Designation: D4106 – 15 D4106 – 20

Standard Test Method Practice for (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Theis Nonequilibrium Method[1]

This standard is issued under the fixed designation D4106; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This test method covers an analytical procedure for determining the transmissivity and storage coefficient of a nonleaky confined aquifer. It is used to analyze data on water-level response collected during radial flow to or from a well of constant discharge or injection.

1.2 This analytical procedure, along with others, is used in conjunction with the field procedure given in Test Method D4050.

1.3 *Limitations*—The limitations of this test method for determination of hydraulic properties of aquifers are primarily related to the correspondence between the field situation and the simplifying assumptions of this test method (see 5.1).

1.4 All observed and calculated values shall conform to the guidelines for significant digits and rounding established in Practice D6026.

1.4.1 The procedures used to specify how data are collected/recorded or calculated, in this standard are regarded as the industry standard. In addition, they are representative of the significant digits that generally should be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any considerations for the user's objectives; and it is common practice to increase or reduce significant digits of reported data to be commensurate with these considerations. It is beyond the scope of this standard to consider significant digits used in analytical methods for engineering design.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

D653 Terminology Relating to Soil, Rock, and Contained Fluids

D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction

D4043 Guide for Selection of Aquifer Test Method in Determining Hydraulic Properties by Well Techniques

D4050 Test Method for (Field Procedure) for Withdrawal and Injection Well Testing for Determining Hydraulic Properties of Aquifer Systems

D6026 Practice for Using Significant Digits in Geotechnical Data

3. Terminology

3.1 *Definitions:*

3.1.1 For definitions of other terms used in this test method, see Terminology D653.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *observation well*—a well open to all or part of an aquifer.

[1] This test method practice is under the jurisdiction of ASTM Committee D18 on Soil and Rock and is the direct responsibility of Subcommittee D18.21 on Groundwater and Vadose Zone Investigations.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

*A Summary of Changes section appears at the end of this standard

3.2.2 *unconfined aquifer*—an aquifer that has a water table.

3.3 *Symbols and Dimensions:*

3.3.1 *K* [LT^{-1}]—hydraulic conductivity.

3.3.2 K_{xy}—hydraulic conductivity in the horizontal plane, radially from the control well.

3.3.3 K_z—hydraulic conductivity in the vertical direction.

3.3.4 *Q* [L^3T^{-1}]—discharge.

3.3.5 *S* [nd]—storage coefficient.

3.3.6 S_s[L^{-1}]—specific storage.

3.3.7 *T* [L^2T^{-1}]—transmissivity.

3.3.8 *W(u)* [nd]—well function of *u*.

3.3.9 *b* [L]—thickness of aquifer.

3.3.10 *r* [L]—radial distance from control well.

3.3.11 *s* [L]—drawdown.

4. Summary of Test Method

4.1 This test method describes an analytical procedure for analyzing data collected during a withdrawal or injection well test. The field procedure (see Test Method D4050) involves pumping a control well at a constant rate and measuring the water level response in one or more observation wells or piezometers. The water-level response in the aquifer is a function of the transmissivity and storage coefficient of the aquifer. Alternatively, this test method can be performed by injecting water at a constant rate into the aquifer through the control well. Analysis of buildup of water level in response to injection is similar to analysis of drawdown of water level in response to withdrawal in a confined aquifer. Drawdown of water level is analyzed by plotting drawdown against factors incorporating either time or distance from the control well, or both, and matching the drawdown response with a type curve.

4.2 *Solution*—The solution given by Theis (**1**)[3] may be expressed as follows:

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-y}}{y}\,dy \tag{1}$$

where:

$$u = \frac{r^2S}{4Tt} \tag{2}$$

$$\int_u^\infty \frac{e^{-y}}{y}\,dy = W(u)$$

$$= -0.577216 - \log_e u + u - \frac{u^2}{2!2} + \frac{u^3}{3!3} - \frac{u^4}{4!4} + \ldots \tag{3}$$

5. Significance and Use

5.1 *Assumptions:*

5.1.1 Well discharges at a constant rate, *Q*.

5.1.2 Well is of infinitesimal diameter and fully penetrates the aquifer.

5.1.3 The nonleaky aquifer is homogeneous, isotropic, and aerially extensive. A nonleaky aquifer receives insignificant contribution of water from confining beds.

5.1.4 Discharge from the well is derived exclusively from storage in the aquifer.

5.1.5 The geometry of the assumed aquifer and well conditions are shown in Fig. 1.

5.2 *Implications of Assumptions:*

5.2.1 Implicit in the assumptions are the conditions of radial flow. Vertical flow components are induced by a control well that partially penetrates the aquifer, that is, the well is not open to the aquifer through its full thickness. If the control well does not fully penetrate the aquifer, the nearest piezometer or partially penetrating observation well should be located at a distance, *r*, beyond which vertical flow components are negligible, where according to Reed (**2**):

$$r = 1.5\frac{b}{\sqrt{\frac{K_z}{K_{xy}}}} \tag{4}$$

[3] The boldface numbers in parentheses refer to a list of references at the end of this standard.

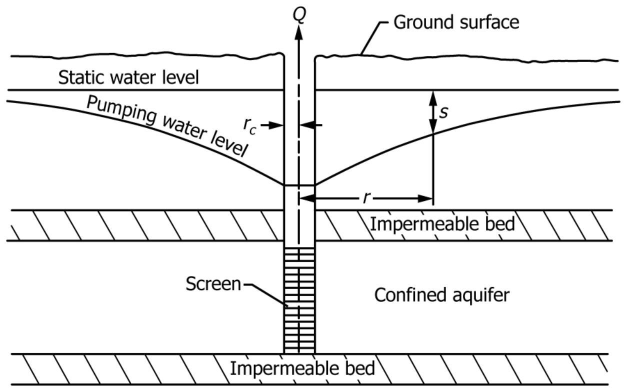

FIG. 1 Cross Section Through a Discharging Well in a Nonleaky Confined Aquifer

This section applies to distance-drawdown calculations of transmissivity and storage coefficient and time-drawdown calculations of storage coefficient. If possible, compute transmissivity from time-drawdown data from wells located within a distance, *r*, of the pumped well using data measured after the effects of partial penetration have become constant. The time at which this occurs is given by Hantush (**3**) by:

$$t = b^2 s/2T\,(K_z/K_r) \tag{5}$$

Fully penetrating observation wells may be placed at less than distance *r* from the control well. Observation wells may be on the same or on various radial lines from the control well.

5.2.2 The Theis method assumes the control well is of infinitesimal diameter. Also, it assumes that the water level in the control well is the same as in the aquifer contiguous to the well. In practice these assumptions may cause a difference between the theoretical drawdown and field measurements of drawdown in the early part of the test and in and near the control well. Control well storage is negligible after a time, *t*, given by the Eq 6 after Weeks (**4**).

$$t = 25 \times \frac{r_c^2}{T} \tag{6}$$

where:

r_c = the radius of the control well in the interval in which the water level changes.

5.2.3 *Application of Theis Method to Unconfined Aquifers:*

5.2.3.1 Although the assumptions are applicable to artesian or confined conditions, the Theis solution may be applied to unconfined aquifers if drawdown is small compared with the saturated thickness of the aquifer or if the drawdown is corrected for reduction in thickness of the aquifer, and the effects of delayed gravity yield are small.

5.2.3.2 *Reduction in Aquifer Thickness*—In an unconfined aquifer dewatering occurs when the water levels decline in the vicinity of a pumping well. Corrections in drawdown need to be made when the drawdown is a significant fraction of the aquifer thickness as shown by Jacob (**5**). The drawdown, *s*, needs to be replaced by *s*', the drawdown that would occur in an equivalent confined aquifer, where:

$$s' = s - \left(\frac{s^2}{2b}\right) \tag{7}$$

5.2.3.3 *Gravity Yield Effects*—In unconfined aquifers, delayed gravity yield effects may invalidate measurements of drawdown during the early part of the test for application to the Theis method. Effects of delayed gravity yield are negligible in partially penetrating observation wells at and beyond a distance, *r*, from the control well, where:

$$r = \frac{b}{\sqrt{\frac{K_z}{K_{xy}}}} \tag{8}$$

After the time, *t*, as given in Eq 9 from Neuman (**6**).

$$t = 10 \times S_y(r^2/T) \tag{9}$$

where:

S_y = the specific yield. For fully penetrating observation wells, the effects of delayed yield are negligible at the distance, *r*, in Eq 8 after one tenth of the time given in the Eq 9.

NOTE 1—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself ensure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.

6. Apparatus

6.1 Analysis of data from the field procedure (see Test Method D4050) by the method specified in this test method requires that the control well and observation wells meet the specifications in the following paragraphs.

6.2 *Construction of Control Well*—Screen the control well in the aquifer to be tested and equip with a pump capable of discharging water from the well at a constant rate for the duration of the test. Preferably, screen the control well throughout the full thickness of the aquifer. If the control well partially penetrates the aquifer, take special precaution in the placement and design of observation wells (see 5.2.1).

6.3 *Construction of Observation Wells*—Construct one or more observation wells at a distance from the control well. Observation wells may be partially open or open throughout the thickness of the aquifer.

6.4 *Location of Observation Wells*—Locate observation wells at various distances from the control well within the area of influence of pumping. However, if vertical flow components are significant and if partially penetrating observation wells are used, locate them at a distance beyond the effect of vertical flow components (see 5.2.1). If the aquifer is unconfined, constraints are imposed on the distance to partially penetrating observation wells and the validity of early time measurements (see 5.2.3).

7. Procedure

7.1 The overall procedure consists of conducting the field procedure for withdrawal or injection well tests (described in Test Method D4050) and analysis of the field data that is addressed in this test method.

7.2 The integral expression in Eq 1 and Eq 2 can not be evaluated analytically. A graphical procedure is used to solve for the two unknown parameters transmissivity and storage coefficient where:

$$s = \frac{Q}{4\pi T} W(u) \tag{10}$$

and:

$$u = \frac{r^2 S}{4Tt} \tag{11}$$

8. Calculation

8.1 The graphical procedure used to calculate test results is based on the functional relations between $W(u)$ and s and between u and t or t/r^2.

8.1.1 Plot values of $W(u)$ versus $1/u$ on logarithmic-scale paper (see Table 1). This plot is referred to as the type curve plot.

8.1.2 On logarithmic tracing paper of the same scale and size as the $W(u)$ versus $1/u$ type curve, plot values of drawdown, s, on the vertical coordinate versus either time on the horizontal coordinate if one observation well is used or versus t/r^2 on the horizontal coordinate if more than one observation well is used.

8.1.3 Overlay the data plot on the type curve plot and, while the coordinate axes of the two plots are held parallel, shift the plot to align with the type curve (see Fig. 2).

8.1.4 Select and record the values of $W(u)$, $1/u$, s, and t at an arbitrary point, referred to as the match point (see Fig. 2), anywhere on the overlapping part of the plots. For convenience the point may be selected where $W(u)$ and $1/u$ are integer values.

NOTE 2—Alternatively, the type curve can be constructed by plotting $W(u)$ against u, then plotting the data as s versus r^2/t.

NOTE 3—Commercially available software is available from several sources that can perform the calculation and plotting.

8.1.5 Using the coordinates of the point, determine the transmissivity and storage coefficient from Eq 12 and Eq 13:

$$T = \frac{QW(u)}{4\pi s} \tag{12}$$

$$S = 4Tu\frac{t}{r^2} \tag{13}$$

8.1.6 To apply the Theis nonequilibrium method to thin unconfined aquifers where the drawdown is a significant fraction of the initial saturated thickness, apply a correction to the drawdown in solving for transmissivity and coefficient of storage (see 5.2.3.2).

9. Report/Record

9.1 Prepare a report including the minimum information described in this section. The report of the analytical procedure will include information from the report on test method selection (see Guide D4043) and the field testing procedure (see Test Method D4050).

9.1.1 *Introduction*—The introductory section is intended to present the scope and purpose of the constant discharge method for determining transmissivity and storativity in a confined nonleaky aquifer under constant flux. Summarize the field hydrogeologic

TABLE 1 Values of Theis Equation *W(u)* for values of 1/u, from Reed (2)

1/*u*	1/*u* × 10 $^{-1}$	1	10	10^2	10^3	10^4	10^5	10^6
1.0	0.00000[A]	0.21938	1.82292	4.03793	6.33154	8.63322	10.93572	13.23830
1.2	0.00003	0.29255	1.98932	4.21859	6.51369	8.81553	11.11804	13.42062
1.5	0.00017	0.39841	2.19641	4.44007	6.73667	9.03866	11.34118	13.64376
2.0	0.00115	0.55977	2.46790	4.72610	7.02419	9.32632	11.62886	13.93144
2.5	0.00378	0.70238	2.68126	4.94824	7.24723	9.54945	11.85201	14.15459
3.0	0.00857	0.82889	2.85704	5.12990	7.42949	9.73177	12.03433	14.33691
3.5	0.01566	0.94208	3.00650	5.28357	7.58359	9.88592	12.18847	14.49106
4.0	0.02491	1.04428	3.13651	5.41675	7.71708	10.01944	12.32201	14.62459
5.0	0.04890	1.22265	3.35471	5.63939	7.94018	10.24258	12.54515	14.84773
6.0	0.07833	1.37451	3.53372	5.82138	8.12247	10.42490	12.72747	15.03006
7.0	0.11131	1.50661	3.68551	5.97529	8.27659	10.57905	12.88162	15.18421
8.0	0.14641	1.62342	3.81727	6.10865	8.41011	10.71258	13.01515	15.31774
9.0	0.18266	1.72811	3.93367	6.22629	8.52787	10.83036	13.13294	15.43551
1/*u*	1/*u* × 10^7	10^8	10^9	10^{10}	10^{11}	10^{12}	10^{13}	10^{14}
1.0	15.54087	17.84344	20.14604	22.44862	24.75121	27.05379	29.36638	31.65897
1.2	15.72320	18.02577	20.32835	22.63094	24.93353	27.23611	29.53870	31.84128
1.5	15.94634	18.24892	20.55150	22.85408	25.15668	27.45926	29.76184	32.06442
2.0	16.23401	18.53659	20.83919	23.14177	25.44435	27.74693	30.04953	32.35211
2.5	16.45715	18.76974	21.06233	23.36491	25.66750	27.97008	30.27267	32.57526
3.0	16.63948	18.94206	21.24464	23.54723	25.84982	28.15240	30.45499	32.75757
3.5	16.79362	19.09621	21.39880	23.70139	26.00397	28.30655	30.60915	32.91173
4.0	16.92715	19.22975	21.53233	23.83492	26.13750	28.44008	30.74268	33.04526
5.0	17.15030	19.45288	21.75548	24.05806	26.36064	28.66322	30.96582	33.26840
6.0	17.33263	19.63521	21.93779	24.24039	26.54297	28.84555	31.14813	33.45071
7.0	17.48677	19.78937	22.09195	24.39453	26.69711	28.99969	31.30229	33.60487
8.0	17.62030	19.92290	22.22548	24.52806	26.83064	29.13324	31.43582	33.73840
9.0	17.73808	20.04068	22.34326	24.64584	26.94843	29.25102	31.55360	33.85619

[A] Value shown as 0.00000 is nonzero but less than 0.000005.

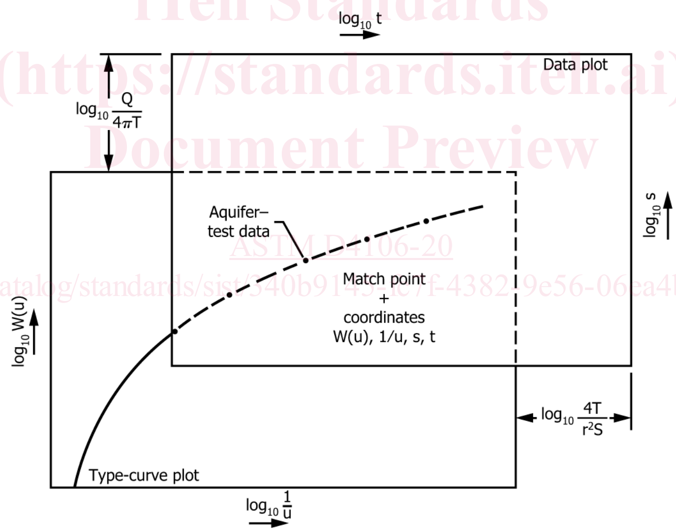

FIG. 2 Relation of 1/u, W(u) Type Curve and t, s Data Plot

~~conditions and the field equipment and instrumentation including the construction of the control well and observation wells or piezometers, or both, the method of measurement of discharge and water levels, and the duration of the test and pumping rate. Discuss rationale for selecting the Theis nonequilibrium method.~~

~~9.1.2 *Hydrogeologic Setting*—Review the information available on the hydrogeology of the site; interpret and describe the hydrogeology of the site as it pertains to the selection of this test method for conducting and analyzing an aquifer test. Compare the hydrogeologic characteristics of the site as it conforms and differs from the assumptions of this test method.~~

~~9.1.3 *Equipment*—Report the field installation and equipment for the aquifer test, including the construction, diameter, depth of screened and gravel packed intervals, and location of control well and pumping equipment, and the construction, diameter, depth, and screened interval of observation wells or piezometers.~~

~~9.1.4 Describe the methods of observing water levels, pumping rate, barometric changes, and other environmental conditions pertinent to the test. Include a list of measuring devices used during the test, the manufacturers name, model number, and basic specifications for each major item, and the name and date and method of the last calibration, if applicable.~~

9.1.5 *Testing Procedures*—State the steps taken in conducting pre-test, drawdown, and recovery phases of the test. Include the date, clock time, and time since pumping started or stopped for measurements of discharge rate, water levels, and other environmental data recorded during the testing procedure.

9.2 *Presentation and Interpretation of Test Results:*

9.2.1 *Data*—Present tables of data collected during the test. Show methods of adjusting water levels for background water-level and barometric changes and calculation of drawdown and residual drawdown.

9.2.2 *Data Plots*—Present data plots used in analysis of the data. Show overlays of data plots and type curve with match points and corresponding values of parameters at match points.

9.2.3 Show calculation of transmissivity and storage coefficient.

9.2.4 Evaluate qualitatively the test on the basis of the adequacy of instrumentation, observations of stress and response, the conformance of the hydrogeologic conditions, and the performance of the test to the assumptions of this test method.

10. Precision and Bias

10.1 *Precision*—Test data on precision is not presented due to the nature of the material (groundwater) tested by this test method. It is either not feasible or too costly at this time to have ten or more laboratories participated in a round-robin testing program. It is not practicable to specify the precision of this test method because the response of aquifer systems during aquifer tests is dependent upon ambient system stresses.

10.2 *Bias*—There is no accepted reference value for this test method, therefore bias cannot be determined. No statement can be made about bias because no true reference values exist.

11. Keywords

11.1 aquifer tests; aquifers; control wells; groundwater; hydraulic conductivity; observation wells; storage coefficient; transmissivity

REFERENCES

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(2) Reed, J. E., "Type Curves for Selected Problems of Flow to Wells in Confined Aquifers," *U.S. Geological Survey Techniques of Water-Resources Investigations*, Book 3, Chapter B3, 1980.

(3) Hantush, M. S., and Jacob, C. E., "Non-Steady Radial Flow in an Infinite Leaky Aquifer," *American Geophysical Union Transactions,* Vol 36, No. 1, 1955, pp. 95–100.

(4) Weeks, E. P., "*Aquifer Tests—The State of the Art in Hydrology*" in Proceedings of the International Well-Testing Symposium, October 19–21, 1977, Berkeley, California, LBL, 7027, Lawrence Berkeley Laboratory, pp 14–26.

(5) Jacob, C. C., "Determining the Permeability of Water-Table Aquifers," *in* Bentall, Ray, compiler, "Methods of Determining Permeability, Transmissibility, and Drawdown," U.S. Geological Survey Water-Supply Paper 1536-I, 1963, pp. 245–271.

(6) Neuman, S. P., "Effect of Partial Penetration on Flow in Unconfined Aquifers Considering Delayed Gravity Response," *Water Resources Research* , Vol 10, No. 2, 1974, pp. 303–312.

(7) Wenzel, L. K., "Methods for Determining Permeability of Water-Bearing Materials, with Special Reference to Discharging Well Methods," U.S. Geological Survey Water Supply Paper 887, 1942.

SUMMARY OF CHANGES

In accordance with Committee D18 policy, this section identifies the location of changes to this standard since the last edition (1996(2008)) that may impact the use of this standard.

(1) Deleted terminology that already appears in Terminology D653.
(2) Added Practices D3740 and D6026 to list of Referenced Documents.
(3) Added SI units notes, Practice D3740 notes.
(4) Revised Precision and Bias to current format.
(5) Edited throughout to comply with D18 Procedures Preparation Manual.
(6) Added new note on commercially available software for calculations and plotting.

1. Scope*

1.1 This practice covers an analytical procedure for determining the transmissivity and storage coefficient of a nonleaky confined aquifer. It is used to analyze data on water-level response collected during radial flow to or from a well of constant discharge or injection.

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1.4 All observed and calculated values shall conform to the guidelines for significant digits and rounding established in Practice D6026.

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1.5 *This practice offers a set of instructions for performing one or more specific operations. This document cannot replace education or experience and should be used in conjunction with professional judgment. Not all aspects of the practice may be applicable in all circumstances. This ASTM standard is not intended to represent or replace the standard of care by which the adequacy of a given professional service must be judged, nor should this document be applied without the consideration of a project's many unique aspects. The word "Standard" in the title of this document means only that the document has been approved through the ASTM consensus process.*

1.6 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.7 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

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