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Addendum modification of the teeth of cylindrical gears for speed-reducing and speed-increasing gear pairs

Déport des dentures des roues cylindriques pour engrenages extérieurs réducteurs et multiplicateurs

The Technical Committee decided, at its meeting in Bad Dürkheim (Germany) in April 1978, to publish this document in the form of a Technical Report.

The draft Technical Report (DTR 4467) submitted to the members of Technical Committee TC 60 led only to some editing comments and has been approved at the Oslo meeting in June 1981. The following Member-bodies were represented at this meeting :

Belgium	Hungary	Sweden
Finland	Japan	Switzerland
France	Netherlands	United Kingdom
Germany	Norway	USA

0 Introduction

Since Lasche showed empirically in 1889 the advantage of an addendum modification of the teeth for the life of tramway gears geometrical, kinematic and dynamic studies of the gear have shown the importance of the addendum modification in relation to all the characteristics of the gear pair (see clause 2).

In 1957, Henri Deby, who was at that time Chairman of TC 60 — Gears, had the standardization of addendum modifications included in the work programme of this Technical Committee.

Working Group WG 5 was set up and Belgium was entrusted with the Secretariat. Mr Deby was the leader. From the main calculations which he carried out at that time, he drew up a draft proposal for a Standard which was put before WG 5 by the Secretariat as a working document, at the meeting in Chez-le-Bart (Switzerland), September 1970.

It then turned out that some member countries who already had national standards which they found satisfactory, could not approve the rather rigid formulation proposed by the Secretariat of the Working Group, as it did not include their own designs to a sufficient degree.

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The majority of experts thought that the choice of an addendum modification for a given application could not follow a general mandatory rule, but that possibilities of choice should be left to the specialists in each of the particular fields of application for gears, based on their own experience and on the use of methods of calculation which progress with regard to electronic instruments has made easier.

However, it was recognised by these same experts that it was necessary to give non-specialist technicians who are concerned with the problems of gears, practical rules enabling them, without a great deal of experience, to choose carefully an addendum modification of teeth suitable for the problem to be resolved.

1 Scope and field of application

This Technical Report constitutes a good general guide regarding the limits of addendum modifications of teeth and the distribution of addendum modifications between mating gears; however, attention is drawn to the fact that it only regards recommendations which do not have any restrictive nature nor do they require the replacement of existing regulations, instructions or standards which have proved their worth in the past.

This Technical Report only applies to external gear pairs with parallel axes in general engineering, used in speed reducing and speed increasing gear pairs and of which the teeth of spur and helical gears are defined by the standard basic rack tooth profile in conformity with ISO 53.

2 Importance of the addendum modification of the teeth

2.1 Cost of manufacture of a gear-pair

The value of the addendum modification of teeth depends solely, during manufacture, on the relative position of the gear to be cut and the cutter.

The adjustment and control operations are identical whatever the value of this addendum modification. Its influence on the cost of toothed gears is therefore nil in the majority of cases.

2.2 Influence on the normal wear of a gear

An addendum modification of the teeth suitable for toothed gears results in a decrease in specific "slipping" of tooth flanks and, consequently in a decrease in the risk of normal wear of these flanks. Experience and theory both confirm this advantage.

2.3 Influence on the load capacity of the gear

An increase in the sum of the addendum modifications of both toothed gears is beneficial to the gear both on the level of its load capacity under contact pressure (pitting) and on that of its load capacity under tension at the root of the teeth.

2.4 Influence on the shape of teeth (figure 1)

An increase in the addendum modification of the teeth of a gear causes an increase in root thickness and a decrease in the tip thickness for one tooth depth.

A given value which is a function of the number of teeth on the gear under consideration, leads to a zero tip thickness and an increase in the addendum modification beyond this limit leads to both a pointed tooth and a shortened tooth.

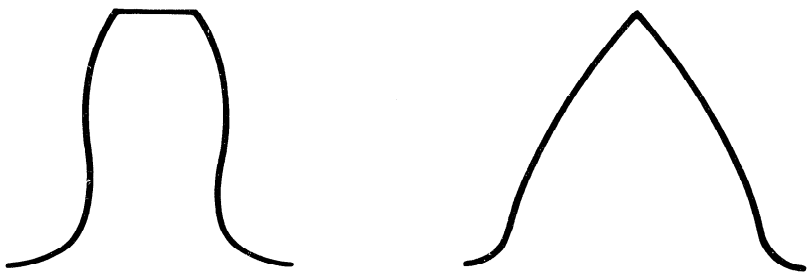

Figure 1

A decrease in the addendum modification of teeth has the reverse effect on tip and root thicknesses and, below a given value which depends on the number of teeth in the gear in question, the addendum modification causes cutter interference which grooves the tooth root. This interference causes weakening of the gear tooth in question and poor functioning of the gear pair of which it is a constituent (noise, rapid wear, abnormal vibrations, excessive heating, loss of transmitted power etc.). For each gear, there is a maximum value for the addendum modification which results in a pointed tooth and a minimum value causing cutter interference.

2.5 Influence on contact

The increase in the sum of addendum modification of the teeth of the two mating gears causes a slight decrease in the contact ratio of the gear pair, which involves a decrease in the regularity of speed of the driven gear and an increase in the level of vibrations. This is all the more noticeable for high pitch speeds. It is necessary to take this into account when selecting addendum modifications for high speed gear-pairs.

2.6 Influence on the clearance at the root of teeth

An increase in the sum of the addendum modifications in conjunction with a small number of teeth in the gear pair may cause an excessive increase in the bottom clearance and cause detrimental phenomena (interference, lubricant pressure, etc.).

2.7 Speed reducing and speed increasing gear pairs

When considering the kinematics of a gear pair it is ascertained that the slipping of the leading tooth, during the approach time, is in the opposite direction to the movement of the point of contact on the tooth flank; this results in a jamming effect which interferes with the correct operation of the gear. A favourable solution is to reduce the approach time by giving the leading tooth as large an addendum modification as possible. For a given sum of addendum modifications, as small an addendum modification as possible must then be chosen for the driven gear.

For a speed-reducing gear pair, the pinion is driving and it is preferable to increase the addendum modification of the latter when the accompanying reduction in the addendum modification of the gear has no detrimental effect. However, for a speed-increasing gear pair, the gear is driving and in this case, a reduction in the addendum modification of the driven pinion is detrimental to the gear pair if it becomes too great. To obtain a suitable speed-increasing gear pair, an addendum modification smaller than in the case of the speed-reducing gear pair must be chosen without however going to an unacceptable value. The reasons for publishing a Technical Report instead of an International Standard are the same as for Technical Report on speed-reducing gear pairs.

2.8 Conclusions

By limiting correctly the addendum modifications of the teeth of toothed gears, the disadvantages which these addendum modifications could cause may be avoided whilst very important properties are modified very favourably by choosing carefully the sum of the addendum modifications and their distribution.

A specific calculation should be carried out for each gear-pair.

The aim of this Technical Report is to give information on the choice of addendum modifications without having to carry out these control calculations.

3 Values of the addendum modifications

3.1 Virtual number of teeth

So as to include straight teeth and helical teeth in an identical formulation, the virtual number of teeth defined by the following equation is used :

$$z_v = \frac{z}{\cos^3\beta}$$

where

z is the normal number of teeth;

β is the reference helix angle.

The virtual number of teeth can be expressed by other, more complex formulae although, depending on the case this does not necessarily lead to more accurate results. However for the addendum modification of the teeth greater accuracy than that given by the simple formula above is not required.

For the straight teeth $\beta = 0$ and $z_v = z$.

NOTES

1 In the formula where the number of teeth of the two mating gears appear by their ratio, the virtual numbers of teeth z_v or the normal numbers of teeth z may be used without discrimination.

2 The formulae in this Technical Report relate to the addendum modification coefficient; the addendum modification is obtained by multiplying this addendum modification coefficient by the normal module m_n.

3.2 Rules for the allocation of the sum of the addendum modification coefficients on the two mating gears

3.2.1 General

The addendum modification coefficients of two mating gears are defined as a function of the number of teeth of these gears and a value for the sum of the addendum modification coefficients (Σx), choosen beforehand as a function of the technical problem raised. For a wise choice of Σx, the sum of the addendum modification coefficients of two mating gears, see 3.3.

3.2.2 General formula

For the allocation of the sum of the addendum modification coefficients on the two mating gears, it is recommended that the following formulae be used :

a) Addendum modification coefficient of the pinion

$$x_1 = \lambda \frac{z_{v2} - z_{v1}}{z_{v2} + z_{v1}} + \Sigma x \frac{z_{v1}}{z_{v2} + z_{v1}}$$

b) Addendum modification coefficient of the gear

$$x_2 = \Sigma x - x_1$$

Bearing in mind note 1 in 3.1 the following may be written :

$$x_1 = \lambda \frac{z_2 - z_1}{z_2 + z_1} + \Sigma x \frac{z_1}{z_2 + z_1}$$

$$x_2 = \Sigma x - x_1$$

or,

$$x_1 = \lambda \frac{u - 1}{u + 1} + \Sigma x \frac{1}{u + 1}$$

$$x_2 = \Sigma x - x_1$$

For gears in which the gear ratio exceeds 5, the allocation of the sum of the addendum modification coefficients is calculated with $u = 5$.

3.2.3 Recommended values for λ

It is recommended that for λ a value such as :

$0{,}5 \leqslant \lambda \leqslant 0{,}75$ for speed-reducing gears

$0 \leqslant \lambda \leqslant 0{,}5$ for speed-increasing gears

be adopted.

3.2.4 Final choice of addendum modification coefficients

Addendum modification coefficients shall be within the limit indicated in 3.4.

If the value calculated for an addendum modification coefficient for one of the gears falls outside these limits, the nearest limit will be adopted as a suitable value for this coefficient. The other addendum modification coefficient shall be calculated by the difference between Σx and the value adopted.

3.3 Sum of addendum modification coefficients

3.3.1 General

The choice of the sum of addendum modification coefficients is arbitrary and depends on the centre distance conditions or the operational conditions desired.

Sums which are too high or too low for addendum modification coefficients may be harmful to the satisfactory functioning of the gear pair. Therefore, in this Technical Report, upper and lower limits are specified for Σx.

— the conventional limits are those which may not be exceeded in any circumstances.

— the recommended limits are those within which there is no risk of faulty operation and consequently no need to carry out any type of verification.

The zones of values between these two types of limit shall be reserved for special cases and the verification of the operating conditions of the gear is essential if one of these values is chosen for Σx.

3.3.2 Conventional limits

The following equations define the conventional limits which shall not be exceeded. Figure 2 is the graphical representation of these limits.

3.3.2.1 Upper limit

for $20 \leqslant \Sigma z_v \leqslant 80$	$\Sigma x = (100 + \Sigma z_v) / 120$
for $80 < \Sigma z_v$	$\Sigma x = 1{,}5$

3.3.2.2 Lower limit

for $20 \leqslant \Sigma z_v \leqslant 40$	$\Sigma x = 0{,}0375\ (40 - \Sigma z_v)$
for $40 < \Sigma z_v \leqslant 160$	$\Sigma x = 0{,}005\ \ (40 - \Sigma z_v)$
for $160 < \Sigma z_v$	$\Sigma x = -0{,}60$

3.3.3 Recommended limits

The following equations define the recommended limits for the sum of addendum modification coefficients. Figure 2 also gives their graphical representation.

3.3.3.1 Upper limit

for $20 \leqslant \Sigma z_v$	$\Sigma x = 1$

3.3.3.2 Lower limit

for $20 \leqslant \Sigma z_v \leqslant 60$	$\Sigma x = 0{,}025\ (60 - \Sigma z_v)$
for $60 < \Sigma z_v$	$\Sigma x = 0$

The shaded parts of figure 2 indicate the zones reserved for special cases.

The sum of the number of teeth (normal or virtual as the case may be) of the gear may in no case be less than 20. It is recommended that this sum be not less than 24.

3.3.4 Selection of the sum of addendum modifications

The choice of Σx will be made taking the following two comments into account :

For each increase in the sum of the addendum modification coefficients there is a corresponding increase in the breaking load capacity.

For each reduction in the sum of the addendum modification coefficients there is a corresponding increase in the contact ratio.

3.4 Limit values of the addendum modification coefficient

3.4.1 General

Too small an addendum modification coefficient gives rise to cutter interference. The exact value which produces this interference for a given virtual number of teeth is the lower geometric limit of the addendum modification coefficient.

Too high an addendum modification coefficient produces a pointed tooth (or even a shortened one). The exact value of the addendum modification coefficient which, for a given virtual number of teeth, results in a pointed tooth is the upper geometric limit of the addendum modification coefficient.

For reasons of ease of use, these geometric limits are replaced by conventional limits regarded as absolute which lie within the geometric limits and which are defined by simple linear ratios between the addendum modification coefficient, x, and the virtual number of teeth, z_v.

As the addendum modification of the teeth has an effect on the shape of the teeth and also on the operating characteristics, there is a more restricted area than the conventional area where the operating conditions are acceptable without any verification being necessary. The limits of this area are the upper and lower limits recommended. They are also given by simple linear ratios between x and z_v.

3.4.2 Conventional limits

The conventional limits outside which no addendum modification coefficient may be chosen are given by the following equations. Their graphical representation is given in figure 3.

3.4.2.1 Upper limit

for $6 \leqslant z_v \leqslant 10$ $\qquad x = 0{,}60$

for $10 < z_v \leqslant 50$ $\qquad x = 0{,}50 + 0{,}01\, z_v$

for $50 < z_v$ $\qquad x = 1{,}00$

3.4.2.2 Lower limit

for $6 \leqslant z_v \leqslant 12$ $\qquad x = 0{,}05\,(18 - z_v)$

for $12 < z_v \leqslant 20$ $\qquad x = 0{,}0375\,(20 - z_v)$

for $20 < z_v \leqslant 50$ $\qquad x = (20 - z_v)\,/\,60$

for $50 < z_v$ $\qquad x = -0{,}50$

3.4.3 Recommended limits

The equations defining the recommended limits are given on the next page. Figure 3 represents these limits graphically.

Within these limits, and only within them, it is not necessary to carry out any calculation of the operating characteristics.

3.4.3.1 Upper limit

for $6 \leqslant z_v$ $\qquad x = 0{,}60$

3.4.3.2 Lower limit

for $6 \leqslant z_v \leqslant 50$ $\qquad x = 0{,}025\,(30 - z_v)$

for $50 < z_v$ $\qquad x = -0{,}50$

The virtual number of teeth of a gear shall always have 6 as its lower limit. The shaded parts of figure 3 represent the zones in which the choice of the addendum modification coefficient necessitates verification of the characteristics of the gear pair.

3.4.4 Tip thickness

Figure 4 is an enlarged part (relating to small numbers of teeth) of figure 3 which also has curves for the constant tip thickness equal to 0,2 m_n, 0,3 m_n and 0,4 m_n for a tooth depth in conformity with the standard basic rack tooth profile given in ISO 53 without any reduction in the addendum.

Table 1 gives the values of the addendum modification coefficient as a function of the virtual number of teeth relating to these curves.

3.5 Reduction of the addendum of teeth

If it is wished to retain a tip thickness s_{an} greater than 0,2 m_n, in some cases it is useful to reduce the addendum of the teeth. For calculating this reduction it is recommended that the following formulae be used :

for $x \leqslant 0{,}6$ $\qquad k = 0{,}01\,(50\,x - 3\,z_v + 6)$

for $x > 0{,}6$ $\qquad k = 0{,}01\,(70\,x - 3\,z_v - 6)$

where

$$d_a = m_n \left[\frac{z}{\cos\beta} + 2\,(1 + x - k) \right]$$

If k is calculated as negative, $k = 0$

In the case of surface heat treatment, the tip thickness will be adapted to these circumstances in conformity with good practice for these treatments.

Annex

A.1 Values of bottom clearance

The sum of the addendum modification coefficients of a gear pair has an effect on the value of the bottom clearance. Figure 5 gives (for teeth without shortening of the addendum) curves representing Σx as a function of Σz_v, the constant clearance for a given curve and taking respectively the five values : 0,05 m_n, 0,1 m_n, 0,15 m_n, 0,20 m_n and 0,25 m_n.

Table 2 gives some values relating to these curves.

A.2 Relations between centre distance, sum of addendum modifications and operating pressure angle

These relations are expressed as follows :

$$z_m = \frac{1}{2}(z_1 + z_2)$$

$$d_m = \frac{1}{2}(d_1 + d_2)$$

$$a \cos \alpha'_t = d_m \cos \alpha_t$$

$$\frac{\Sigma x}{z_m} = (\text{inv } \alpha'_t - \text{inv } \alpha_t)/\tan \alpha_o$$

$$\tan \alpha_t = \frac{\tan \alpha_o}{\cos \beta}$$

Table 1 — Values of x as a function of z_v giving the tip thicknesses s_{an}

z_v	s_{an}		
	0,2 m_n	0,3 m_n	0,4 m_n
	x		
8	0,385		
9	0,443		
10	0,499	0,388	
11	0,552	0,436	0,309
12	0,602	0,481	0,350
13	0,650	0,525	0,389
14		0,568	0,427
15		0,609	0,464
16		0,648	0,499
17		0,687	0,533
18			0,567
19			0,599
20			0,631
21			0,662
22			0,692
23			0,722
24			0,751

Table 2 — Values of Σx for various values of bottom clearance without any reduction in the addendum

Σz_v	c			
	0,05 m_n	0,1 m_n	0,15 m_n	0,2 m_n
	Σx			
20	1,016	0,849	0,658	0,435
40	1,318	1,108	0,873	0,588
60	1,552	1,311	1,038	0,703
80	—	1,479	1,175	0,800
100	—	1,628	1,297	0,886
120	—	—	1,392	0,958
140	—	—	1,508	1,033
160	—	—	1,600	1,094

In the tables, the intermediate values can be obtained by linear interpolation.

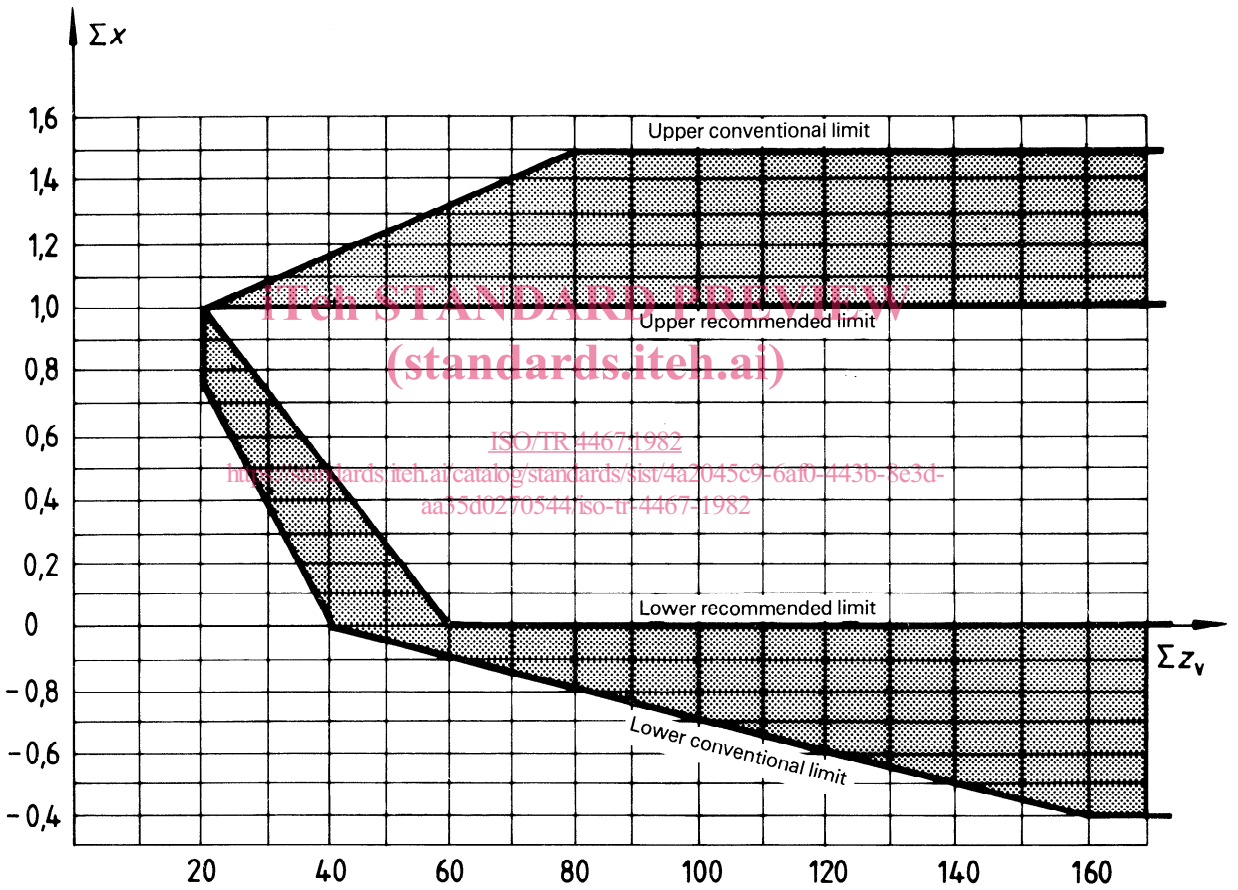

Figure 2 — Conventional and recommended limits for the sum of the addendum modification coefficients and zones for special cases.