4.743416E-02 | -9.989368E-01 | 4.743416E + 00 | -2.520498E-01 |
| 5.375872E-02 | -9.986336E-01 | 5.375872E + 00 | -2.144593E-01 |
| 6.324555E-02 | -9.981127E-01 | 6.324555E + 00 | -1.702739E-01 |
| 7.115125E-02 | -9.976193E-01 | 7.115125E + 00 | -1.419167E-01 |
| 7.905694E-02 | -9.970732E-01 | 7.905694E + 00 | -1.193325E-01 |
| 8.696264E-02 | -9.964754E-01 | 8.696264E + 00 | -1.012090E-01 |
| 9.486833E-02 | -9.958272E-01 | 9.486833E + 00 | -8.656077E-02 |
| 1.106797E-01 | -9.943835E-01 | 1.106797E + 01 | -6.487489E-02 |
| 1.264911E-01 | -9.927509E-01 | 1.264911E + 01 | -5.014254E-02 |
| 1.423025E-01 | -9.909379E-01 | 1.423025E + 01 | -3.9878338E-02 |
| 1.581139E-01 | -9.889526E-01 | 1.581139E + 01 | -3.254993E-02 |
| 1.739253E-01 | -9.868033E-01 | 1.739253E + 01 | -2.719151E-02 |
| 1.897367E-01 | -9.844978E-01 | 1.897367E + 01 | -2.318242E-02 |
| 2.213594E-01 | -9.794484E-01 | 2.213594E + 01 | -1.772176E-02 |
| 2.529822E-01 | -9.738631E-01 | 2.529822E + 01 | -1.426997E-02 |
| 2.846050E-01 | -9.677964E-01 | 2.846050E + 01 | -1.193176E-02 |
| 3.162278E-01 | -9.612998E-01 | 3.162278E + 01 | -1.025528E-02 |
| 3.636619E-01 | -9.508522E-01 | 3.636619E + 01 | -8.479138E-03 |
| 3.952847E-01 | -9.448418E-01 | 3.952847E + 01 | -7.606080E-03 |
| 4.269075E-01 | -9.319204E-01 | 4.269075E + 01 | -6.898443E-03 |
| 5.375872E-01 | -9.073899E-01 | 4.743416E + 01 | -6.056228E-03 |
| 6.324555E-01 | -8.816423E-01 | 5.375872E + 01 | -5.210778E-03 |
| 7.115125E-01 | -8.597084E-01 | 6.324555E + 01 | -4.310538E-03 |
| 7.905694E-01 | -8.376400E-01 | 7.115125E + 01 | -3.768776E-03 |
| 8.696264E-01 | -8.156531E-01 | 7.905694E + 01 | -3.348259E-03 |
| 9.486833E-01 | -7.939129E-01 | 8.696264E + 01 | -3.012294E-03 |
| 1.106797E + 00 | -6.891964E-01 | 9.486833E + 01 | -2.739666E-03 |
| 1.264911E + 00 | -6.328903E-01 | 1.106797E + 02 | -2.315479E-03 |
| 1.423025E + 00 | -5.794237E-01 | 1.264911E + 02 | -2.006129E-03 |
| 1.581139E + 00 | -5.294147E-01 | 1.423025E + 02 | -1.769696E-03 |
| 1.739253E + 00 | -4.759468E-01 | 1.581139E + 02 | -1.583084E-03 |
| 1.897367E + 00 | -4.408436E-01 | 1.739253E + 02 | -1.432076E-03 |
| 2.213594E + 00 | -3.675417E-01 | 1.897367E + 02 | -1.307343E-03 |
| 2.529822E + 00 | -3.213633E-01 | 2.213594E + 02 | -1.113382E-03 |
| 2.846050E + 00 | -2.602688E-01 | 2.529822E + 02 | -9.695169E-04 |
| ... | ... | 2.846050E + 02 | -8.585304E-04 |

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