



Designation: D6034 – 20

# Standard Practice for (Analytical Procedure) Determining the Efficiency of a Production Well in a Confined Aquifer from a Constant Rate Pumping Test<sup>1</sup>

This standard is issued under the fixed designation D6034; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon ( $\epsilon$ ) indicates an editorial change since the last revision or reapproval.

## 1. Scope\*

1.1 This practice describes an analytical procedure for determining the hydraulic efficiency of a production well in a confined aquifer. It involves comparing the actual drawdown in the well to the theoretical minimum drawdown achievable and is based upon data and aquifer coefficients obtained from a constant rate pumping test.

1.2 This analytical practice is used in conjunction with the field procedure, Test Method [D4050](#).

1.3 The values stated in inch-pound units are to be regarded as standard, except as noted below. The values given in parentheses are mathematical conversions to SI units, which are provided for information only and are not considered standard. The reporting of results in units other than inch-pound shall not be regarded as nonconformance with this standard.

1.3.1 The gravitational system of inch-pound units is used when dealing with inch-pound units. In this system, the pound (lbf) represents a unit of force (weight), while the unit for mass is slugs.

1.4 *Limitations*—The limitations of the technique for determination of well efficiency are related primarily to the correspondence between the field situation and the simplifying assumption of this practice.

1.5 All observed and calculated values shall conform to the guidelines for significant digits and round established in Practice [D6026](#), unless superseded by this standard.

1.5.1 The procedures used to specify how data are collected/recorded or calculated, in this standard are regarded as the industry standard. In addition, they are representative of the significant digits that generally should be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any consider-

<sup>1</sup> This practice is under the jurisdiction of ASTM Committee [D18](#) on Soil and Rock and is the direct responsibility of Subcommittee [D18.21](#) on Groundwater and Vadose Zone Investigations.

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ations for the user's objectives; and it is common practice to increase or reduce significant digits of reported data to be commensurate with these considerations. It is beyond the scope of this standard to consider significant digits used in analysis method for engineering design.

1.6 This practice offers a set of instructions for performing one or more specific operations. This document cannot replace education or experience and should be used in conjunction with professional judgment. Not all aspects of the practice may be applicable in all circumstances. This ASTM standard is not intended to represent or replace the standard of care by which the adequacy of a given professional service must be judged, nor should this document be applied without the consideration of a project's many unique aspects. The word "Standard" in the title of this document means only that the document has been approved through the ASTM consensus process.

1.7 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.8 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

## 2. Referenced Documents

2.1 *ASTM Standards*:<sup>2</sup>

[D653 Terminology Relating to Soil, Rock, and Contained Fluids](#)

[D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction](#)

<sup>2</sup> For referenced ASTM standards, visit the ASTM website, [www.astm.org](http://www.astm.org), or contact ASTM Customer Service at [service@astm.org](mailto:service@astm.org). For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

\*A Summary of Changes section appears at the end of this standard

- D4050 Test Method for (Field Procedure) for Withdrawal and Injection Well Testing for Determining Hydraulic Properties of Aquifer Systems
- D5521/D5521M Guide for Development of Groundwater Monitoring Wells in Granular Aquifers
- D6026 Practice for Using Significant Digits in Geotechnical Data

### 3. Terminology

3.1 *Definitions*—For definitions of common technical terms used in this standard, refer to Terminology D653.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *well efficiency, n*—the ratio, usually expressed as a percentage, of the measured drawdown inside the control well divided into the theoretical drawdown which would occur in the aquifer just outside the borehole if there were no drilling damage, that is, no reduction in the natural permeability of the sediments in the vicinity of the borehole.

3.3 *Symbols:*

3.3.1 *Symbols and Dimensions:*

3.3.2 *K*—hydraulic conductivity [ $LT^{-1}$ ].

3.3.2.1 *Discussion*—The use of the symbol *K* for the term hydraulic conductivity is the predominant usage in ground water literature by hydrogeologists, whereas the symbol *k* is commonly used for this term in soil and rock mechanics and soil science.

3.3.3 *K<sub>r</sub>*—hydraulic conductivity in the plane of the aquifer, radially from the control well (horizontal hydraulic conductivity) [ $LT^{-1}$ ].

3.3.4 *K<sub>z</sub>*—hydraulic conductivity normal to the plane of the aquifer (vertical hydraulic conductivity) [ $LT^{-1}$ ].

3.3.5 *K<sub>0</sub>(x)*—modified Bessel function of the second kind and zero order [nd].

3.3.6 *Q*—discharge [ $L^3T^{-1}$ ].

3.3.7 *S*—storage coefficient [nd].

3.3.8 *T*—transmissivity [ $L^2T^{-1}$ ].

3.3.9 *s<sub>r</sub>*—drawdown in the aquifer at a distance *r* from the control well [*L*].

3.3.10 *s<sub>f</sub>*—drawdown which would occur in response to pumping a fully penetrating well [*L*].

3.3.11 *r<sub>w</sub>*—borehole radius of control well [*L*].

3.3.12 *s<sub>r,w</sub>*—theoretical drawdown which would occur in the aquifer just outside the borehole if there were no drilling damage, that is, no reduction in the natural permeability of the sediments in the vicinity of the borehole [*L*].

3.3.13 *s<sub>w</sub>*—drawdown measured inside the control well [*L*].

3.3.14 *u*— $(r^2S)/(4Tt)$  [nd].

3.3.15 *W(u)*—an exponential integral known in hydrology as the Theis well function of *u* [nd].

3.3.16 *A*— $K_z/K_r$ , anisotropy ratio [nd].

3.3.17 *b*—thickness of aquifer [*L*].

3.3.18 *d*—distance from top of aquifer to top of screened interval of control well [*L*].

3.3.19 *d'*—distance from top of aquifer to top of screened interval of observation well [*L*].

3.3.20 *f<sub>s</sub>*—incremental dimensionless drawdown component resulting from partial penetration [nd].

3.3.21 *l*—distance from top of aquifer to bottom of screened interval of control well [*L*].

3.3.22 *l'*—distance from top of aquifer to bottom of screened interval of observation well [*L*].

3.3.23 *r*—radial distance from control well [*L*].

3.3.24 *t*—time since pumping began [*T*].

3.3.25 *E*—well efficiency [nd].

### 4. Summary of Practice

4.1 This practice uses data from a constant rate pumping test to determine the well efficiency. The efficiency is calculated as the ratio of the theoretical drawdown in the aquifer just outside the well bore (*s<sub>r,w</sub>*) to the drawdown measured inside the pumped well (*s<sub>w</sub>*). The theoretical drawdown in the aquifer (*s<sub>r,w</sub>*) is determined from the field pumping test data by either extrapolation or direct calculation.

4.2 During the drilling of a well, the hydraulic conductivity of the sediments in the vicinity of the borehole wall is reduced significantly by the drilling operation. Damaging effects of drilling include mixing of fine and coarse formation grains, invasion of drilling mud, smearing of the borehole wall by the drilling tools, and compaction of sand grains near the borehole. The added head loss (drawdown) associated with the permeability reduction due to drilling damage increases the drawdown in the pumped well and reduces its efficiency (see Fig. 1). Well development procedures help repair the damage (see Guide D5521/D5521M) but generally cannot restore the sediments to their original, natural permeability.

4.2.1 Additional drawdown occurs from head loss associated with flow through the filter pack, through the well screen and vertically upward inside the well casing to the pump intake. While these drawdown components contribute to inefficiency, they usually are minor in comparison to the head loss resulting from drilling damage.

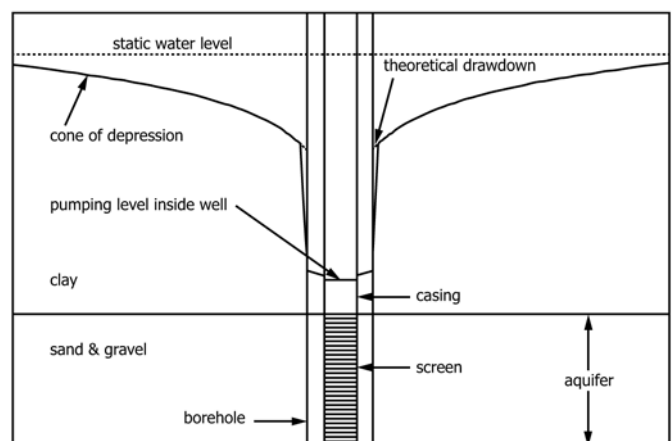


FIG. 1 Illustration of Drawdown Inside and Outside Pumping Well

4.2.2 The well efficiency, usually expressed as a percentage, is defined as the theoretical drawdown, also called aquifer drawdown, which would have occurred just outside the well if there were no drilling damage divided by the actual drawdown inside the well. The head losses contributing to inefficiency generally are constant with time while aquifer drawdown gradually increases with time. This causes the computed efficiency to increase slightly with time. Because the efficiency is somewhat time dependent, usually it is assumed that the well efficiency is the calculated drawdown ratio achieved after one day of continuous pumping. It is acceptable, however, to use other pumping times, as long as the time that was used in the efficiency calculation is specified. The only restriction on the pumping time is that sufficient time should have passed so that wellbore storage effects are insignificant. In the vast majority of cases, after one day of pumping, the effects of wellbore storage have long since become negligible.

4.2.3 Efficiency is also somewhat discharge dependent. Both the aquifer drawdown and the inefficiency drawdown can include both laminar (first order) and turbulent (approximately second order) components. Because the proportion of laminar versus turbulent flow can be different in the undisturbed aquifer than it is in the damaged zone and inside the well, the aquifer drawdown and inefficiency drawdown can increase at different rates as  $Q$  increases. When this happens, the calculated efficiency is different for different pumping rates. Because of this discharge dependence, efficiency testing usually is performed at or near the design discharge rate.

4.3 The drawdown in the aquifer around a well pumped at a constant rate can be described by one of several equations.

4.3.1 For fully penetrating wells, the Theis equation (1)<sup>3</sup> is used.

$$s_r = \frac{Q}{4\pi T} W(u) \quad (1)$$

where:

$$W(u) = \int_u^\infty \frac{e^{-x}}{x} dx \quad (2)$$

and

$$u = \frac{r^2 S}{4Tt} \quad (3)$$

4.3.2 For sufficiently small values of  $u$ , the Theis equation may be approximated by the Cooper-Jacob equation (2).

$$s_r = \frac{2.3Q}{4\pi T} \log\left(\frac{2.25Tt}{r^2 S}\right) \quad (4)$$

4.3.2.1 Examples of errors in this approximation for some  $u$  values are as follows:

$u$	Error
0.01	0.25 %
0.03	1.01 %
0.05	2.00 %
0.10	5.35 %

<sup>3</sup> The boldface numbers in parentheses refer to the list of references at the end of this practice.

4.3.3 For partially penetrating wells, the drawdown can be described by either the Hantush equation (3-5) or the Kozeny equation (6).

4.3.3.1 The Hantush equation is similar to the Theis equation but includes a correction factor for partial penetration.

$$s_r = \frac{Q}{4\pi T} (W(u) + f_s) \quad (5)$$

4.3.3.2 According to Hantush, at late pumping times, when  $t > b^2 S / (2TA)$ ,  $f_s$  can be expressed as follows:

$$f_s = \frac{4b^2}{\pi^2(l-d)(l'-d')} \sum_{n=1}^{\infty} \left(\frac{1}{n^2}\right) K_0\left(\frac{n\pi r \sqrt{K_c/K_r}}{b}\right) \left[ \sin\left(\frac{n\pi l}{b}\right) - \sin\left(\frac{n\pi d}{b}\right) \right] \left[ \sin\left(\frac{n\pi l}{b}\right) - \sin\left(\frac{n\pi d}{b}\right) \right] \quad (6)$$

4.3.3.3 The Kozeny equation is as follows:

$$s_r = \frac{s_f}{\frac{l-d}{b} \left( 1 + 7 \sqrt{\frac{r}{2(l-d)} \cos \frac{\pi(l-d)}{2b}} \right)} \quad (7)$$

4.3.3.4 In this equation,  $s_f$  is the drawdown for a fully penetrating well system and can be computed from Eq 1-4. While easier to compute than the Hantush equation, the Kozeny equation is not as accurate. It does not incorporate pumping time or anisotropy and assumes that the screen in the control well reaches either the top or the bottom of the aquifer.

4.3.4 The presence of a positive boundary (for example, recharge) causes the drawdown in the aquifer to be less than predicted by Eq 1-6, while a negative boundary (for example, the aquifer pinching out) results in more drawdown. The boundary-induced increases or decreases in drawdown usually can be determined from the field pumping test data. These increases/decreases can be combined with calculations using Eq 1-7 to determine the drawdown just outside the well bore.

4.4 The efficiency of a production well is calculated as follows:

$$E = \frac{s_{r_w}}{s_w} \quad (8)$$

where:

$s_w$  = denominator, the drawdown measured inside the well, and

$s_{r_w}$  = numerator, determined from field data.

Two procedures are available for determining  $s_{r_w}$ —extrapolation and direct calculation.

4.4.1 *Extrapolation*—Extrapolation can be used to determine  $s_{r_w}$  if data from two or more observation wells are available. Distance drawdown data can be plotted from these wells on either log-log or semilog graphs. If a log-log plot is used, the Theis type curve is used to extrapolate the drawdown data to the borehole radius to determine  $s_{r_w}$ . If a semilog plot is used, extrapolation is done using a straight line of best fit. The semilog method can be used only if the  $u$  value for each observation well is sufficiently small that the error introduced by the log approximation to the Theis equation is minimal.

4.4.1.1 For partially penetrating wells, the observation wells should be located beyond the zone affected by partial penetration, that is, at a distance  $r$  from the pumped well such that:

$$r \geq \frac{1.5b}{\sqrt{K_z/K_r}} \quad (9)$$

4.4.1.2 The extrapolated drawdown obtained in this case is  $s_f$ , the theoretical drawdown, which would have occurred just outside the borehole of a fully penetrating pumped well. The aquifer drawdown corresponding to partial penetration is then computed with the Hantush equation as follows:

$$s_{r_w} = s_f + \frac{Q}{4\pi T} f_s \quad (10)$$

4.4.1.3 The second term on the right-hand side of **Eq 10** represents the incremental aquifer drawdown caused by partial penetration.

4.4.1.4 Using the Kozeny equation, the aquifer drawdown for partial penetration is computed from **Eq 7** with  $r$  set equal to the borehole radius  $r_w$ :

$$s_{r_w} = \frac{l-d}{b} \left( 1 + 7 \sqrt{\frac{s_f}{2(l-d)} \cos \frac{\pi(l-d)}{2b}} \right) \quad (11)$$

4.4.1.5 If the extrapolation method is used for determining aquifer drawdown, it is not necessary to make a separate adjustment to account for boundaries or recharge.

4.4.2 *Direct Calculation*—If the aquifer drawdown  $s_{r_w}$  cannot be obtained by extrapolation, direct calculation should be used to determine its value.

4.4.2.1 For fully penetrating wells,  $s_{r_w}$  can be obtained by direct calculation using either the Theis or Cooper-Jacob equations (**Eq 1-4**).

4.4.2.2 For partially penetrating wells,  $s_{r_w}$  is calculated from the Hantush equation (**Eq 5 and 6**) or the Kozeny equation (**Eq 11**).

4.4.2.3 The presence of aquifer boundaries or recharge will tend to increase or decrease, respectively, the drawdown in and around the pumped well. When they are present, the calculated value of  $s_{r_w}$  should be adjusted to reflect the impact of the boundary conditions.

## 5. Significance and Use

5.1 This practice allows the user to compute the true hydraulic efficiency of a pumped well in a confined aquifer from a constant rate pumping field test. The procedures described constitute the only valid method of determining well efficiency. Some practitioners have confused well efficiency with percentage of head loss associated with laminar flow, a parameter commonly determined from a step-drawdown test. Well efficiency, however, cannot be determined from a step-drawdown test but only can be determined from a constant rate test.

### 5.2 Assumptions:

- 5.2.1 Control well discharges at a constant rate,  $Q$ .
- 5.2.2 Control well is of infinitesimal diameter.

5.2.3 Data are obtained from the control well and, if available, a number of observation wells.

5.2.4 The aquifer is confined, homogeneous, and extensive. The aquifer may be anisotropic, and if so, the directions of maximum and minimum hydraulic conductivity are horizontal and vertical, respectively.

5.2.5 Discharge from the well is derived exclusively from storage in the aquifer.

5.3 *Calculation Requirements*—For the special case of partially penetrating wells, application of this practice may be computationally intensive. The function  $f_s$  shown in **Eq 6** should be evaluated using arbitrary input parameters. It is not practical to use existing, somewhat limited, tables of values for  $f_s$  and, because this equation is rather formidable, it may not be tractable by hand. Because of this, it is assumed the practitioner using this practice will have available a computerized procedure for evaluating the function  $f_s$ . This can be accomplished using commercially available mathematical software including some spreadsheet applications. If calculating  $f_s$  is not practical, it is recommended to substitute the Kozeny equation for the Hantush equation as previously described.

NOTE 1—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice **D3740** are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice **D3740** does not in itself assure reliable results. Reliable results depend on many factors; Practice **D3740** provides a means of evaluating some of those factors.

NOTE 2—Commercially available software is available for the calculating, graphing, plotting, and analyses of this practice. The user is responsible for verifying the correctness of the formulas, graphs, plots and analyses of the software.

## 6. Apparatus

6.1 Apparatus for field withdrawal tests is given in Test Method **D4050**. The following apparatus are those components of the apparatus that require special attributes for this specific test.

6.2 *Construction of the Control Well*—Install the control well in the aquifer and equip with a pump capable of discharging water from the well at a constant rate for the duration of the test. A fully penetrating control well is preferred.

6.3 *Construction and Placement of Observation Wells*—If observation wells are used, they should be located on a straight line extending from the control well and positioned at different distances so that they span a good portion of the anticipated cone of depression. It is preferable that the wells be fully penetrating. If the control well and observation wells are partially penetrating, the extrapolation method of determining well efficiency can be used only if the observation wells are located outside the zone effected by partial penetration.

## 7. Procedure

7.1 Pretest preparations, pumping test guidelines, and post-test procedures associated with the pumping test itself are described in Test Method **D4050**.

7.2 Verify the quality of the data set. Review the record of measured flow rates to make sure the rate was held constant during the test. Check to see that hand measurements of drawdown agree well with electronically measured values. Check the background water-level fluctuations observed prior to or following the pumping test to see if adjustments should be made to the observed drawdown values to account for background fluctuations. If appropriate, adjust the observed drawdown values accordingly.

7.3 Analysis of the field data is described in Section 8.

## 8. Calculation and Interpretation of Results

### 8.1 Methods:

8.1.1 *Extrapolation*—This practice relies on extrapolating observation well drawdown data to estimate the theoretical drawdown just outside the well bore. It requires a single drawdown observation for the control well and each observation well used in the field test, preferably after one day of continuous pumping. If the wells are penetrating partially, the observation wells should be located outside the zone effected by partial penetration as described by Eq 9.

8.1.1.1 *Log-Log Method*—Plot the observation well distance drawdown data on a log-log graph with drawdown on the vertical axis and the reciprocal of the distance squared on the horizontal axis. On a separate graph having the same scale as the data graph, prepare a standard Theis type curve by plotting  $W(u)$  on the vertical axis versus  $1/u$  on the horizontal axis (Fig. 2). Overlay the data plot on the type curve, and while keeping the coordinate axes of the two plots parallel, shift the data plot to align with the type curve effecting a match position. On the data graph, follow the type curve to a horizontal axis coordinate of  $1/r_w^2$  and read  $s_{rw}$  from the graph. For partially penetrating wells, the extrapolated value should be corrected for partial penetration using Eq 10 or Eq 11. Calculate well efficiency using Eq 8.

8.1.1.2 *Semilog Method*—This method can be used if the  $u$  value for each observation well is sufficiently small that the Cooper-Jacob equation represents an adequate approximation to the Theis equation. Plot the observation well distance drawdown data on a semilog graph with drawdown on the

linear scale and distance on the log scale. Construct a straight line of best fit through the data points and extrapolate it to a radius value of  $r_w$ . Read  $s_{rw}$  from the graph. If the control well is partially penetrating, the extrapolated value should be corrected for partial penetration using Eq 10 or Eq 11. Calculate well efficiency using Eq 8.

8.1.2 *Direct Calculation*—Aquifer parameters including transmissivity, storage coefficient, and anisotropy ratio ( $T, S, A$ ) are determined using conventional pumping test analysis techniques. Then  $s_{rw}$  is computed directly from Eq 1-7 and Eq 11.

8.1.2.1 *Fully Penetrating Wells*—Determine  $T$  and  $S$  from the field pumping test. If no observation wells are available,  $S$  cannot be determined from the test data. In this case,  $S$  should be estimated.

NOTE 3—An acceptable procedure for estimating  $S$  is to multiply the aquifer thickness in feet by a factor between  $10^{-5}$  and  $10^{-6}$ . Determine the aquifer drawdown,  $s_{rw}$ , by direct calculation using either Eq 1-3 or Eq 4. The time parameter used in the calculation should be the time at which  $s_w$  was measured inside the control well. Determine well efficiency using Eq 8.

8.1.2.2 *Partially Penetrating Wells*—Determine  $T, S,$  and  $A$  from the field pumping test. Often it is difficult to determine the anisotropy ratio,  $A$ , accurately from the pumping test data. If this is the case,  $A$  should be estimated. Likewise, if  $S$  cannot be calculated from the data, it should be estimated. Calculate  $s_{rw}$  from Eq 5 and Eq 6 or Eq 11 and well efficiency from Eq 8.

8.1.2.3 *Boundary Conditions*—If boundary conditions affect the magnitude of the observed drawdown, follow 8.1.2.1 or 8.1.2.2 to calculate an initial value for  $s_{rw}$ . This value then should be increased or decreased by the magnitude of the boundary effect. Determine this value in accordance with 8.1.2.4.

8.1.2.4 Use the time drawdown graph for either the control well or an observation well where the  $u$  value is sufficiently small (approximately  $u < 0.05$ ). Extrapolate the early time drawdown trend to a pumping time of one day to obtain the drawdown that would have been observed if no boundary had been present. Determine the difference between this value and the actual drawdown at one day. Increase (negative boundary) or decrease (positive boundary) the initial value of  $s_{rw}$  by this amount to obtain a value for  $s_{rw}$ . Use Eq 8 to compute well efficiency.

### 8.2 Example Calculations:

#### 8.2.1 Semilog Extrapolation:

8.2.1.1 Table 1 shows example distance drawdown data obtained from a 24-h constant rate pumping test incorporating three observation wells located 30 ft, 100 ft, and 400 ft from

TABLE 1 Distance-Drawdown Data After 24 h of Continuous Pumping at 600 gpm (115 000 cfd)

Well	Distance, ft	Drawdown at 24 h, ft
Control well	1 <sup>A</sup>	46.2
Observation Well 1	30	20.3
Observation Well 2	100	15.5
Observation Well 3	400	9.7

<sup>A</sup>Borehole radius.

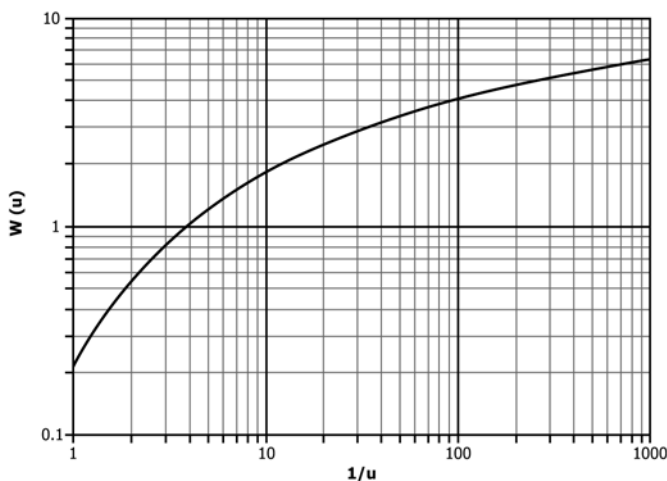


FIG. 2 Theis Type Curve