Designation: D6029 – 17 D6029/D6029M – 20

Standard Test Method (Analytical Procedure) Practice for (Analytical Procedures) Determining Hydraulic Properties of a Confined Aquifer and a Leaky Confining Bed with Negligible Storage by the Hantush-Jacob Method[1]

This standard is issued under the fixed designation D6029;D6029/D6029M; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This test method covers an analytical procedure for determining the transmissivity and storage coefficient of a confined aquifer and the leakance value of an overlying or underlying confining bed for the case where there is negligible change of water in storage in a confining bed. This test method is used to analyze water-level or head data collected from one or more observation wells or piezometers during the pumping of water from a control well at a constant rate. With appropriate changes in sign, this test method also can be used to analyze the effects of injecting water into a control well at a constant rate.

1.2 This analytical procedure is used in conjunction with Test Method D4050.

1.3 *Limitations*—The valid use of the Hantush-Jacob method is limited to the determination of hydraulic properties for aquifers in hydrogeologic settings with reasonable correspondence to the assumptions of the Theis nonequilibrium method (Test Method D4106) with the exception that in this case the aquifer is overlain, or underlain, everywhere by a confining bed having a uniform hydraulic conductivity and thickness, and in which the gain or loss of water in storage is assumed to be negligible, and that bed, in turn, is bounded on the distal side by a zone in which the head remains constant. The hydraulic conductivity of the other bed confining the aquifer is so small that it is assumed to be impermeable (see Fig. 1).

1.4 The values stated in SI units are to be regarded as standard. The values given in parentheses are mathematical conversions to inch-pound units, which are provided for information only and are not considered standard.

1.4.1 The converted inch-pound units use the gravitational system of units. In this system, the pound (lbf) represents a unit of force (weight), while the unit for mass is slugs. The converted slug unit is not given, unless dynamic (F = ma) calculations are involved.

1.5 All observed and calculated values shall conform to the guidelines for significant digits and round established in Practice D6026, unless superseded by this standard.

1.5.1 The procedures used to specify how data are collected/recorded or calculated, in this standard are regarded as the industry standard. In addition, they are representative of the significant digits that generally should be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any considerations for the user's objectives; and it is common practice to increase or reduce significant digits of reported date to be commensurate with these considerations. It is beyond the scope of this standard to consider significant digits used in analysis method for engineering design.

1.6 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

D653 Terminology Relating to Soil, Rock, and Contained Fluids

D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction

[1] This test method practice is under the jurisdiction of ASTM Committee D18 on Soil and Rock and is the direct responsibility of Subcommittee D18.21 on Groundwater and Vadose Zone Investigations.

Current edition approved Jan. 1, 2017June 1, 2020. Published January 2017June 2020. Originally approved in 1996. Last previous edition approved in 20102017 as D6029–96(2010)D6029$^{\varepsilon1}$.–17. DOI: 10.1520/D6029-17.10.1520/D6029_D6029M-20.

[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

***A Summary of Changes section appears at the end of this standard**

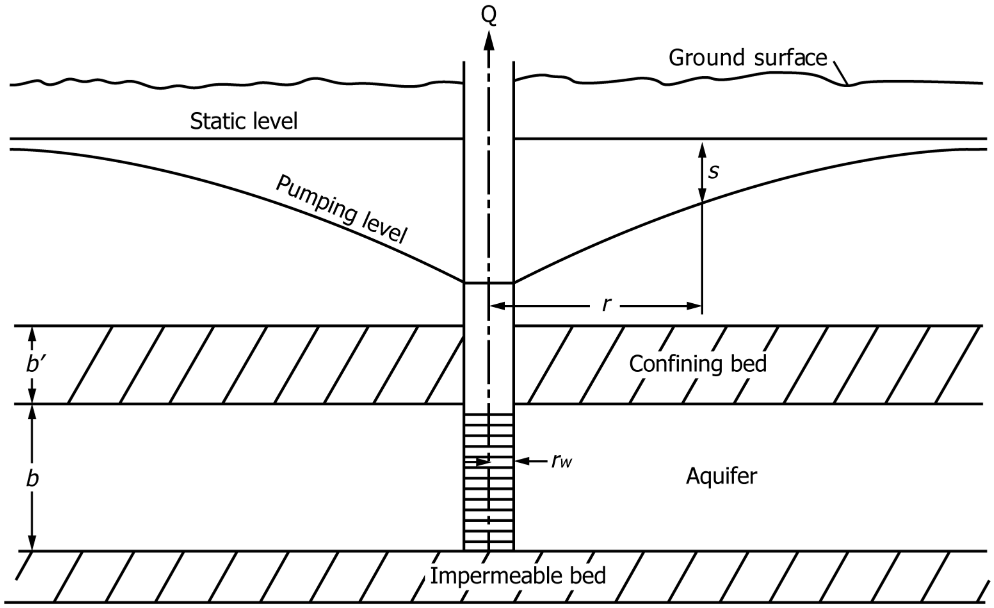


FIG. 1 Cross Section Through a Discharging Well in a Leaky Aquifer (from Reed (1)[3]). The Confining and Impermeable Bed Locations Can Be Interchanged

D4050 Test Method for (Field Procedure) for Withdrawal and Injection Well Testing for Determining Hydraulic Properties of Aquifer Systems

D4106 Practice for (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Theis Nonequilibrium Method

D6026 Practice for Using Significant Digits in Geotechnical Data

D6028 Practice for (Analytical Procedure) Determining Hydraulic Properties of a Confined Aquifer Taking into Consideration Storage of Water in Leaky Confining Beds by Modified Hantush Method

3. Terminology

3.1 *Definitions:*

3.1.1 For definitions of common terms used in this test method, see Terminology D653.

3.2 *Symbols and Dimensions:*

3.2.1 *K*—hydraulic conductivity of the aquifer [LT^{-1}].

3.2.1.1 *Discussion*—

The use of the symbol *K* for the term hydraulic conductivity is the predominant usage in groundwater literature by hydrogeologists, whereas the symbol *k* is commonly used for this term in soil and rock mechanics and soil science.

3.2.2 *K′*—vertical hydraulic conductivity of the confining bed through which leakage can occur [LT^{-1}].

3.2.3 *L(u,v)*—leakance function of *u,v* [*nd*]; equal to *W(u,r/B)*.

3.2.4 *Q*—discharge [L^3T^{-1}].

3.2.5 $S = bS_S$—storage coefficient [*nd*].

3.2.6 S_s—specific storage of the aquifer [L^{-1}].

3.2.7 S'_s—specific storage of the confining bed [L^{-1}].

3.2.8 *T*—transmissivity [L^2T^{-1}].

3.2.9 $u = \frac{r^2S}{4Tt}$ [*nd*].

3.2.10 *W(u,r/B)*—well function for leaky aquifer systems with negligible storage changes in confining beds [*nd*].

3.2.11 *b*—thickness of aquifer [*L*]. *b′*—thickness of the confining bed through which leakage can occur [*L*].

3.2.12 *r*—radial distance from control well [*L*].

3.2.13 r_c—radius of the control well casing, or hole if uncased [*L*].

3.2.14 *s*—drawdown [*L*].

3.2.15 $v = \frac{r}{2B} = \frac{r}{2}\sqrt{\frac{K'}{Tb'}}$ *v*—defined by Eq 7 [*nd*].

3.2.16 $\sqrt{\frac{Tb'}{K'}}$ [*L*].

3.2.17 *t*—time since pumping or injection began [*T*].

3.2.18 $K_0(x)$—zero-order modified Bessel function of the second kind [*nd*].

3.2.19 $\beta = \frac{r}{4b}\sqrt{\frac{K'S'_S}{KS_S}}$

4. Summary of Test Method

4.1 This test method involves pumping a control well that is fully screened through the confined aquifer and measuring the water-level response in one or more observation wells or piezometers. The well is pumped at a constant rate. The water-level response in the aquifer is a function of the transmissivity and storage coefficient of the aquifer and the leakance coefficient of a confining bed. The other confining bed is assumed to be impermeable. Alternatively, the test method can be performed by injecting water at a constant rate into the control well. Analysis of buildup of water level in response to injection is similar to analysis of drawdown of the water level in response to withdrawal in a confined aquifer. The water-level response data may be analyzed in two ways. The time variation of the water-level response in any one well can be analyzed using one set of type curves, or the water-level responses measured at the same time but in observation wells at different distances from the control well can be analyzed using another set of type curves.

NOTE 1—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself assure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.

4.2 *Solution*—Hantush and Jacob (**2**)[3] give two mathematically equivalent expressions for the solution which can be written as follows:

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{1}{z} exp\left(-z - \frac{r^2}{4B^2 z}\right) dz \tag{1}$$

where *z* is the variable of integration and

$$s = \frac{Q}{4\pi T}\left[2K_0\left(\frac{r}{B}\right) - \int_{\frac{r^2}{4B^2 u}}^\infty \frac{1}{z} exp\left(-z - \frac{r^2}{4B^2 z}\right) dz\right] \tag{2}$$

where:

$$u = \frac{r^2 S}{4Tt} \tag{3}$$

$$B^2 = \frac{Tb'}{K'} \tag{4}$$

4.2.1 Because a closed-form expression of the integrals that appear in Eq 1 or Eq 2 are not known, Hantush and Jacob developed equivalent expressions that involve infinite series that can be numerically evaluated. The infinite series for Eq 1 converges more rapidly for early times and the infinite series for Eq 2 converges more rapidly for late times.

4.2.2 Hantush (**3**) expressed Eq 1 and Eq 2 as follows:

$$s = \frac{Q}{4\pi T} W\left(u, \frac{r}{B}\right) \tag{5}$$

where $W\left(u, \frac{r}{B}\right)$ was called the well function for leaky systems. Hantush tabulated values of this function for a practical range of the parameters *u* and $\frac{r}{B}$.

4.2.3 Cooper (**4**) opted to express the Hantush-Jacob solution in the following form:

$$s = \frac{Q}{4\pi T} L(u, v) \tag{6}$$

[3] The boldface numbers in parentheses refer to a list of references at the end of this ~~test method.~~ practice.

where Cooper's v = Hantush's $\frac{r}{2B}$

or

$$v = \frac{r}{2B} = \frac{r}{2\sqrt{\frac{Tb'}{K'}}} \tag{7}$$

4.2.4 Cooper prepared two families of type curves. One set of Cooper's curves allow the head changes as a function of time at a fixed distance to be analyzed for the aquifer parameters, and the other set of curves allow the head changes at different distances at some fixed time to be analyzed.

5. Significance and Use

5.1 *Assumptions:*

5.1.1 The control well discharges at a constant rate, *Q*.

5.1.2 The control well is of infinitesimal diameter and fully penetrates the aquifer.

5.1.3 The aquifer is homogeneous, isotropic, and areally extensive.

5.1.4 The aquifer remains saturated (that is, water level does not decline below the top of the aquifer).

5.1.5 The aquifer is overlain, or underlain, everywhere by a confining bed having a uniform hydraulic conductivity and thickness. It is assumed that there is no change of water storage in this confining bed and that the hydraulic gradient across this bed changes instantaneously with a change in head in the aquifer. This confining bed is bounded on the distal side by a uniform head source where the head does not change with time.

5.1.6 The other confining bed is impermeable.

5.1.7 Leakage into the aquifer is vertical and proportional to the drawdown, and flow in the aquifer is strictly horizontal.

5.1.8 Flow in the aquifer is two-dimensional and radial in the horizontal plane.

5.2 The geometry of the well and aquifer system is shown in Fig. 1.

5.3 *Implications of Assumptions:*

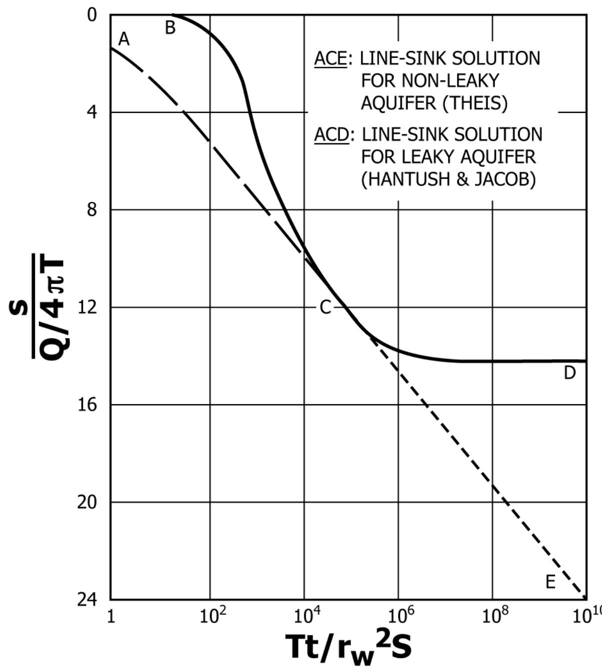
5.3.1 Paragraph 5.1.1 indicates that the discharge from the control well is at a constant rate. Section 8.1 of Test Method D4050 discusses the variation from a strictly constant rate that is acceptable. A continuous trend in the change of the discharge rate could result in misinterpretation of the water-level change data unless taken into consideration.

5.3.2 The leaky confining bed problem considered by the Hantush-Jacob solution requires that the control well has an infinitesimal diameter and has no storage. Abdul Khader and Ramadurgaiah (**5**) developed graphs of a solution for the drawdowns in a large-diameter control well discharging at a constant rate from an aquifer confined by a leaky confining bed. Fig. 2 (Fig. 3 of Abdul Khader and Ramadurgaiah (**5**)) gives a graph showing variation of dimensionless drawdown with dimensionless time in the control well assuming the aquifer storage coefficient, $S = 10^{-3}$, and the leakage parameter, $\frac{r_w}{B}$ Note that at early dimensionless times the curve for a large-diameter well in a non-leaky aquifer (BCE) and in a leaky aquifer (BCD) are coincident. At later dimensionless times, the curve for a large diameter well in a leaky aquifer coalesces with the curve for an infinitesimal diameter well (ACD) in a leaky aquifer. They coalesce about one logarithmic cycle of dimensionless time before the drawdown becomes sensibly constant. For a value of r_w/B smaller than 10^{-3}, the constant drawdown (*D*) would occur at a greater value of dimensionless drawdown and there would be a longer period during which well-bore storage effects are negligible (the period where ACD and BCD are coincident) before a steady drawdown is reached.

For values of $\frac{r_w}{B}$ greater than 10^{-3}, the constant drawdown (*D*) would occur at a smaller value of drawdown and there would be a shorter period of dimensionless time during which well-storage effects are negligible (the period where ACD and BCD are coincident) before a steady drawdown is reached. Abdul Khader and Ramadurgaiah (**5**)present graphs of dimensionless time versus dimensionless drawdown in a discharging control well for values of $S = 10^{-1}$, 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5} and $r_w/B = 10^{-2}$, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, and 0. These graphs can be used in an analysis prior to the aquifer test making use of estimates of the hydraulic properties to estimate the time period during which well-bore storage effects in the control well probably will mask other effects and the drawdowns would not fit the Hantush-Jacob solution.

5.3.2.1 The time needed for the effects of control-well bore storage to diminish enough that drawdowns in observation wells should fit the Hantush-Jacob solution is less clear. But the time adopted for when drawdowns in the discharging control well are no longer dominated by well-bore storage affects probably should be the minimum estimate of the time to adopt for observation well data.

5.3.3 The assumption that the aquifer is bounded, above or below, by a leaky layer on one side and a nonleaky layer on the other side is not likely to be entirely satisfied in the field. Neuman and Witherspoon (**6**, p. 1285) have pointed out that because the Hantush-Jacob formulation uses water-level change data only from the aquifer being pumped (or recharged) it can not be used to distinguish whether the leaking beds are above or below (or from both sides) of the aquifer. Hantush (**7**) presents a refinement that allows the parameters determined by the aquifer test analysis to be interpreted as composite parameters that reflect the combined



Curve *BCE*: Solution for well of large diameter in a non-leaky aquifer (Papadopulos and Cooper (6)).
Curve *BCD*: Solution for well of large diameter in a leaky aquifer (Abdul Khader and Ramadurgaiah).

FIG. 2 Time—Drawdown Variation in the Control Well for $S = \delta = 10^{-3}$ (from Abdul Khader and Ramadurgaiah (5))

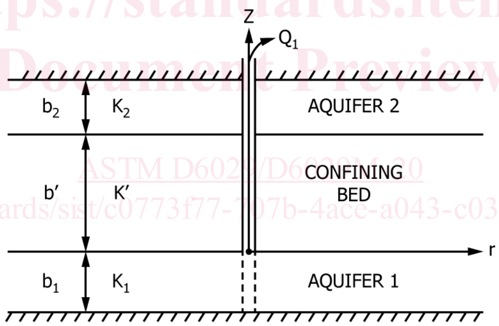


FIG. 3 Schematic Diagram of Two-Aquifer System

effects of overlying and underlying confined beds. Neuman and Witherspoon (6) describe a method to estimate the hydraulic properties of a confining layer by using the head changes in that layer.

5.3.4 The Hantush-Jacob theoretical development requires that the leakage into the aquifer is proportional to the drawdown, and that the drawdown does not vary in the vertical in the aquifer. These requirements are sometimes described by stating that the flow in the confining beds is essentially vertical and in the aquifer is essentially horizontal. Hantush's (8) analysis of an aquifer bounded only by one leaky confining bed suggested that this approximation is acceptably accurate wherever

$$\frac{K}{K'} > 100\,\frac{b}{b'} \tag{8}$$

5.3.5 The Hantush-Jacob method requires that there is no change in water storage in the leaky confining bed. Weeks (9) states that if the "leaky" confining bed is thin and relatively permeable and incompressible, the solution of Hantush and Jacob (2) will apply, whereas the solution of Hantush (7), which is described in Test Method D6028, that considers storage in confining beds will apply in most cases if one confining bed is thick, of low permeability, and highly compressible. For the case where one layer confining the aquifer is sensibly impermeable, and the other confining bed is leaky and bounded on the distal side by a layer in which the head is constant it follows from Hantush (7) that when time, t, satisfies

$$t > \frac{5(b')^2 S'_s}{K'} \tag{9}$$

the drawdowns in the aquifer will be described by the equation

$$s = \frac{Q}{4\pi T} W\left(u\delta, r \sqrt{\frac{K'}{Tb'}} \right) \quad (10)$$

where

$$\delta = 1 + \frac{S'}{3S} \quad (11)$$

Note that in Hantush's (7) solution, the term

$$u\delta = u\left(1+\frac{S'}{3S}\right) = \frac{r^2 S}{4Tt}\left(1+\frac{S'}{3S}\right) = \frac{r^2}{4Tt}\left(S+\frac{S'}{3}\right) \quad (12)$$

appears instead of the expression given for u in Eq 3, namely

$$u = \frac{r^2 S}{4Tt} \quad (13)$$

The implication being from Hantush (7) that after the time criterion given by Eq 9 is satisfied, the apparent storage coefficient of the aquifer will include the aquifer storage coefficient and one third of the storage coefficient for the confining bed. If the storage coefficient of the confining bed is very much less than that of the aquifer, then the effect of storage in the confining bed will be very small or sensibly nil. To illustrate the use of Hantush's time criterion, suppose a confining bed is characterized by $b' = 3$ m, $K' = 0.001$ m/day, and $S'_s = 3.6 \times 10^{-6}$ m^{-1}, then the Hantush-Jacob solution Eq 10 would apply everywhere when

$$t > \frac{5(b')^2 S'_s}{K'} = \frac{5(3\ \text{m})^2 (3.6 \times 10^{-6}\ \text{m}^{-1})}{0.001\ (\text{m/day})} \quad (14)$$

or

$$t > 0.162\ \text{day} = 233\text{min} \quad (15)$$

If the vertical hydraulic conductivity of the confining bed was an order of magnitude larger, $K' = 0.01$ m/day, then the Hantush-Jacob (2) solution would apply when $t > 23$ min.

5.3.5.1 It should be noted that the Hantush (7) analysis assumes that well bore storage is negligible.

5.3.5.2 Moench (10) presents numerical results that give insight into the effects of control well storage and changes in storage in the confining bed on drawdowns in the aquifer for various parameter values. However, Moench does not offer an explicit formula for when those effects diminish enough for subsequent drawdown data to fit the Hantush-Jacob solution.

5.3.6 The assumption stated in 5.1.5, that the leaky confining bed is bounded on the other side by a uniform head source, the level of which does not change with time, was considered by Neuman and Witherspoon (11, p. 810). They considered a confined system of two aquifers separated by a confining bed as shown schematically in Fig. 3. Their analysis concluded that the drawdowns in an aquifer in response to discharging from a well in that aquifer would not be affected by the properties of the other, unpumped, aquifer for times that satisfy

$$t \le 0.1 \frac{S'_s b'^2}{K'} \quad (16)$$

6. Apparatus

6.1 Analysis of data from the field procedure (see Test Method D4050) by this test method requires that the control well and observation wells meet the requirements specified in the following paragraphs.

6.2 *Construction of Control Well*—Install the control well in the aquifer and equip with a pump capable of discharging water from the well at a constant rate for the duration of the test. Preferably, the control well should be open throughout the full thickness of the aquifer. If the control well partially penetrates the aquifer, take special precaution in the placement or design of observation wells.

6.3 *Construction and Location of Observation Wells and Piezometers*—Construct one or more observation wells or piezometers screened only in the pumped aquifer at a distance from the control well. Observation wells may be open through all or part of the thickness of the aquifer. Hantush (12, p. 350) indicates that the effects of a partially penetrating control well can be neglected for

$$r > 1.5b \sqrt{\frac{K_r}{K_z}} \quad (17)$$

where K_r and K_z are the aquifer hydraulic conductivities in the horizontal and vertical directions, respectively. If an observation well fully penetrates the aquifer, its drawdown is not affected by a partially penetrating control well and it reacts as if the control well completely penetrated the aquifer (Hantush, 12, p. 351).

7. Procedure

7.1 Pretest preparations are described in detail in Test Method D4050. The overall test procedure consists of (*1*) conducting the field procedure for withdrawal or injection well tests (described in Test Method D4050) and (*2*) analysis of the field data, which is addressed in Section 8.

8. Calculation and Interpretation of Test Data

8.1 Aquifer-test data may be plotted in two ways (Cooper (**4**), p. C51). Cooper (**4**) prepared two families of type curves that are plots of *L(u,v)* versus 1/*u*.Fig. 4 is a plot of a family of solid-type curves involving the parameter *v* (recall Eq 7, $v = \frac{r}{2}\sqrt{\frac{K'}{Tb'}}$) that are useful for a plot of drawdown versus time at some constant distance, *r*. For the other family of type curves, v^2/u (this is equal to $K't/Sb'$) there is the parameter for which type curves having different values are plotted (see Fig. 4, the dashed-line curves are the v^2/u curves). These curves are useful for a plot of drawdown versus $1/r^2$ at some constant time, *t*. Note that the parent curve of both families of curves is the Theis nonequilibrium type curve that corresponds to a nonleaky confined aquifer. Either family of type curves can be used to compute values of *T, S*, and *K'*/*b'*.

8.2 Except for a change in the notation used for the leakage coefficient, change of the equation numbers, and deletion of a small amount of text, the following description of the method of use of type curves is taken directly from Cooper (**4**, p. C51–C53).

8.2.1 To compute *T, S*, and *K'*/*b'* by use of the $v = \frac{r}{2}\sqrt{\frac{K'}{Tb'}}$ curves (solid-line type curves on Fig. 4), proceed as follows:

8.2.1.1 Plot *s* versus t/r^2 for each observation well on logarithmic graph paper having the same scale as the graph of the type curves.

8.2.1.2 Superpose this time-drawdown plot on the *v* curves and, keeping the coordinate axes of the two graphs parallel, translate the data plot to the position where the earliest data approach the limiting curve labeled *W*(*u*) and the remaining data for each well fall either between one pair of the curves labeled *v* = 2.2, *v* = 2.0, and so forth, or along one of them.

8.2.1.3 Select a convenient match point and note its coordinates (*s*, t/r^2 , *L*(*u,v*), and 1/*u*).

8.2.1.4 Determine the value of *v* that corresponds to the value of *r* for each observation well. If the later data do not lie along one of the *v*-curves, estimate the value of *v* by interpolation.

8.2.1.5 Compute the hydraulic constants of the aquifer by making appropriate substitutions in the following equations:

$$T = \frac{Q}{4\pi}\frac{L(u,v)}{s} \tag{18}$$

$$S = 4T\frac{(t/r^2)}{1/u} \tag{19}$$

and

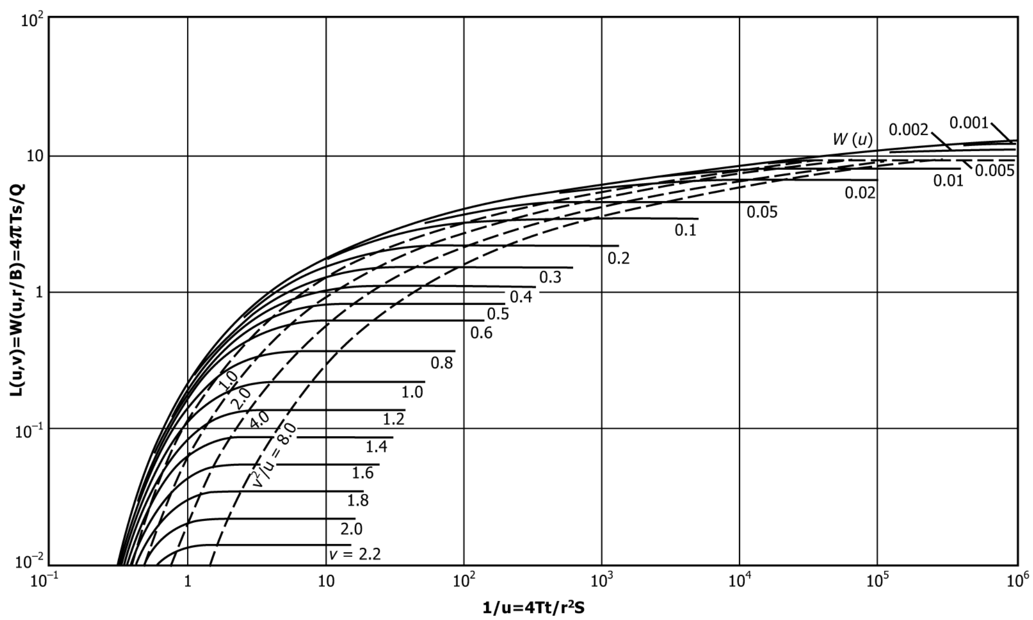


FIG. 4 Type Curve of L(u,v) versus 1/u (from Cooper (4)). The type curves for the region *v* ≤ 1.2 are based on data computed by H. H. Cooper, Jr., and Yvonne Clarke of the U.S. Geological Survey; those for the region *v* ≥ 1.4 are based on data graphically interpolated from a table computed by Hantush ((3), p. 707–711)

$$\frac{K'}{b'} = 4T\frac{v^2}{r^2} \tag{20}$$

8.2.2 To compute T, S, and K'/b' by use of $\frac{v^2}{u}$ curves (the dashed-line type curves on Fig. 4), proceed as follows:

8.2.2.1 Plot values of s, each from a different observation well but for identical values of t, versus t/r^2 on logarithmic graph paper having the same scale as the graph of the type curves.

8.2.2.2 Superpose this distance-drawdown plot on the v^2/u type curves and, keeping the coordinate axes of the two graphs parallel, translate the data plot to the position where the data fall between one pair of the type curves or along one of them.

8.2.2.3 Select a convenient match point and note its coordinates (s, t/r^2, $L(u,v)$, and $1/u$).

8.2.2.4 Determine the value of v^2/u that corresponds to the value of t at which the drawdowns occurred. If the data do not lie along one of the type curves, estimate the value of v^2/u by interpolation.

8.2.2.5 Compute the values of T and S from Eq 17 and Eq 18. If the value is to be expressed in units consistent with those for T and S in Eq 17 and Eq 18, use

$$\frac{K'}{b'} = S\frac{v^2/u}{t} \tag{21}$$

If, when superposed on the v^2/u (dashed-line) type curves, the plotted data fall in the region $v^2/u \geq 8$ and $L(u,v) \geq 10^{-2}$, steady state conditions have been reached and the method of analysis suggested by Jacob (**13**) and described by Ferris et al (**14**, p. 112–115) is applicable.

8.2.3 Type curves of the Hantush-Jacob solution in the form developed by Cooper are available in numerous publications at scales convenient for matching against data plots. Some available sources of those type curves are Cooper (**4**), Lohman (**15**), and Reed (**1**). Cooper (**4**) illustrates the type curve procedure using hypothetical field data involving drawdowns at selected times for observation points at three different distances from a control well (see Note 4).

8.2.3.1 A procedure for analyzing data for steady-state conditions is described in 8.3.

8.2.3.2 Table 1 gives a tabulation of selected values of $W(u, r/B)$. If a set of type curves are not available these data can be used to develop a type curve plot. More detailed tabulations of the Hantush-Jacob solution are available from Hantush (**3**,**12**), and Walton (**16**).

TABLE 1 Values of *W(u,r/B)* for Selected Values of *u* and *r/B* (from Hantush (12))

u	r/B 0.001	0.003	0.01	0.03	0.1	0.3	1	3
1×10^{-6}	13.0031	11.8153	9.4425	7.2471	4.8541	2.7449	0.8420	0.0695
2	12.4240	11.6716						
3	12.0581	11.5098	9.4425					
5	11.5795	11.2248	9.4413					
7	11.2570	10.9951	9.4361					
1×10^{-5}	10.9109	10.7228	9.4176					
2	10.2301	10.1332	9.2961	7.2471				
3	9.6288	9.7635	9.1499	7.2470				
5	9.3213	9.2618	8.8827	7.2450				
7	8.9863	8.9580	8.6625	7.2371				
1×10^{-4}	8.6308	8.6109	8.3983	7.2122				
2	7.9390	7.9290	7.8192	7.0685				
3	7.5340	7.5274	7.4534	6.9068	4.8541			
5	7.0237	7.0197	6.9750	6.6219	4.8530			
7	6.6876	6.6848	6.6527	6.3923	4.8478			
1×10^{-3}	6.3313	6.3293	6.3069	6.1202	4.8292			
2	5.6393	5.6383	5.6271	5.5314	4.7079	2.7449		
3	5.2348	5.2342	5.2267	5.1627	4.5622	2.7448		
5	4.7260	4.7256	4.7212	4.6829	4.2960	2.7428		
7	4.3916	4.3913	4.3882	4.3609	4.0771	2.7350		
1×10^{-2}	4.0379	4.0377	4.0356	4.0167	3.8150	2.7104		
2	3.3547	3.3546	3.3536	3.3444	3.2442	2.5688		
3	2.9591	2.9590	2.9584	2.9523	2.8873	2.4110	0.8420	
5	2.4679	2.4679	2.4675	2.4642	2.4271	2.1371	0.8409	
7	2.1508	2.1508	2.1506	2.1483	2.1232	1.9206	0.8360	
1×10^{-1}	1.8229	1.8229	1.8227	1.8213	1.8050	1.6704	0.8190	
2	1.2226	1.2226	1.2226	1.2220	1.2155	1.1602	0.7148	0.0695
3	0.9057	0.9057	0.9056	0.9053	0.9018	0.8713	0.6010	0.0694
5	0.5598	0.5598	0.5598	0.5596	0.5581	0.5453	0.4210	0.0681
7	0.3738	0.3738	0.3738	0.3737	0.3729	0.3663	0.2996	0.0639
1×10^{0}	0.2194	0.2194	0.2194	0.2193	0.2190	0.2161	0.1855	0.0534
2	0.0489	0.0489	0.0489	0.0489	0.0488	0.0485	0.0444	0.0210
3	0.0130	0.0130	0.0130	0.0130	0.0130	0.0130	0.0122	0.0071
5	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0008
7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001

NOTE 2—Commercial software is available to calculate and plot these values and curve.

8.2.3.3 Because the *v* curves represent different values of *r/B*, there is an advantage to having more than one observation well and for such wells to be at different distances from the control well so that a composite data-matching process can be used. Weeks (**9**) states of a composite data-curve matching process that:

> Such a match should be made when data from more than one observation well are available, and single values of transmissivity, storage coefficient, and other hydraulic properties are to be determined from that match. The ability or lack thereof of the data from observation wells at different distances to fit type curves having proportional distance-based parameters, will do much to confirm or deny the validity of the selected type-curve model. Moreover, the time-drawdown plot for a given observation well is affected by many extraneous factors, such as storage and inertial effects in the observation well, deviations of natural water-level fluctuations from those predicted from the pretest trend, barometric or loading effects on the water levels, and effects of local aquifer heterogeneity. Because most type-curve families include curves exhibiting a wide range of shapes, the chance of fortuitously fitting one of them is high when data for only a single well are matched. Thus, the composite data-curve matching process is useful both in confirming the validity of the selected model and in screening the data for extraneous effects.

NOTE 3—Spane and Wurstner (**17**) discussed the advantage of supplementing the type-curve plots of drawdown versus time by plots of the derivative of drawdown (with respect to an appropriate time function) versus time as an aide in selecting an aquifer interpretation model and in estimating the aquifer parameters. To apply the derivative methods requires that measurements be spaced closely enough that numerically developed time derivatives can be reasonably approximated.

8.2.4 Cooper (**4**) expressed some reservations about the use of this test method to determine values of the leakance, K'/b', for confining beds other than those that are sufficiently thin and for which the confining bed diffusivity K'/S'_s is sufficiently large. He noted that for confining beds that have a relatively large specific storage, much of the water yielded to the aquifer for a certain period of time would be derived from storage in the confining bed. For these reasons, the values of leakance obtained by this test method should be scrutinized considering independent geologic and hydrologic information.

NOTE 4—Following is an application of the type-curve method that Cooper (**4**) presented using "postulated" measurements of water-level drawdowns in observation wells at 100, 500, and 1000 ft from a well being pumped at a constant rate of 1000 gal/min for 1000 min. The aquifer in which the pumped well is screened is confined by a thin bed of materials whose lithologic character suggests that its ability to transmit water vertically through it may be an important factor that should be estimated. Fig. 5 is a log-log plot of the postulated drawdown data for each observation well plotted against values of t/r^2. Cooper's hypothetical "data" plot is superposed on a plot of the type curves (see Fig. 4) of $L(u,v)$ versus $1/u$.

8.2.4.1 For convenience, a match point is selected on the type-curve plot where $L(u,v) = 1.0$ and $1/u = 1.0$. For that choice, the corresponding point on the data plot gives $s = 1.15$ ft and t/r^2 of 1.87×10^{-9} (day/ft^2). Substitution of those values into Eq 17 and Eq 18 is as follows:

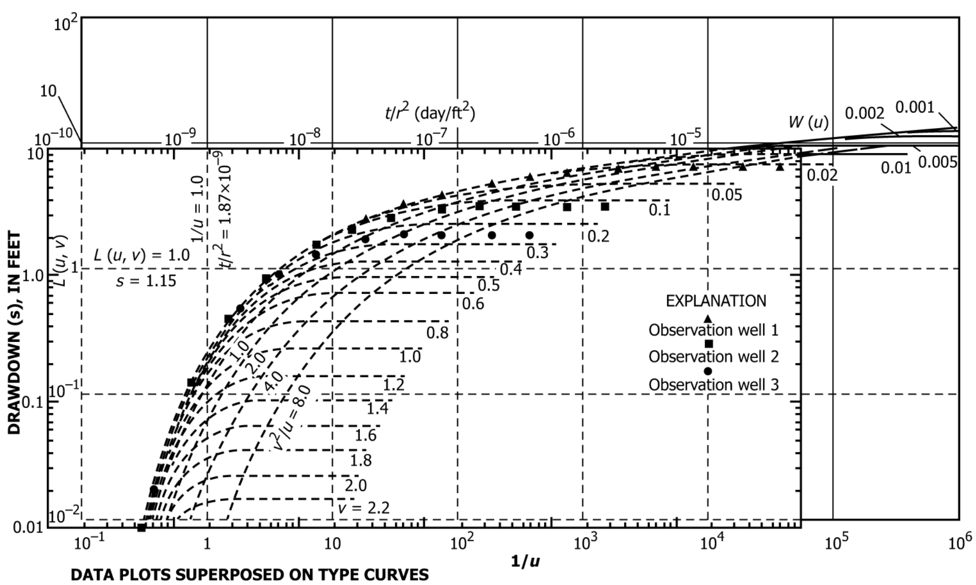


FIG. 5 Data Plot of Drawdown *s* versus Corresponding Values of t/r^2 (time/distance^2) Superposed on the Type Curves Plot of $L(u,v)$ versus $1/u$

$$T = \frac{Q}{4\pi}\frac{L(u,v)}{s}$$

$$= \frac{1000 \text{ gal/min} \times (1440\text{min/day})}{4\pi \times 7.48 \text{ (gal/ft}^3)}$$

$$\times \frac{1.0}{1.15 \text{ ft}}$$

$$= 13\,320 \text{ ft}^2/\text{day} \quad (22)$$

and

$$S = 4T\frac{(t/r^2)}{1/u} = \frac{4(13\,320 \text{ ft}^2/\text{day})\,(1.87 \times 10^{-9} \text{ day/ft}^2)}{1.0} = 0.0001 \quad (23)$$

For the match position selected, Cooper estimated that the v curves that best fit the drawdown data are $v = 0.025$ for Observation Well 1 ($r = 100$ ft), $v = 0.125$ for Observation Well 2 ($r = 500$ ft), and $v = 0.25$ for Observation Well 3 ($r = 1000$ ft). Considering the data at Observation Well 1, using Eq 19 the following is obtained:

$$\frac{K'}{b'} = 4T\frac{v^2}{r^2} = 4(13\,320 \text{ ft}^2/\text{day})\frac{(0.025)^2}{(100 \text{ ft})^2} = 3.3(10^{-3}) \text{ day}^{-1} \quad (24)$$

The same value for K'/b' would be calculated for Observation Wells 2 and 3 because note that the ratios of v/r for Observation Wells 1, 2, and 3 turn out to be

$$\frac{v}{r} = \frac{0.025}{100 \text{ ft}} = 0.00025 \text{ ft}^{-1} \quad (25)$$

$$\frac{v}{r} = \frac{0.125}{500 \text{ ft}} = 0.00025 \text{ ft}^{-1}$$

$$\frac{v}{r} = \frac{0.25}{1000 \text{ ft}} = 0.00025 \text{ ft}^{-1}$$

respectively. This exact agreement reflects that the data used for the illustration are hypothetical and idealized. That the ratio of the v's selected for the observation wells are in the same proportion as the distances to the observation wells is a desirable property to seek as Weeks (**9**) stresses because it is useful "in confirming the validity of the selected model and in screening the data for extraneous effects."

8.3 For the case where drawdowns in the vicinity of a control well have essentially reached a steady state, Jacob (**13**, p. 204) suggested a graphical type-curve method to analyze drawdowns at different distances to obtain an estimate of transmissivity and the coefficient of leakage. The steady-state drawdown is given by the following equation (Jacob, **13**,Eq 16):

$$s = \frac{Q}{2\pi T}K_0(x) \quad (26)$$

where

$$x = \frac{r}{B} = r\sqrt{\frac{K'}{Tb'}} \quad (27)$$

and where $K_0(x)$ is the zero-order modified Bessel function of the second kind.

8.3.1 The graphical type-curve procedure used to calculate aquifer test results is based on the functional relations between $K_0(x)$ and s and between x and r.

8.3.1.1 Plot values of $K_0(x)$ versus x on logarithmic paper (see Table 2 and Fig. 6). This plot is referred to as the type-curve plot.

TABLE 2 Values of the Bessel Function $K_o(x)$ for Selected Values of x (from Hantush (3, p. 704))

N	N × 10^{-2}	10^{-1}	1
1	4.7212	2.4271	0.4210
1.5	4.3159	2.0300	0.2138
2	4.0285	1.7527	0.1139
3	3.6235	1.3725	0.0347
4	3.3365	1.1145	0.0112
5	3.1142	0.9244	0.0037
6	2.9329	0.7775	...
7	2.7798	0.6605	...
8	2.6475	0.5653	...
9	2.5310	0.4867	...

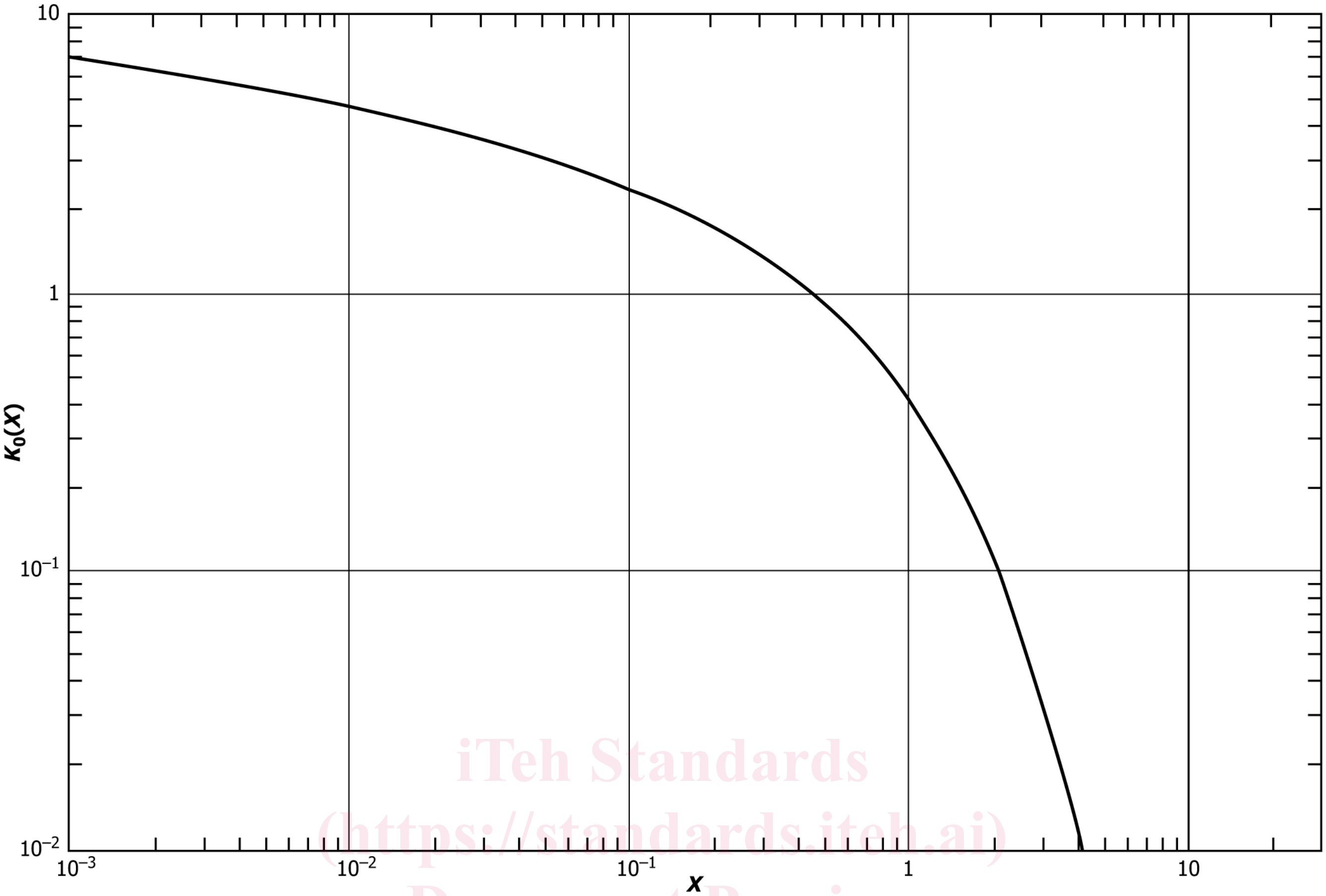


FIG. 6 Type Curve of the Bessel Function K_0, (x) as a Function of x (from Reed (1))

For convenience plot values of $K_0(x)$ on the vertical coordinate.

8.3.1.2 On logarithmic tracing paper of the same scale and size as the $K_0(x)$ versus x type curve, plot values of drawdown, s, on the vertical coordinate versus the distance r on the horizontal coordinate.

8.3.1.3 Overlay the data plot on the type-curve plot and, while holding the coordinate axes of the two plots parallel, translate the data plot to a position that gives the best match to the type curve. Select and record the values of x, $K_0(x)$, r, and s at an arbitrary match point anywhere on the overlapping part of the two matched plots. It is convenient to select a match point where x and $K_0(x)$ are integers.

8.3.1.4 Using the coordinates of the arbitrarily selected point, the transmissivity and leakance are computed from Eq 21 and Eq 22:

$$T = \frac{Q}{2\pi s} K_0(x) \tag{28}$$

$$\frac{K'}{b'} = \frac{x^2 T}{r^2} \tag{29}$$

8.3.2 Hantush (3, see p. 703) notes that where $r/B \leq 0.05$ the Bessel function $K_0\left(\frac{r}{B}\right)$ is well approximated by a logarithmic function. Thus, in that range, a semilogarithmic plot of drawdown, s, versus distance, r, with r on the logarithmic scale, will give a straight-line relationship. The slope of that line, $\Delta s/\Delta \log_{10} r$, is equal to $(2.303Q)/(2\pi T)$, from which the transmissivity, T, can be calculated. This relationship indicates that in the region $r/B \leq 0.05$, the shape of the drawdown curve is not affected by the effects of leakage. Hantush also noted that the intercept, r_0, of this straight line at the zero-drawdown axis, has the property that $B = \sqrt{\frac{Tb'}{K'}}$ from which the leakance $\frac{K'}{b'}$ can be calculated.

8.4 Qualitatively assess the test results considering the correspondence of the hydrogeologic conditions to the assumptions associated with the Hantush-Jacob (2) solution and the adequacy of the measurements of discharge and water-level changes.