**Designation: D6028/D6028M − 20**

# Standard Practice for (Analytical Procedure) Determining Hydraulic Properties of a Confined Aquifer Taking into Consideration Storage of Water in Leaky Confining Beds by Modified Hantush Method[1]

This standard is issued under the fixed designation D6028/D6028M; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

## 1. Scope*

1.1 This practice covers an analytical procedure for determining the transmissivity and storage coefficient of a confined aquifer taking into consideration the change in storage of water in overlying or underlying confining beds, or both. This practice is used to analyze water-level or head data collected from one or more observation wells or piezometers during the pumping of water from a control well at a constant rate. With appropriate changes in sign, this practice also can be used to analyze the effects of injecting water into a control well at a constant rate.

1.2 This analytical procedure is used in conjunction with Test Method D4050.

1.3 *Limitations*—The valid use of the modified Hantush method (**1**)[2] is limited to the determination of hydraulic properties for aquifers in hydrogeologic settings with reasonable correspondence to the assumptions of the Hantush-Jacob method (Practice D6029/D6029M) with the exception that in this case the gain or loss of water in storage in the confining beds is taken into consideration (see 5.1). All possible combinations of impermeable beds and source beds (for example, beds in which the head remains uniform) are considered on the distal side of the leaky beds that confine the aquifer of interest (see Fig. 1).

1.4 All observed and calculated values shall conform to the guidelines for significant digits and rounding established in Practice D6026.

1.4.1 The procedures used to specify how data are collected/recorded and calculated in the standard are regarded as the industry standard. In addition, they are representative of the significant digits that generally should be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any considerations for the user's objectives; and it is common practice to increase or reduce significant digits of reported data to be commensurate with these considerations. It is beyond the scope of these test methods to consider significant digits used in analysis methods for engineering data.

1.5 The values stated in SI units or inch-pound units are to be regarded separately as standard. The values stated in each system may not be exact equivalents; therefore, each system shall be used independently of the other. Combining values for the two systems may result in nonconformance with the standard. Reporting of results in units other than SI shall not be regarded as nonconformance with this standard.

1.6 This practice offers a set of instructions for performing one or more specific operations. This document cannot replace education or experience and should be used in conjunction with professional judgment. Not all aspects of the practice may be applicable in all circumstances. This ASTM standard is not intended to represent or replace the standard of care by which the adequacy of a given professional service must be judged, nor should this document be applied without the consideration of a project's many unique aspects. The word "Standard" in the title of this document means only that the document has been approved through the ASTM consensus process.

1.7 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.8 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

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[1] This practice is under the jurisdiction of Committee D18 on Soil and Rock and is the direct responsibility of Subcommittee D18.21 on Groundwater and Vadose Zone Investigations.

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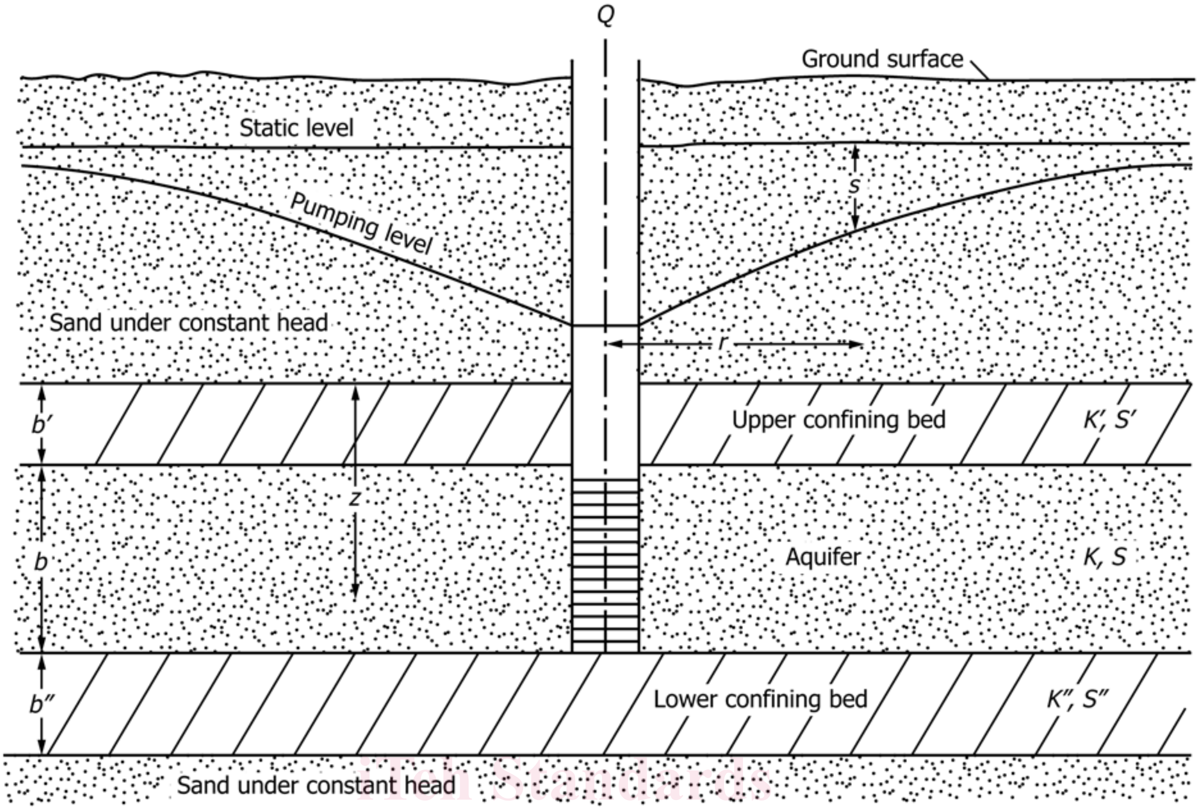
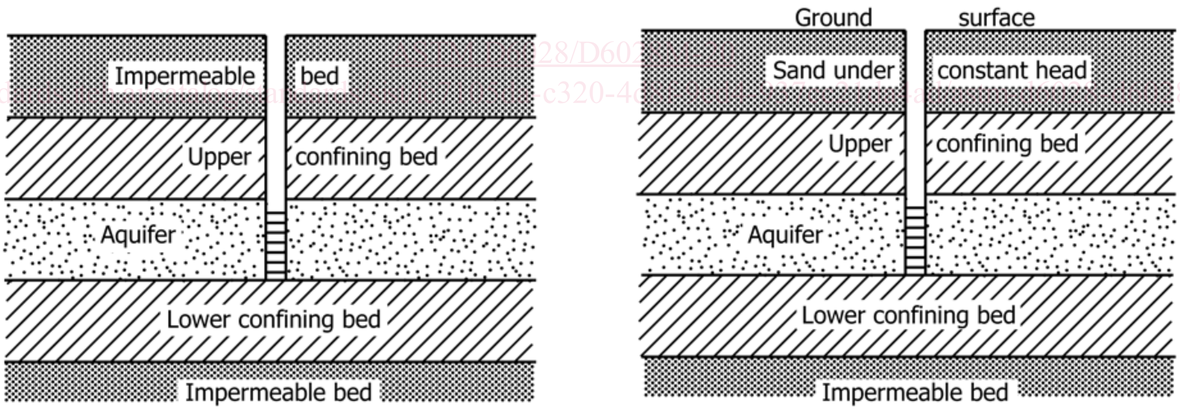
[2] The boldface numbers in parentheses refer to a list of references at the end of this practice.

***A Summary of Changes section appears at the end of this standard**



**CASE 1**



**CASE 2** **CASE 3**

FIG. 1 Cross Sections Through Discharging Wells in Leaky Aquifers with Storage of Water in the Confining Beds, Illustrating Three Different Cases of Boundary Conditions (from Reed (2) )

## 2. Referenced Documents

2.1 *ASTM Standards:*[3]

D653 Terminology Relating to Soil, Rock, and Contained Fluids

D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction

D4050 Test Method for (Field Procedure) for Withdrawal and Injection Well Testing for Determining Hydraulic Properties of Aquifer Systems

D4106 Practice for (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Theis Nonequilibrium Method

D6026 Practice for Using Significant Digits in Geotechnical Data

D6029/D6029M Practice for (Analytical Procedures) Determining Hydraulic Properties of a Confined Aquifer and a Leaky Confining Bed with Negligible Storage by the Hantush-Jacob Method

## 3. Terminology

3.1 *Definitions*—For definitions of common technical terms used in this standard, refer to Terminology D653.

3.2 *Symbols and Dimensions:*

3.2.1 *H (u,β)*—well function for leaky systems where water storage in confining beds is important [*nd*].

3.2.2 *K*—hydraulic conductivity of the aquifer [$LT^{-1}$].

3.2.2.1 *Discussion*—The use of the symbol *K* for the term hydraulic conductivity is the predominant usage in groundwater literature by hydrogeologists, whereas the symbol *k* is commonly used for this term in soil and rock mechanics and soil science.

3.2.3 *K′, K″*—vertical hydraulic conductivities of the confining beds through which leakage can occur [$LT^{-1}$].

3.2.4 *Q*—discharge [$L^3T^{-1}$].

3.2.5 $S = bS_s$—storage coefficient of the aquifer [*nd*].

3.2.6 $\underline{S'=b'S'_s}$—storage coefficients of the confining beds [*nd*].

$\underline{S''=b''S''_s}$

3.2.7 $S_s$—specific storage of the aquifer [$L^{-1}$].

3.2.8 $\underline{S'_s\ S''_s}$—specific storages of the confining beds. [$\underline{L}^{-1}$]

3.2.9 *T*—transmissivity [$L^2T^{-1}$].

3.2.10 $u = \frac{r^2s}{4Tt}$ [*nd*].

3.2.11 *W(u,r/B)*—well function for leaky aquifer systems with negligible storage changes in confining beds [ *nd*].

3.2.12 *W(u)*—well function for nonleaky aquifer systems [*nd*].

3.2.13 *b*—thickness of aquifer [ *L*].

3.2.14 *b′, b″*—thicknesses of the confining beds through which leakage can occur [*L*].

3.2.15 *r*—radial distance from control well [*L*].

3.2.16 *s*—drawdown [*L*].

3.2.17 $B=\sqrt{\frac{Tb'}{K'}}\,[L]$ .

3.2.18 *t*—time since pumping or injection began [*T*].

3.2.19 $\beta=\frac{r}{4b}\left(\sqrt{\frac{K'S'}{b'KS_s}}+\right)\sqrt{\frac{K''S''}{b''KS_s}}\,[nd]$.

## 4. Summary of Practice

4.1 This practice involves pumping a control well that is fully screened through the confined aquifer and measuring the water-level response in one or more observation wells or piezometers. The well is pumped at a constant rate. The water-level response in the aquifer is a function of the transmissivity and storage coefficient of the aquifer and the leakance coefficients and storage coefficients of the confining beds. Alternatively, the practice can be performed by injecting water at a constant rate into the control well. Analysis of buildup of water level in response to injection is similar to analysis of drawdown of water level in response to withdrawal in a confined aquifer. The water-level response data are analyzed using a set of type curves.

4.2 *Solution*—Hantush (**1**) gave solutions applicable to each of Cases 1, 2, and 3 shown in Fig. 1 for "relatively small" values of time and for "relatively large" values of time. The solution applicable for each case for relatively small values of time can be written as follows

$$s = \frac{Q}{4\pi T} H\,(u,\beta) \qquad (1)$$

where:

$$u = \frac{r^2S}{4Tt} \qquad (2)$$

and

$$\beta = \frac{r}{4b}\left(\frac{K'S'}{b'KS_s}+\frac{K''\ S''}{b''KS_s}\right) \qquad (3)$$

$$H\,(u,\beta) = \int_u^\infty \frac{e^{-y}}{y}\, erfc\, \frac{\beta\sqrt{u}}{\sqrt{y\,(y-u)}}\, dy \qquad (4)$$

$$erfc\,(x) = \frac{2}{\sqrt{\pi}}\int_u^\infty e^{-y2}\ dy \qquad (5)$$

where *y* is the variable of integration.

4.2.1 The "relatively small" times when Eq 1 is applicable are when:

$$t<\frac{b'S'}{10K'} \text{ and } t<\frac{b''S''}{10K''} \qquad (6)$$

Equation 1 is applicable at early times for each of the cases shown in Fig. 1 even though the conditions on the distal sides of the confining beds are quite different because for early times the solution in the aquifer is essentially independent of conditions on the distal side of the confining beds. The effects of those distant boundary conditions are not felt in the aquifer

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[3] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

for a while. Eq 1-5 are the basis for the type curve solution that is described by this practice.

4.2.2 For relatively large values of time the solutions given by Hantush **(1)** can be written as:

4.2.2.1 *Case 1*—Heads in zones on the distal side of the confining beds remain constant and are unaffected by discharge of the pumped well. For times when

$$t > 5\frac{b'S'}{K'} \text{ and } t > 5\frac{b''S''}{K''} \tag{7}$$

are both satisfied, then

$$s = \frac{Q}{4\pi T} W(u\delta_1, \alpha) \tag{8}$$

where:

$$\delta_1 = 1 + \frac{(S' + S'')}{3S} \text{ and } \alpha = r\sqrt{\frac{K'}{Tb'} + \frac{K''}{Tb''}} \tag{9}$$

Hantush **(1)** notes that if $K''$, $S'$, and $S''$ are taken as zero in the flow systems shown in Fig. 1 as Case 1 or Case 3, the resulting flow system is that of a confined aquifer overlying an impermeable bed and the aquifer being overlain by a confining bed in which the storage is negligible. Hantush gives the solution for that special case as follows:

$$s = \frac{Q}{4\pi T} W(u, r/B) \tag{10}$$

where:

$$\frac{r}{B} = r\sqrt{\frac{K'}{Tb'}}$$

Note that *W*(*u,r/B*) is the well function for leaky systems with negligible storage in the confining beds given by Hantush and Jacob **(3)** and described in Practice D6029/D6029M. That function is defined as follows:

$$W(u, r/B) = \int_u^\infty \exp(-y - r^2/(4B^2y))\,\frac{dy}{y} \tag{11}$$

4.2.2.2 *Case 2*—The materials in the zones on the distal sides of the confining beds are impermeable. For times when

$$t > 10\frac{b'S'}{K'} \text{ and } t > 10\frac{b''S''}{K''} \tag{12}$$

are both satisfied, then

$$s = \frac{Q}{4\pi T} W(u, \delta_2) \tag{13}$$

where:

$$\delta_2 = 1 + \frac{(S' + S'')}{S}$$

and where the function *W*(*u*) is the well function for non-leaky aquifers that appears in the solution given by Theis **(4)** described in Practice D4106 for drawdowns in response to a well pumped at a constant rate from a non-leaky aquifer.

4.2.2.3 *Case 3*—The materials on the distal side of one confining bed are impermeable and the heads on the distal sides of the other confining bed remain constant and are unaffected by discharge of the pumped well. For times when

$$t > \frac{5b'S'}{K'} \text{ and } t > \frac{10b''S''}{K''} \tag{14}$$

are both satisfied, then

$$s = \frac{Q}{4\pi T} W\left(u\delta_3, r\sqrt{\frac{K'}{Tb'}}\right) = \frac{Q}{4\pi t} W(u\delta_3, r/B) \tag{15}$$

where:

$$\delta_3 = 1 + (S'' + S'/3)S \tag{16}$$

and *W*(*u,r/B*) is defined in Case 1 (see Eq 11).

Hantush **(1)** did not develop expressions for the solutions to these cases for intermediate times (between“ small” and “large” times). Reed (**(2)** p. 26) notes that Neuman and Witherspoon (**(5)**, p. 250) developed a complete (that is, applicable for all times) solution for Case 1 (source beds on the distal sides of both confining beds) but did not tabulate it.

## 5. Significance and Use

5.1 *Assumptions:*

5.1.1 The control well discharges at a constant rate, *Q*.

5.1.2 The control well is of infinitesimal diameter and fully penetrates the aquifer.

5.1.3 The aquifer is homogeneous, isotropic, and areally extensive.

NOTE 1—Slug and pumping tests implicitly assume a porous medium. Fractured rock and carbonate settings may not provide meaningful data and information.

5.1.4 The aquifer remains saturated (that is, water level does not decline below the top of the aquifer).

5.1.5 The aquifer is overlain or underlain, or both, everywhere by confining beds individually having uniform hydraulic conductivities, specific storages, and thicknesses. The confining beds are bounded on the distal sides by one of the cases shown in Fig. 1.

5.1.6 Flow in the aquifer is two-dimensional and radial in the horizontal plane.

5.2 The geometry of the well and aquifer system is shown in Fig. 1.

5.3 *Implications of Assumptions:*

5.3.1 Paragraph 5.1.1 indicates that the discharge from the control well is at a constant rate. Paragraph 8.1 of Test Method D4050 discusses the variation from a strictly constant rate that is acceptable. A continuous trend in the change of the discharge rate could result in misinterpretation of the water-level change data unless taken into consideration.

NOTE 2—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself assure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.

5.3.2 The leaky confining bed problem considered by the modified Hantush method requires that the control well has an infinitesimal diameter and has no storage. Moench **(6)** generalized the field situation addressed by the modified Hantush **(1)** method to include the well bore storage in the pumped well. The mathematical approach that he used to obtain a solution for that more general problem results in a Laplace transform solution whose analytical inversion has not been developed and probably would be very complicated, if possible, to evaluate. Moench (**6**) used a numerical Laplace inversion algorithm to develop type curves for selected situations. The situations considered by Moench indicate that large well bore storage may mask effects of leakage derived from storage changes in the confining beds. The particular combinations of aquifer and confining bed properties and well radius that result in such masking is not explicitly given. However, Moench ((**6**), p. 1125) states "Thus observable effects of well bore storage are maximized, for a given well diameter, when aquifer transmissivity $Kb$ and the storage coefficient $S_sb$ are small." Moench (p. 1129) notes that "...one way to reduce or effectively eliminate the masking effect of well bore storage is to isolate the aquifer of interest with hydraulic packers and repeat the pump test under pressurized conditions. Because well bore storage $C$ will then be due to fluid compressibility rather than changing water levels in the well"..."the dimensionless well bore storage parameter may be reduced by 4 to 5 orders of magnitude."

5.3.3 The modified Hantush method assumes, for Cases 1 and 3 (see Fig. 1), that the heads in source layers on the distal side of confining beds remain constant. Neuman and Witherspoon (**7**) developed a solution for a case that could correspond to Hantush's Case 1 with $K'' = O = S''$ except that they do not require the head in the unpumped aquifer to remain constant. For that case, they concluded that the drawdowns in the pumped aquifer would not be affected by the properties of the other, unpumped, aquifer when (Neuman and Witherspoon (**7**) p. 810) time satisfies:

$$t \leq 0.1 \frac{S'b'}{K'} \tag{17}$$

5.3.4 Implicit in the assumptions are the conditions that the flow in the confining beds is essentially vertical and in the aquifer is essentially horizontal. Hantush's (**8**) analysis of an aquifer bounded only by one leaky confining bed suggested that these assumptions are acceptably accurate wherever

$$\frac{K}{K'} > 100 \frac{b}{b'} \tag{18}$$

That form of relation between aquifer and confining bed properties may also be a useful guide for the case of two leaky confining beds.

## 6. Apparatus

6.1 Analysis of data from the field procedure (see Test Method D4050) by this practice requires that the control well and observation wells meet the requirements specified in the following paragraphs.

6.2 *Construction of Control Well*—Install the control well in the aquifer and equip with a pump capable of discharging water from the well at a constant rate for the duration of the test. Preferably, the control well should be open throughout the full thickness of the aquifer. If the control well partially penetrates the aquifer, take special precaution in the placement or design of observation wells.

6.3 *Construction and Location of Observation Wells and Piezometers*—Construct one or more observation wells or piezometers screened only in the pumped aquifer at a distance from the control well. Observation wells may be open through all or part of the thickness of the aquifer. Hantush ((**9**) p. 350) indicates that the effects of a partially penetrating control well can be neglected for

$$r > 1.5b \sqrt{\frac{K_r}{K_z}} \tag{19}$$

where $K_r$ and $K_z$ are the aquifer hydraulic conductivities in the horizontal and vertical directions, respectively. Although that relationship was developed for an aquifer confined by a leaky confining bed in which storage is neglected, it may be a useful guideline for the cases where storage in the confining beds is important. If an observation well fully penetrates the aquifer, it's drawdown is not affected by a partially penetrating control well and it reacts as if the control well completely penetrated the aquifer (Hantush (**9**) p. 351).

## 7. Procedure

7.1 Pretest preparations are described in detail in Test Method D4050. The overall test procedure consists of (*1*) conducting the field procedure for withdrawal or injection well tests (described in Test Method D4050) and (*2*) analysis of the field data, which is addressed in Section 8.

## 8. Calculation and Interpretation of Test Data

8.1 *Aquifer*—Field test data for "relatively small" values of time are analyzed using Eq 1-3. The graphical procedure used to calculate test results is based on the functional relations between $H(u,\beta)$ and $s$ and between $u$ and $t/r^2$.

NOTE 3—Because the $H(u,\beta)$ type curve method is based on the assumption that the duration of the test is such that the boundary conditions on the distal sides of the confining beds have not yet affected drawdowns in the pumped aquifer, only the relatively early-time drawdown data should be used in fitting the $H(u,\beta)$ curves. "Relatively late-time" drawdown data can be analyzed using Eq 8, Eq 13, or Eq 15 for field conditions described by Cases 1, 2, or 3, respectively. Equations 8 and Equations 15 correspond to the condition that there are no further changes in storage in the leaky confining beds bounded by constant head layers and leakage into the pumped aquifer though those confining beds by those times correspond entirely to water transmitted from the source (constant head) layers. That situation is discussed in Practice D6029/D6029M. Reed ((**4**) p. 28–29) notes that the late-time data for Cases 1 and 3 will fall on the flat part of the $W(u,r/B)$ type curves and a time-drawdown plot match would be indeterminate. Equation 13 corresponds to non-leaky confined aquifers, and that situation is discussed in Practice D4106. Spane and Wurstner (**10**) discuss the advantage of supplementing the type curve plots of drawdown versus time by plots of the derivative of drawdown (with respect to an appropriate time function) versus time as an aid in selecting an aquifer interpretation model and in estimating the aquifer parameters. They discuss also an approach that transforms water-level recovery (that is, the response of water levels when the pump is shut off) data plots to a form that can be analyzed with drawdown data in constructing derivative plots. To apply the derivative methods requires that measurements be spaced closely enough that numerically developed time derivatives can be reasonably approximated.

8.1.1 Plot values of $H(u,\beta)$ versus $1/u$ for selected values of $\beta$ on logarithmic-scale paper. This plot is referred to as the type curve plot. Table 1 gives a tabulation of values of $H(u,\beta)$ for selected values of $u$ and $\beta$. Fig. 2 is a logarithmic plot of $H(u,\beta)$

**TABLE 1 Values of *H*(*u*,β) for Selected Values of *u* and β (from Reed ).**

NOTE 1—From Hantush . Numbers in parentheses are powers of 10 by which the other numbers are multiplied (for example 963(−4) = 0.0963)

| u | β | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0.03 | 0.1 | 0.2 | 1 | 3 | 10 | 30 | 100 |
| $1 \times 10^{-9}$ | 12.3088 | 11.1051 | 10.0066 | 8.8030 | 7.7051 | 6.5033 | 5.4101 | 4.2221 |
| 2 | 11.9622 | 10.7585 | 9.6602 | 8.4566 | 7.3590 | 6.1579 | 5.0666 | 3.8839 |
| 3 | 11.7593 | 10.5558 | 9.4575 | 8.2540 | 7.1565 | 5.9561 | 4.8661 | 3.6874 |
| 5 | 11.5038 | 10.3003 | 9.2021 | 7.9987 | 6.9016 | 5.7020 | 4.6142 | 3.4413 |
| 7 | 11.3354 | 10.1321 | 9.0339 | 7.8306 | 6.7337 | 5.5348 | 4.4487 | 3.2804 |
| $1 \times 10^{-8}$ | 11.1569 | 9.9538 | 8.8556 | 7.6525 | 6.5558 | 5.3578 | 4.2737 | 3.1110 |
| 2 | 10.8100 | 9.6071 | 8.5091 | 7.3063 | 6.2104 | 5.0145 | 3.9352 | 2.7858 |
| 3 | 10.6070 | 9.4044 | 8.3065 | 7.1039 | 6.0085 | 4.8141 | 3.7383 | 2.5985 |
| 5 | 10.3511 | 9.1489 | 8.0512 | 6.8490 | 5.7544 | 4.5623 | 3.4919 | 2.3662 |
| 7 | 10.1825 | 8.9806 | 7.8830 | 6.6811 | 5.5872 | 4.3969 | 3.3307 | 2.2159 |
| $1 \times 10^{-7}$ | 10.0037 | 8.8021 | 7.7048 | 6.5032 | 5.4101 | 4.2221 | 3.1609 | 2.0591 |
| 2 | 9.6560 | 8.4554 | 7.3585 | 6.1578 | 5.0666 | 3.8839 | 2.8348 | 1.7633 |
| 3 | 9.4524 | 8.2525 | 7.1560 | 5.9559 | 4.8661 | 3.6874 | 2.6469 | 1.5966 |
| 5 | 9.1955 | 7.9968 | 6.9009 | 5.7018 | 4.6141 | 3.4413 | 2.4137 | 1.3944 |
| 7 | 9.0261 | 7.8283 | 6.7329 | 5.5346 | 4.4486 | 3.2804 | 2.2627 | 1.2666 |
| $1 \times 10^{-6}$ | 8.8463 | 7.6497 | 6.5549 | 5.3575 | 4.2736 | 3.1110 | 2.1051 | 1.1361 |
| 2 | 8.4960 | 7.3024 | 6.2091 | 5.0141 | 3.9350 | 2.7857 | 1.8074 | 0.8995 |
| 3 | 8.2904 | 7.0991 | 6.0069 | 4.8136 | 3.7382 | 2.5984 | 1.6395 | 0.7725 |
| 5 | 8.0304 | 6.8427 | 5.7523 | 4.5617 | 3.4917 | 2.3661 | 1.4354 | 0.6256 |
| 7 | 7.8584 | 6.6737 | 5.5847 | 4.3962 | 3.3304 | 2.2158 | 1.3061 | 0.5375 |
| $1 \times 10^{-5}$ | 7.6754 | 6.4944 | 5.4071 | 4.2212 | 3.1606 | 2.0590 | 1.1741 | 0.4519 |
| 2 | 7.3170 | 6.1453 | 5.0624 | 3.8827 | 2.8344 | 1.7632 | 0.9339 | 0.3091 |
| 3 | 7.1051 | 5.9406 | 4.8610 | 3.6858 | 2.6464 | 1.5965 | 0.8046 | 0.2402 |
| 5 | 6.8353 | 5.6821 | 4.6075 | 3.4394 | 2.4131 | 1.3943 | 0.6546 | 0.1635 |
| 7 | 6.6553 | 5.5113 | 4.4408 | 3.2781 | 2.2619 | 1.2664 | 0.5643 | 0.1300 |
| $1 \times 10^{-4}$ | 6.4623 | 5.3297 | 4.2643 | 3.1082 | 2.1042 | 1.1359 | 0.4763 | 963(−4) |
| 2 | 6.0787 | 4.9747 | 3.9220 | 2.7819 | 1.8062 | 0.8992 | 0.3287 | 494(−4) |
| 3 | 5.8479 | 4.7655 | 3.7222 | 2.5937 | 1.6380 | 0.7721 | 0.2570 | 315(−4) |
| 5 | 5.5488 | 4.4996 | 3.4711 | 2.3601 | 1.4335 | 0.6252 | 0.1818 | 166(−4) |
| 7 | 5.3458 | 4.3228 | 3.3062 | 2.2087 | 1.3039 | 0.5370 | 0.1412 | 103(−4) |
| $1 \times 10^{-3}$ | 5.1247 | 4.1337 | 3.1317 | 2.0506 | 1.1715 | 0.4513 | 0.1055 | 390(−5) |
| 2 | 4.6753 | 3.7598 | 2.7938 | 1.7516 | 0.9305 | 0.3084 | 551(−4) | 169(−5) |
| 3 | 4.3993 | 3.5363 | 2.5969 | 1.5825 | 0.8006 | 0.2394 | 355(−4) | 713(−6) |
| 5 | 4.0369 | 3.2483 | 2.3499 | 1.3767 | 0.6498 | 0.1677 | 190(−4) | 205(−6) |
| 7 | 3.7893 | 3.0542 | 2.1877 | 1.2460 | 0.5589 | 0.1292 | 120)−4) | 821(−7) |
| $1 \times 10^{-2}$ | 3.5195 | 2.8443 | 2.0164 | 1.1122 | 0.4702 | 955(−4) | 695(−5) | 274(−7) |
| 2 | 2.9759 | 2.4227 | 1.6853 | 0.8677 | 0.3214 | 487(−4) | 205(−5) | 226(−8) |
| 3 | 2.6487 | 2.1680 | 1.4932 | 0.7353 | 0.2491 | 308)−4) | 888(−6) | |
| 5 | 2.2312 | 1.8401 | 1.2535 | 0.5812 | 0.1733 | 160)−4) | 261(−6) | |
| 7 | 1.9558 | 1.6213 | 1.0979 | 0.4880 | 0.1325 | 982(−5) | 106(−6) | |
| $1 \times 10^{-1}$ | 1.6667 | 1.3893 | 0.9358 | 0.3970 | 966(−4) | 552(−5) | 365(−7) | |
| 2 | 1.1278 | 0.9497 | 0.6352 | 0.2452 | 468(−4) | 149(−5) | 307(−8) | |
| 3 | 0.8389 | 0.7103 | 0.4740 | 0.1729 | 281(−4) | 592(−6) | | |
| 5 | 0.5207 | 0.4436 | 0.29556 | 0.1006 | 130(−4) | 151(−6) | | |
| 7 | 0.3485 | 0.2980 | 0.1985 | 646(−4) | 714(−5) | 534(−7) | | |
| $1 \times 1$ | 0.2050 | 0.1758 | 0.1172 | 365(−4) | 337(−5) | 151(−7) | | |
| 2 | 458(−4) | 395(−4) | 264(−4) | 760(−5) | 487(−6) | | | |
| 3 | 122(−4) | 106(−4) | 707(−5) | 196(−5) | 102(−6) | | | |
| 5 | 108(−5) | 934(−6) | 624(−6) | 167(−6) | 672(−8) | | | |
| 7 | 109(−6) | 941(−7) | 629(−7) | 165(−7) | | | | |
| $1 \times 10$ | 391(−8) | 339(−8) | 227(−8) | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | |
| 5 | | | | | | | | |
| 7 | | | | | | | | |

versus 1/*u* for selected values of β (from Kruseman and deRidder (**11**)). If a set of type curves are inaccessible, these data can be used to develop type curves. A more extensive tabulation of *H*(*u*,β) is given in Hantush (**12**). Some readily available sources of these type curves are Lohman (**13**) and Reed (**2**). Commercially available software is available to calculate and plot these values and curves.

NOTE 4—Commercial software is available to perform the calculations and plotting described in this practice. The user should verify the correctness of the formulas, calculations, plotting and analyses of the software.

8.1.2 On logarithmic tracing paper of the same scale and size as the *H*(*u*,β) versus 1/*u* type curves, plot values of drawdown, *s*, for each observation well on the vertical coordinate versus time divided by distance between the control well and the observation well squared, $t/r^2$, on the horizontal coordinate. This plot is referred to as the data plot.

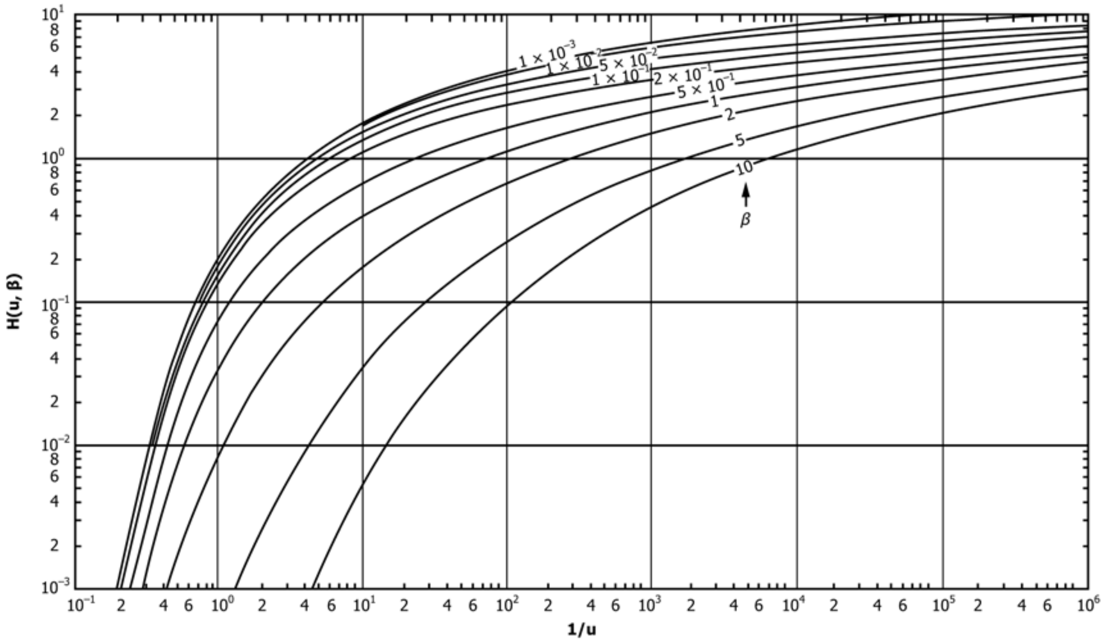

FIG. 2 Family of Curves of H(u,β) versus 1/u for Selected Values of β (from Kruseman and deRidder (**11**))

8.1.3 Overlay the data plot on the type curve plot and, keeping the coordinate axes of the two plots parallel, shift the plot to the position where the data for each observation well falls either between one pair of the β curves, or along one of them. It is preferable for two or more observation wells to be at different distances from the control well. Recall the definition of β (see Eq 3). The advantages of having two or more observation wells is that the distance values, *r*, for the observation wells should fall on curves having proportional β values. For example, if data are available from three observation wells at 100, 200, and 800 ft from the control well, the data plots for the three wells should match curves having corresponding β values having the ratios 1:2:8. Weeks (**14**) notes that for values of β ranging from zero (this is the Theis curve which corresponds to a non-leaky case) to about 0.7, there is virtually no difference in the shape of the curves on the $H(u,\beta)$ versus $1/u$ plot. Weeks states that if β falls within this range for a given observation well it is impossible to determine unique values of transmissivity and storativity for the aquifer and β using only that well. The use of a composite plot involving more than one observation well at different distances, *r*, may permit a unique fit to be obtained.

NOTE 5—Moench (**6**) notes that it is desirable to also obtain data on water-level changes in the pumped well because it can "...be helpful in determining the presence or absence of leakage when compared with observation well data." However, data from the pumped well are affected by variations in the pumping rate, effects of well-bore storage, and the "skin" (a zone around the well hydraulically different from the native materials because of disturbance and alteration caused by well drilling and construction).

8.1.4 Select and record the values of $H(u,\beta)$, $1/u$, $s$, and $t/r^2$ at an arbitrary point, referred to as the match point, anywhere on the overlapping part of the type curve plot and the data plot. For convenience, the match point may be selected where $H(u,\beta)$ and $1/u$ are integer values. Record the value of β for each observation well's data.

8.1.5 Using the selected values, determine the transmissivity and storage coefficient from Eq 1 and Eq 2:

$$T = \frac{Q}{4\pi s} H(u,\beta) \tag{20}$$

$$S = 4Tu \frac{t}{r^2} \tag{21}$$

Equation 3 indicates that if the aquifer of interest is overlain and underlain by leaky confining beds, the value of β characterizes a composite of the properties of the individual confining beds.

8.1.6 Reed ((**2**) p. 26–27) notes that for certain special situations, the β values may be used to characterize individual confining bed properties. For example, suppose that the hydrogeologic information for an area suggests that the value of $K''S''$ for the underlying confining bed is negligible. This would occur if the bed is effectively impermeable and incompressible. For that situation Eq 3 reduces to:

$$\beta = \frac{r}{4b}\sqrt{\frac{K'S'}{b'KS_S}} \tag{22}$$

which can be manipulated to give that

$$K'S' = \frac{16\beta^2 b^2 KS_S}{r_2} b' \tag{23}$$

Recalling that $T = bK$ and $S = bS_s$ this can be rewritten as

$$K'S' = \frac{16\beta^2\, TS}{r^2} b' \tag{24}$$

Note that $b'$ and $r$ are measured and $T$, $S$, and β are estimated from the test analysis so that a value for $K'S'$ can be calculated.