



Designation: D5785/D5785M – 20

Standard Practice for (Analytical Procedure) for Determining Transmissivity of Confined Nonleaky Aquifers by Underdamped Well Response to Instantaneous Change in Head (Slug Test)¹

This standard is issued under the fixed designation D5785/D5785M; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This practice covers determination of transmissivity from the measurement of the damped oscillation about the equilibrium water level of a well-aquifer system to a sudden change of water level in a well. Underdamped response of water level in a well to a sudden change in water level is characterized by oscillatory fluctuation about the static water level with a decrease in the magnitude of fluctuation and recovery to initial water level. Underdamped response may occur in wells tapping highly transmissive confined aquifers and in deep wells having long water columns.

1.2 This analytical procedure is used in conjunction with the field procedure Test Method **D4044/D4044M** for collection of test data.

1.3 *Limitations*—Slug tests are considered to provide an estimate of transmissivity of a confined aquifer. This test method requires that the storage coefficient be known. Assumptions of this practice prescribe a fully penetrating well (a well open through the full thickness of the aquifer), but the slug test method is commonly conducted using a partially penetrating well. Such a practice may be acceptable for application under conditions in which the aquifer is stratified and horizontal hydraulic conductivity is much greater than vertical hydraulic conductivity. In such a case the test would be considered to be representative of the average hydraulic conductivity of the portion of the aquifer adjacent to the open interval of the well. The method assumes laminar flow and is applicable for a slug test in which the initial water-level displacement is less than 0.1 or 0.2 of the length of the static water column.

1.4 This practice for analysis presented here is derived by van der Kamp **(1)**² based on an approximation of the underdamped response to that of an exponentially damped sinusoid.

¹ This practice is under the jurisdiction of ASTM Committee **D18** on Soil and Rock and is the direct responsibility of Subcommittee **D18.21** on Groundwater and Vadose Zone Investigations.

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² The boldface numbers given in parentheses refer to a list of references at the end of the text.

A more rigorous analysis of the response of wells to a sudden change in water level by Kipp **(2)** indicates that the method presented by van der Kamp **(1)** matches the solution of Kipp **(2)** when the damping parameter values are less than about 0.2 and time greater than that of the first peak of the oscillation **(2)**.

1.5 *Units*—The values stated in either SI units or inch-pound units are to be regarded separately as standard. The values in each system may not be exact equivalents; therefore each system shall be used independently of the other. Combining values from the two systems may result in nonconformance with the standard. Reporting of test results in units other than SI shall not be regarded as nonconformance with this practice.

1.6 All observed and calculated values shall conform to the guidelines for significant digits and rounding established in Practice **D6026**.

1.7 *This practice offers a set of instructions for performing one or more specific operations. This document cannot replace education or experience and should be used in conjunction with professional judgment. Not all aspects of the practice may be applicable in all circumstances. This ASTM standard is not intended to represent or replace the standard of care by which the adequacy of a given professional service must be judged, nor should this document be applied without the consideration of a project's many unique aspects. The word "Standard" in the title of this document means only that the document has been approved through the ASTM consensus process.*

1.8 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.9 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

*A Summary of Changes section appears at the end of this standard

2. Referenced Documents

2.1 ASTM Standards:³

- D653** Terminology Relating to Soil, Rock, and Contained Fluids
- D3740** Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction
- D4043** Guide for Selection of Aquifer Test Method in Determining Hydraulic Properties by Well Techniques
- D4044/D4044M** Test Method for (Field Procedure) for Instantaneous Change in Head (Slug) Tests for Determining Hydraulic Properties of Aquifers
- D6026** Practice for Using Significant Digits in Geotechnical Data

3. Terminology

3.1 *Definitions*—For definitions of common technical terms used in this practice, refer to Terminology **D653**.

3.1.1 *observation well*—a well open to all or part of an aquifer.

3.1.2 *storage coefficient*—the volume of water an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head. For a confined aquifer, the storage coefficient is equal to the product of specific storage and aquifer thickness. For an unconfined aquifer, the storage coefficient is approximately equal to the specific yield.

3.1.3 *transmissivity*—the volume of water at the existing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit width of the aquifer.

3.2 Symbols and Dimensions:

3.2.1 T —transmissivity [L^2T^{-1}].

3.2.2 S —storage coefficient [nd].

3.2.3 L —effective length of water column, equal to $L_c + \frac{w}{(r_c^2/r_s^2)}(m/2)$.

3.2.3.1 *Discussion*—This expression for the effective length is given by Kipp (2). The expression for the effective length of the water column from Cooper et al. (3) is given as $L_c + 3/8L_s$ and assumes that the well screen and well casing have the same diameter.

3.2.4 L_c —length of water column within casing [L].

3.2.5 L_s —length of water column within well screen [L].

3.2.6 g —acceleration of gravity [LT^{-2}].

3.2.7 h —hydraulic head in the aquifer [L].

3.2.8 h_o —initial hydraulic head in the aquifer [L].

3.2.9 h_s —hydraulic head in the well screen [L].

3.2.10 r_c —radius of well casing [L].

3.2.11 r_s —radius of well screen [L].

3.2.12 t —time [T].

3.2.13 w —water level displacement from the initial static level [L].

3.2.14 w_o —initial water level displacement [L].

3.2.15 γ —damping constant [T^{-1}].

3.2.16 τ —wavelength [T].

3.2.17 ω —angular frequency [T^{-1}].

3.2.18 m —aquifer thickness, [L].

4. Summary of Practice

4.1 This practice describes the analytical procedure for analyzing data collected during an instantaneous head (slug) test using a well in which the response is underdamped. The field procedures in conducting a slug test are given in Test Method **D4044/D4044M**. The analytical procedure consists of analyzing the response of water level in the well following the change in water level induced in the well.

4.2 *Theory*—The equations that govern the response of well to an instantaneous change in head are treated at length by Kipp (2). The flow in the aquifer is governed by the following equation for cylindrical flow:

$$\frac{S}{T} \frac{dh}{dt} = \frac{1}{r} \frac{d}{dr} \left(r \frac{dh}{dr} \right) \quad (1)$$

where:

h = hydraulic head,

T = aquifer transmissivity, and

S = storage coefficient.

4.2.1 The initial condition is at $t=0$ and $h = h_o$ and the outer boundary condition is as $r \rightarrow \infty$ and $h \rightarrow h_o$.

4.3 The flow rate balance on the well bore relates the displacement of the water level in the well-riser to the flow into the well:

$$\pi r_c^2 \frac{dw}{dt} = 2\pi r_s T \left. \frac{\partial h}{\partial r} \right|_{r=r_s} \quad (2)$$

where:

r_c = radius of the well casing, and

w = displacement of the water level in the well from its initial position.

4.3.1 The third equation describing the system, relating h_s and w , comes from a momentum balance of Bird et al. (4) as referenced in Kipp (2).

$$\frac{d}{dt} \int_{-m}^0 \pi r_s^2 \rho v_d z = [-\rho v^2 + p_1 - p_2 - \rho g m] \pi r_s^2 \quad (3)$$

where:

v = velocity in the well screen interval,

m = aquifer thickness,

p = pressure,

ρ = fluid density,

g = gravitational acceleration, and

r_s = well screen radius. Well and aquifer geometry are shown in Fig. 1.

Atmospheric pressure is taken as zero.

5. Solution

5.1 The method of van der Kamp (1) assumes the water level response to a sudden change for the underdamped case, except near critical damping conditions, can be approximately described as an exponentially damped cyclic fluctuation that decays exponentially. The water-level fluctuation would then be given by:

³ For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For Annual Book of ASTM Standards volume information, refer to the standard's Document Summary page on the ASTM website.

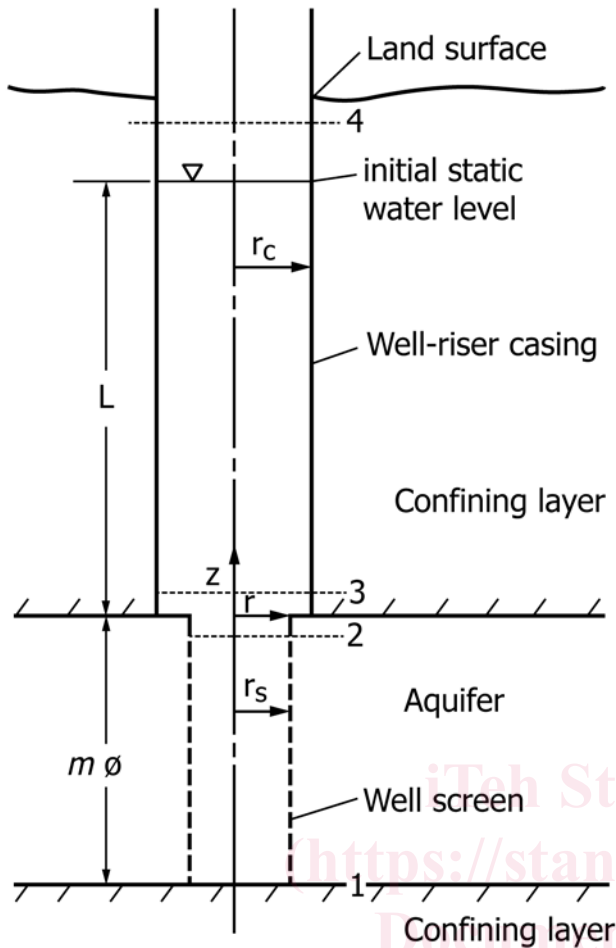


FIG. 1 Well and Aquifer Geometry

$$w(t) = w_0 e^{-\gamma t} \cos \omega t \quad (4)$$

5.1.1 The following solution is given by van der Kamp (1).

$$d = \frac{-r_c^2 (g/L)^{1/2} \ln[0.79 r_s^2 (S/T) (g/L)^{1/2}]}{8T} \quad (5)$$

that may be written as:

$$T = b + a \ln T \quad (6)$$

where:

$$b = a \ln[0.79 r_s^2 S (g/L)^{1/2}] \quad (7)$$

$$a = \frac{r_c^2 (g/L)^{1/2}}{8d} \quad (8)$$

$$d = \gamma / (g/L)^{1/2} \quad (9)$$

and

$$L = g / (\omega^2 + \gamma^2) \quad (10)$$

NOTE 1—Other analytical solutions are proposed by Kipp (2); Kraus (5); Kruseman and de Ridder (6); and Kabala, Pinder, and Milly (7).

NOTE 2—Commercial software is available to perform the calculations/plotting described in this practice. Users should verify the correctness of the software and results.

6. Significance and Use

6.1 The assumptions of the physical system are given as follows:

6.1.1 The aquifer is of uniform thickness and confined by impermeable beds above and below.

6.1.2 The aquifer is of constant homogeneous porosity and matrix compressibility and of homogeneous and isotropic hydraulic conductivity.

6.1.3 The origin of the cylindrical coordinate system is taken to be on the well-bore axis at the top of the aquifer.

6.1.4 The aquifer is fully screened.

6.2 The assumptions made in defining the momentum balance are as follows:

6.2.1 The average water velocity in the well is approximately constant over the well-bore section.

6.2.2 Flow is laminar and frictional head losses from flow across the well screen are negligible.

6.2.3 Flow through the well screen is uniformly distributed over the entire aquifer thickness.

6.2.4 Change in momentum from the water velocity changing from radial flow through the screen to vertical flow in the well are negligible.

6.2.5 The system response is an exponentially decaying sinusoidal function.

NOTE 3—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself assure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.

7. Procedure

7.1 The overall procedure consists of:

7.1.1 Conducting the slug test field procedure (see Test Method D4044/D4044M), and

7.1.2 Analyzing the field data, that is addressed in this practice.

NOTE 4—The initial displacement of water level should not exceed 0.1 or 0.2 of the length of the static water column in the well, because of considerations for calculating L_w . Practically, the displacement should be small, a few times larger than the well radius, to minimize frictional losses. The measurement of displacement should be within 1% of the initial water-level displacement. The water-level displacement needs to be calculated independently for comparison to the observed initial displacement.

8. Calculation and Interpretation of Test Data

8.1 Plot the water-level response in the well to the sudden change in head, as in Fig. 2.

8.2 Calculate the angular frequency, ω :

$$\omega = 2\pi/\tau \quad (11)$$

where:

$\tau = t_1 - t_2$, and t_1 and t_2 are times of successive maxima or minima of the oscillatory wave.

8.3 Calculate the damping factor, γ :

$$\gamma = \ln[w(t_1)/w(t_2)]/t_2 - t_1 \quad (12)$$

where:

$w(t_1)$ and $w(t_2)$ are the water-level displacements at times t_1 and t_2 , respectively.

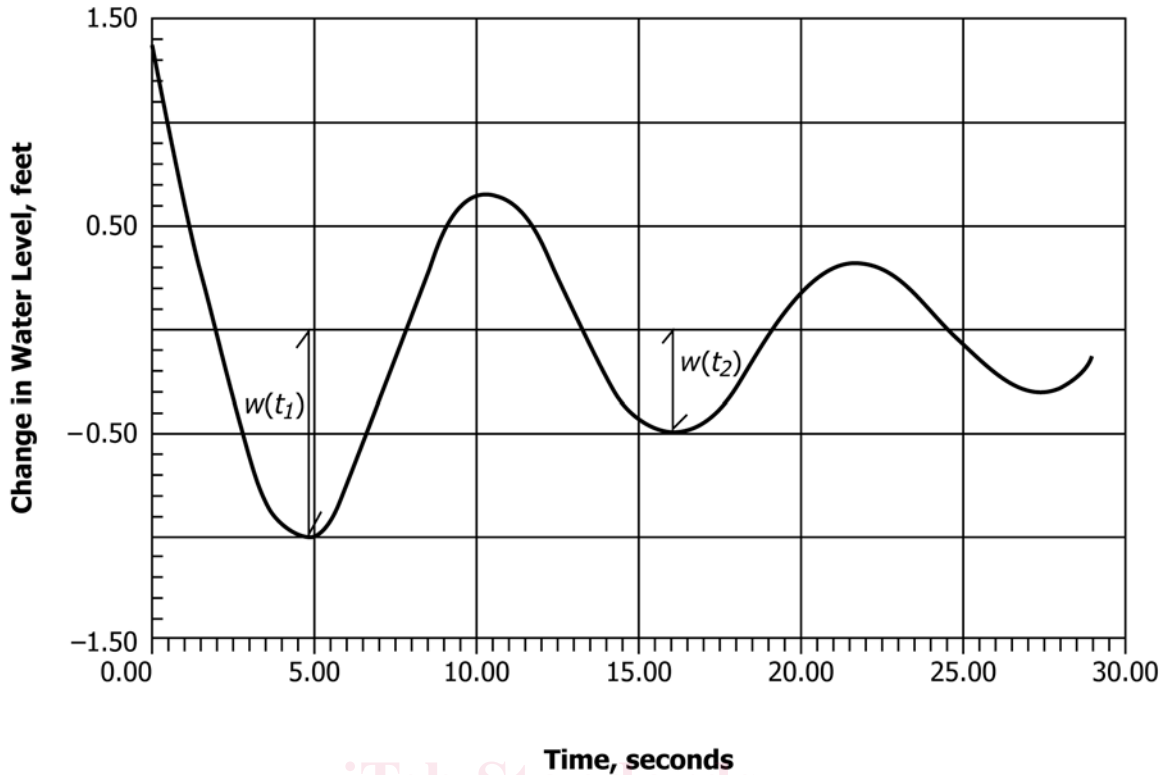


FIG. 2 Underdamped Response of Water Level to a Sudden Change in Head

8.4 Determine transmissivity, T ,

$$T = b + a \ln T \quad (13)$$

where:

$$a = [r_c^2 (g/L)^{1/2}] / 8d \quad (14)$$

$$d = \gamma / (g/L)^{1/2} \quad (15)$$

$$L = g / (\omega^2 + \gamma^2) \quad (16)$$

and:

$$b = -a \ln[0] \quad (17)$$

8.4.1 Solve for transmissivity iteratively using an initial estimate value for transmissivity, T , and a known or estimated value of storage coefficient, S .

8.5 Check the results.

8.5.1 Compare the effective length of the water column, L , calculated by the following two relationships:

$$L = g / (\omega^2 + \gamma^2) \quad (18)$$

and:

$$L = L_c + (r_c^2 / r_s^2) m / 2 \quad (19)$$

The values of L should agree within 20 %.

8.5.2 Check to see that the value of $\alpha \ll 0.1$, where:

$$\alpha = 0.89(S/T)^{1/2} (\omega^2 + \gamma^2)^{1/4} r_s < 0.1 \quad (20)$$

8.5.3 Check to see that the value of $d \ll 0.7$, where:

$$d = \gamma / (g/L)^{1/2} \quad (21)$$

8.5.4 Example—The following data are taken from the underdamped response to a slug test shown in Fig. 2:

- $w(t_1) = -1.0$ ft
- $w(t_2) = -0.5$ ft
- $t_1 = 4.9$ s
- $t_2 = 16.9$ s
- $r_c = 0.25$ ft
- $r_s = 0.25$ ft
- $L_c = 95$ ft
- $L_s = 55$ ft
- $\tau = t_2 - t_1 = 16.9 - 4.9 = 12$ s
- $\omega = 2\pi / \tau = 2 * 3.1416 / 12.0 = 0.5236$ s⁻¹
- $\gamma = 1n(w(t_1)/w(t_2)) / \tau = 1n(-1.0/-0.5) / 12 = 0.6931 / 12 = 0.05776$ s⁻¹
- $T = b + a \ln T$
- $(g/L)^{1/2} = (\omega^2 + \gamma^2)^{1/2} = ((0.5236)^2 + (0.05776)^2)^{1/2} = ((0.2742) + (0.0033362))^{1/2} = (0.2775)^{1/2} = 0.5268$
- $d = \gamma / (g/L)^{1/2} = 0.05776 / 0.5268 = 0.1096$
- $a = (r_c^2 (g/L)^{1/2}) / 8d = (0.25)^2 (0.5268) / 8(0.1096) = 0.03755$ ft²/s

Assume $S = 1.5 \times 10^{-5}$

$$b = a \ln(0.79 r_s^2 S (g/L)^{1/2}) = (-0.03755) \ln(0.79 (0.25)^2 (0.000015) (0.5268)) = 0.5541 \text{ ft}^2/\text{s}$$

$$T_1 = b + a \ln T_0$$

Assume $T_0 \cong b$,