Designation: ~~D5881 – 18~~ D5881 – 20

Standard Practice for (Analytical ~~Procedure)~~Procedures) Determining Transmissivity of Confined Nonleaky Aquifers by Critically Damped Well Response to Instantaneous Change in Head (Slug)[1]

This standard is issued under the fixed designation D5881; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This practice covers determination of transmissivity from the measurement of water-level response to a sudden change of water level in a well-aquifer system characterized as being critically damped or in the transition range from underdamped to overdamped. Underdamped response is characterized by oscillatory changes in water level; overdamped response is characterized by return of the water level to the initial static level in an approximately exponential manner. Overdamped response is covered in Guide D4043; underdamped response is covered in Practice D5785/D5785M, Guide D4043.

1.2 The analytical procedure in this practice is used in conjunction with Guide D4043 and the field procedure in Test Method D4044/D4044M for collection of test data.

1.3 *Limitations*—Slug tests are considered to provide an estimate of the transmissivity of an aquifer near the well screen. The method is applicable for systems in which the damping parameter, ζ, is within the range from 0.2 through 5.0. The assumptions of the method prescribe a fully penetrating well (a well open through the full thickness of the aquifer) in a confined, nonleaky aquifer.

1.4 All observed and calculated values shall conform to the guidelines for significant digits and rounding established in Practice D6026.

1.4.1 The procedures used to specify how data are collected/recorded and calculated in this standard are regarded as the industry standard. In addition, they are representative of the significant digits that should generally be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any considerations for the user's objectives; and it is common practice to increase or reduce significant digits of reported data to commensurate with these considerations. It is beyond the scope of this standard to consider significant digits used in analysis methods for engineering design.

1.5 *Units*—The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard. Reporting of ~~test~~ results in units other than SI shall not be regarded as nonconformance with this standard.

1.6 This practice offers a set of instructions for performing one or more specific operations. This document cannot replace education or experience and should be used in conjunction with professional judgment. Not all aspects of the practice may be applicable in all circumstances. This ASTM standard is not intended to represent or replace the standard of care by which the adequacy of a given professional service must be judged, nor should this document be applied without the consideration of a project's many unique aspects. The word "Standard" in the title of this document means only that the document has been approved through he ASTM consensus process.

1.7 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.8 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

[1] This test method is under the jurisdiction of ASTM Committee D18 on Soil and Rock and is the direct responsibility of Subcommittee D18.21 on Groundwater and Vadose Zone Investigations.

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***A Summary of Changes section appears at the end of this standard**

2. Referenced Documents

2.1 *ASTM Standards:*[2]

D653 Terminology Relating to Soil, Rock, and Contained Fluids

D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction

D4043 Guide for Selection of Aquifer Test Method in Determining Hydraulic Properties by Well Techniques

D4044/D4044M Test Method for (Field Procedure) for Instantaneous Change in Head (Slug) Tests for Determining Hydraulic Properties of Aquifers

D5717 Guide for Design of Ground-Water Monitoring Systems in Karst and Fractured-Rock Aquifers (Withdrawn 2005)[3]

D5785/D5785M Practice for (Analytical Procedure) for Determining Transmissivity of Confined Nonleaky Aquifers by Underdamped Well Response to Instantaneous Change in Head (Slug Test)

D6026 Practice for Using Significant Digits in Geotechnical Data

3. Terminology

3.1 *Definitions*—For definitions of common technical terms in this standard, refer to Terminology D653.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *aquifer, confined, n—in ground water*, an aquifer bounded above and below by confining beds and in which the static head is above the top of the aquifer.

3.2.2 *critically damped well response, n—in ground water*, characterized by the water level responding in a transitional range between underdamped and overdamped following a sudden change in water level.

3.2.3 *observation well, n—in ground water*, a well open to all or part of an aquifer.

3.3 *Symbols and Dimensions:*

3.3.1 T—transmissivity [L^2T^{-1}].

3.3.2 S—storage coefficient [*nd*].

3.3.3 L—static water column length above top of aquifer [L].

3.3.4 L_e—effective length of water column in a well, equal to $L_c + (r_c^2/r_s^2)(b/2)$ [L].

3.3.5 L_c—length of water column within casing [L].

3.3.6 L_s—length of water column within well screen [L].

3.3.7 g—acceleration of gravity [LT^{-2}].

3.3.8 h—hydraulic head in the aquifer [L].

3.3.9 h_o—initial hydraulic head in the aquifer [L].

3.3.10 h_s—hydraulic head in the well screen [L].

3.3.11 r_c—radius of well casing [L].

3.3.12 r_s—radius of well screen [L].

3.3.13 t—time [T].

3.3.14 t'—dimensionless time [*nd*].

3.3.15 $\hat{t}$—dimensionless time [*nd*].

3.3.16 w—water level displacement from the initial static level [L].

3.3.17 w_o—initial water level displacement [L].

3.3.18 α—dimensionless storage parameter [*nd*].

3.3.19 β—dimensionless inertial parameter [*nd*].

3.3.20 γ—damping constant [T^{-1}].

3.3.21 τ—wavelength [T].

3.3.22 ω—angular frequency [T^{-1}].

3.3.23 ζ—dimensionless damping factor [*nd*].

[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] The last approved version of this historical standard is referenced on www.astm.org.

4. Summary of Practice

4.1 This practice describes the analytical procedure for analyzing data collected during an instantaneous head (slug) test for well and aquifer response at and near critical damping. Procedures in conducting a slug test are given in Test Method D4044/D4044M. The analytical procedure consists of analyzing the response of water level in the well following the change in water level induced in the well.

4.2 *Theory*—The equations that govern the response of well to an instantaneous change in head are treated at length by Kipp (**1**).[4] The flow in the aquifer is governed by the following equation for cylindrical flow:

$$\frac{S}{T}\frac{dh}{dt} = \frac{1}{r}\frac{d}{dr}\left(r\frac{dh}{dr}\right) \quad (1)$$

where:

h = hydraulic head,
T = aquifer transmissivity, and
S = storage coefficient.

4.2.1 The initial condition is at $t = 0$ and $h = h_o$, and the outer boundary condition is as $r -$ and $h - h_o$.

4.2.1.1 An equation is given by Kipp (**1**) for the skin factor, that is, the effect of aquifer damage during drilling of the well. However, this factor is not treated by Kipp (**1**) and is not considered in this procedure.

4.2.2 The flow rate balance on the well bore relates the displacement of the water level in the well riser to the flow into the well:

$$\pi r_c^2 \frac{dw}{dt} = 2\pi r_s T \frac{dh}{dr}\Big|_{r=rs} \quad (2)$$

where:

r_c = radius of the well casing, and
w = displacement of the water level in the well from its initial position.

4.2.3 The fourth equation describing the system relating h_s and w, comes from a momentum balance equation of Bird et al (**2**) as referenced in Kipp (**1**):

$$\frac{d}{dt}\int_{-b}^{0} \pi r_s^2 pvdz = \left(-pv_2^2 + p_1 - p_2 - \rho gb\right)\pi r_s^2 \quad (3)$$

where:

v = velocity in the well screen interval,
b = aquifer thickness,
p = pressure,
ρ = fluid density,
g = gravitational acceleration, and
r_s = well screen radius.

The numerical subscripts refer to the planes described above and shown in Fig. 1. Atmospheric pressure is taken as zero.

5. Solution

5.1 Kipp (**1**) derives the following differential equation to represent for the response of the displacement of water level in the well:

$$\frac{d^2w}{dt^2} + \left(\frac{g}{L_e}\right) w = \frac{g}{\left(h_s - h_o\right)}/L_e \quad (4)$$

where:

L_e = effective water column length, defined as:

$$L_e = L + \left(r_c^2/r_s^2\right)\left(b/2\right) \quad (5)$$

where:

b = aquifer thickness with initial conditions:

$$at\ t = 0, w = w_o \quad (6)$$

$$dw/dt = w_o{}^* \quad (7)$$

$$h_s = L = h_o \quad (8)$$

[4] The boldface numbers in parentheses refer to a list of references at the end of this standard.

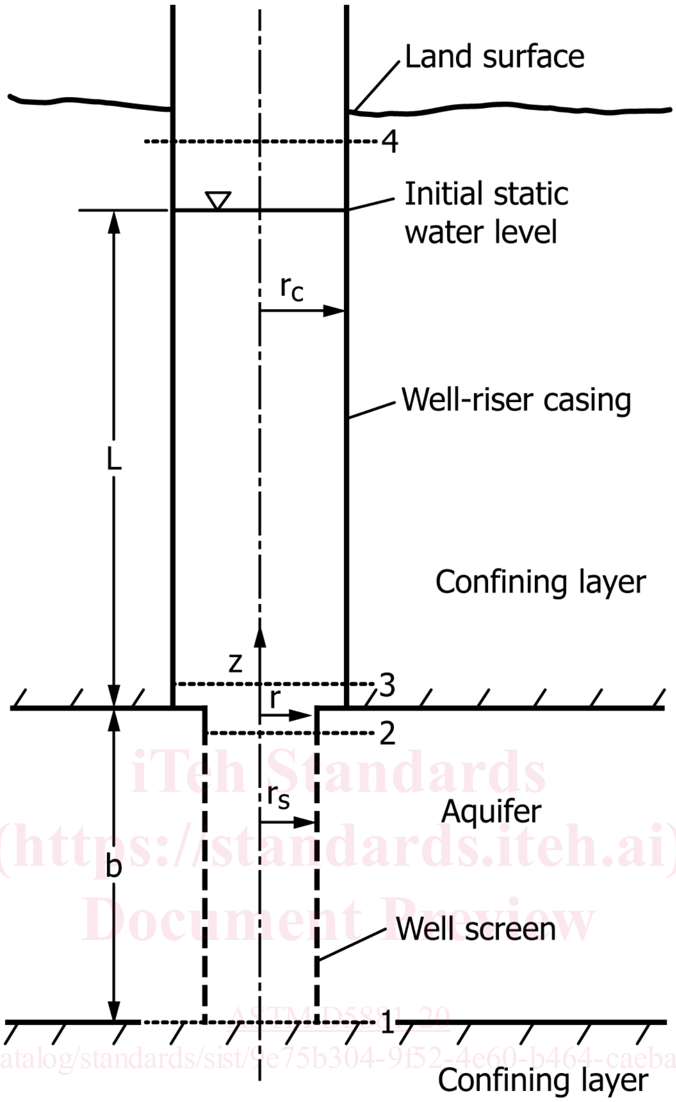

FIG. 1 Well and Aquifer Geometry from Kipp (1)

5.2 Kipp (**1**) introduces dimensionless variables and parameters in converting these equations to dimensionless form, solves the equations by Laplace transforms, and inverts the solution by a Laplace-transform-inversion algorithm.

5.2.1 The following dimensionless parameters are among those given by Kipp (**1**):
dimensionless water-level displacement:

$$w' = -w/w_o \tag{9}$$

dimensionless time:

$$t' = (tT)/(r_s{}^2S) \tag{10}$$

and:

$$\hat{t} = t'/\beta^{1/2} \tag{11}$$

dimensionless storage:

$$\alpha = (r_c{}^2)(2r_s{}^2S) \tag{12}$$

dimensionless inertial parameter:

$$\beta = (Le/g)(T/(r_s{}^2S))^2 \tag{13}$$

dimensionless skin factor:

$$\sigma = f/r_s \tag{14}$$

dimensionless frequency parameter:

$$\omega = \frac{\left[-d^2(\sigma + \frac{1}{4}\ 1n\beta) + 4\beta\right]^{1/2}}{2\beta} \tag{15}$$

dimensionless decay parameter:

$$\gamma = \frac{\alpha(\sigma + \frac{1}{4}\ 1n\beta)}{2\beta} \tag{16}$$

and dimensionless damping factor:

$$\zeta = \frac{\alpha(\sigma + \frac{1}{4}\ 1n\beta)}{2\beta^{1/2}} \tag{17}$$

5.3 For ζ less than one, the system is underdamped; for ζ greater than one, the system is overdamped. For ζ equal to one, the system is critically damped, yet the inertial effects are quite important (**1**). For ζ greater than about five, the system responds as if the inertial effects can be neglected and the solution of Cooper et al. (**3**) (given in Guide D4043) is applicable. For ζ about 0.2 or less, the approximate solution of van der Kamp (**4**) is valid (given in ~~Test Method~~ Practice D5785/D5785M). The solution of Kipp (**1**), the subject of this ~~test method,~~ practice, is applicable for the transition zone between systems that are underdamped and overdamped. Solutions are given here for ζ ranging from 0.2 to 5.0.

6. Significance and Use

6.1 The assumptions of the physical system are given as follows:

6.1.1 The aquifer is of uniform thickness, with impermeable upper and lower confining boundaries.

6.1.2 The aquifer is of constant homogeneous porosity and matrix compressibility and constant homogeneous and isotropic hydraulic conductivity.

6.1.3 The origin of the cylindrical coordinate system is taken to be on the well-bore axis at the top of the aquifer.

6.1.4 The aquifer is fully screened.

6.1.5 The well is 100 % efficient, that is, the skin factor, f, and dimensionless skin factor, σ, are zero.

6.2 The assumptions made in defining the momentum balance are as follows:

6.2.1 The average water velocity in the well is approximately constant over the well-bore section.

6.2.2 Frictional head losses from flow in the well are negligible.

6.2.3 Flow through the well screen is uniformly distributed over the entire aquifer thickness.

6.2.4 Change in momentum from the water velocity changing from radial flow through the screen to vertical flow in the well are negligible.

NOTE 1—Slug and pumping tests implicitly assume a porous medium. Fractured rock and carbonate settings may not provide meaningful data and information.

NOTE 2—The function of wells in any unconfined setting in a fractured terrain might make the determination of k problematic because the wells might only intersect tributary or subsidiary channels or conduits. The problems determining the k of a channel or conduit notwithstanding, the partial penetration of tributary channels may make a determination of a meaningful number difficult. If plots of k in carbonates and other fractured settings are made and compared, they may show no indication that there are conduits or channels present, except when with the lowest probability one maybe intersected by a borehole and can be verified, such problems are described by (**5**) Smart (1999). Additional guidance can be found in Guide D5717.

NOTE 3—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself assure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.

7. Procedure

7.1 The overall procedure consists of conducting the slug test field procedure (see Test Method D4044/D4044M) and analysis of the field data using this practice.

NOTE 4—The initial displacement of water level should not exceed 0.1 or 0.2 of the static water column in the well, the measurement of displacement should be within 1 % of the initial water-level displacement and the water-level displacement needs to be calculated independently.

8. Calculation and Interpretation of Results

8.1 Plot the normalized water-level displacement in the well versus the logarithm of time.

8.2 Prepare a set of type curves from Tables 1-10 by plotting dimensionless water level displacement, w', versus dimensionless time, $\hat{t}$, using the same scale as in plotting the observed water-level displacement.

NOTE 5—Commercially available software can be used for the calculations and plotting. The user should verify the correctness of the software and results.

8.3 Match the semilog plot of water-level displacement to the type curves by translation of the time axis.

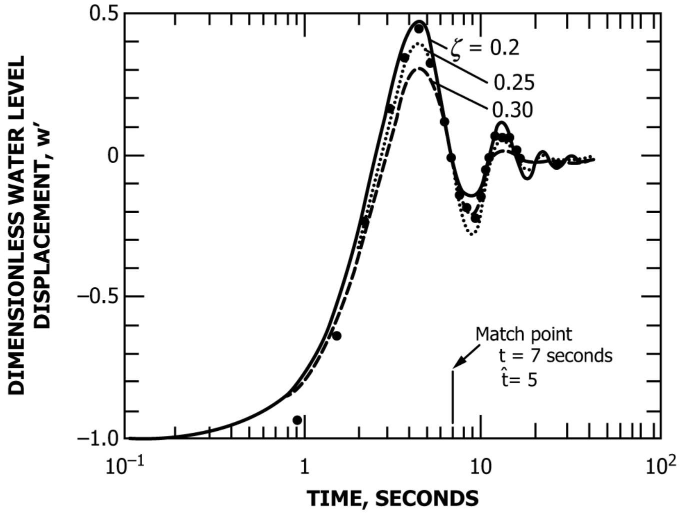

FIG. 2 Slug-Test Data Overlaid on Type Curves for Three Different Damping Factors, Modified from Kipp (1)

TABLE 1 Values of the Dimensionless Water Level Displacement, *w*', Versus Dimensionless Time, *t*, for Construction of Type Curves, ζ = 0.1 and α = 9988.1

t	*w*'	*t*	*w*'
3.162278E−02	−9.994887E−01	3.162278E + 00	7.100277E−01
3.636619E−02	−9.993281E−01	3.636619E + 00	6.204110E−01
3.952847E−02	−9.992086E−01	3.952847E + 00	4.871206E−01
4.269075E−02	−9.990793E−01	4.269075E + 00	3.138511E−01
4.743416E−02	−9.988666E−01	4.743416E + 00	2.218683E−02
5.375872E−02	−9.985483E−01	5.375872E + 00	−3.226809E−01
6.324555E−02	−9.979965E−01	6.324555E + 00	−5.191564E−01
7.115125E−02	−9.974688E−01	7.115125E + 00	−3.413663E−01
7.905694E−02	−9.968794E−01	7.905694E + 00	3.445623E−05
8.696264E−02	−9.962284E−01	8.696264E + 00	2.889492E−01
9.486833E−02	−9.955161E−01	9.486833E + 00	3.712172E−01
1.106797E−01	−9.939077E−01	1.106797E + 01	−1.758246E−02
1.264911E−01	−9.920552E−01	1.264911E + 01	−2.697976E−01
1.423025E−01	−9.899599E−01	1.423025E + 01	2.109260E−02
1.581139E−01	−9.876230E−01	1.581139E + 01	1.919487E−01
1.739253E−01	−9.850456E−01	1.739253E + 01	−2.455328E−02
1.897367E−01	−9.822293E−01	1.897367E + 00	−1.392019E−01
2.213594E−01	−9.758851E−01	2.213594E + 01	9.826209E−02
2.529822E−01	−9.686026E−01	2.529822E + 01	−7.129166E−02
2.846050E−01	−9.603946E−01	2.846050E + 01	4.976069E−02
3.162278E−01	−9.512748E−01	3.162278E + 01	−3.626029E−02
3.636619E−01	−9.359183E−01	3.636619E + 01	−9.997386E−03
3.952847E−01	−9.259452E−01	3.952847E + 01	7.200932E−03
4.269075E−01	−9.084819E−01	4.743416E + 01	5.892951E−03
4.743416E−01	−8.947298E−01	5.375872E + 01	2.737128E−03
5.375872E−01	−8.632514E−01	6.324555E + 01	−1.254582E−03
6.324555E−01	−8.135785E−01	7.115125E + 01	2.961127E−04
7.115125E−01	−7.673017E−01	7.905694E + 01	−5.757717E−05
7.905694E−01	−7.169702E−01	8.696264E + 01	−2.991356E−04
8.696264E−01	−6.629659E−01	9.486833E + 01	−1.835296E−04
9.486833E−01	−6.056883E−01	1.106797E + 02	−1.426791E−04
1.106797E + 00	−4.829810E−01	1.264911E + 02	−1.249977E−04
1.264911E + 00	−3.522848E−01	1.423025E + 02	−1.115579E−04
1.423025E + 00	−2.171309E−01	1.581139E + 02	−1.001696E−04
1.581139E + 00	−8.105198E−02	1.739253E + 02	−9.109389E−05
1.739253E + 00	5.974766E−02	1.897367E + 02	−8.347056E−05
1.897367E + 00	1.802728E−01	2.213594E + 02	−7.152232E−05
2.213594E + 00	4.066508E−01	2.529822E + 02	−6.256450E−05
2.529822E + 00	5.647406E−01	2.846050E + 02	−5.560200E−05
2.846050E + 00	6.811030E−01	...	...

8.4 From the type curve, record the value of ζ; from the match point, record the values of $\hat{t}$, and *w*' from the type curve. From the data plot, record the values of time, *t*, and water-level displacement, *w*.

8.5 Calculate the effective static water column length, L_e, from the following:

TABLE 2 Values of the Dimensionless Water Level Displacement, *w*', Versus Dimensionless Time, *t*, for Construction of Type Curves, ζ = 0.2 and α = 19976

t	*w*'	*t*	*w*'
3.162278E−02	−9.994902E−01	3.162278E + 00	4.939368E−•
3.636619E−02	−9.993263E−01	3.636619E + 00	4.349310E−•
3.952847E−02	−9.992107E−01	3.952847E + 00	3.465758E−•
4.269075E−02	−9.990815E−01	4.269075E + 00	2.343067E−•
4.743416E−02	−9.988695E−01	4.743416E + 00	5.160353E−•
5.375872E−02	−9.985520E−01	5.375872E + 00	−1.543438E−•
6.324555E−02	−9.980024E−01	6.324555E + 00	−2.671865E−•
7.115125E−02	−9.974810E−01	7.115125E + 00	−1.818502E−•
7.905694E−02	−9.968908E−01	7.905694E + 00	−2.600650E−•
8.696264E−02	−9.962437E−01	8.696264E + 00	9.764360E−•
9.486833E−02	−9.955360E−01	9.486833E + 00	1.324266E−•
1.106797E−01	−9.939399E−01	1.106797E + 01	3.871680E−•
1.264911E−01	−9.921040E−01	1.264911E + 01	−7.304361E−•
1.423025E−01	−9.900304E−01	1.423025E + 01	−3.623751E−•
1.581139E−01	−9.877207E−01	1.581139E + 01	3.430765E−•
1.739253E−01	−9.851770E−01	1.739253E + 01	−2.397516E−•
1.897367E−01	−9.824014E−01	1.897367E + 01	−2.051297E−•
2.213594E−01	−9.761622E−01	2.213594E + 01	8.187383E−•
2.529822E−01	−9.690205E−01	2.529822E + 01	−6.259136E−•
2.846050E−01	−9.609942E−01	2.846050E + 01	1.402892E−•
3.162278E−01	−9.521021E−01	3.162278E + 01	−2.331164E−•
3.636619E−01	−9.371834E−01	3.636619E + 01	−1.031248E−•
3.952847E−01	−9.262139E−01	3.952847E + 01	−7.347959E−•
4.269075E−01	−9.105352E−01	4.269075E + 01	−8.050596E−•
4.743416E−01	−8.975464E−01	4.743416E + 01	−6.352422E−•
5.375872E−01	−8.673412E−01	5.375872E + 01	−5.870822E−•
6.324555E−01	−8.201831E−01	6.324555E + 01	−5.087767E−•
7.115125E−01	−7.766091E−01	7.115125E + 01	−4.500425E−•
7.905694E−01	−7.295735E−01	7.905694E + 01	−4.046973E−•
8.696264E−01	−6.794859E−01	8.696264E + 01	−3.675505E−•
9.486833E−01	−6.267637E−01	9.486833E + 01	−3.366208E−•
1.106797E + 00	−5.151022E−01	1.106797E + 02	−2.881191E−•
1.264911E + 00	−3.979593E−01	1.264911E + 02	−2.518280E−•
1.423025E + 00	−2.786373E−01	1.423025E + 02	−2.236385E−•
1.581139E + 00	−1.602887E−01	1.581139E + 02	−2.011471E−•
1.739253E + 00	−3.860371E−02	1.739253E + 02	−1.827551E−•
1.897367E + 00	6.204784E−02	1.897367E + 02	−1.674534E−•
2.213594E + 00	2.492937E−01	2.213594E + 02	−1.434090E−•
2.529822E + 00	3.742380E−01	2.529822E + 02	−1.254123E−•
2.846050E + 00	4.694111E−01	2.846050E + 02	−1.113734E−•

$$\hat{t} = \frac{t}{(L_e/g)^{1/2}} \quad (18)$$

$$L_e = (t/\hat{t})^2 g \quad (19)$$

The effective static water column length should agree, within 20 %, with the effective length calculated from the system geometry (Eq 5).

8.6 Calculate the dimensionless inertial parameter, β, iteratively from the following expression:

$$\beta = [(\alpha \, 1n \, \beta)/8\zeta]^2 \quad (20)$$

where:

ζ = damping parameter,
α = dimensionless storage parameter as given in Eq 12.

8.7 Calculate transmissivity from the following:

$$T = [(\beta g)/L_e]^{1/2} r_s^2 S \quad (21)$$

8.7.1 Kipp **(1)** gives an example application of the method, using data from van der Kamp **(4)** for York Point well 6-2. This well has casing and screen. The well-bore (or casing) radii of 0.051 m, a water column above the aquifer of 6.5 m, an aquifer thickness of 15 m, and an independently estimated storage coefficient of 8×10^{-5}.

8.7.2 A type curve of dimensionless water-level displacement, *w*', plotted against the log of dimensionless time, tˆ, for three values of the dimensionless damping factor, ζ, was prepared. Water-level displacement was calculated using an estimated initial displacement of 3.45 cm, and plotted against the log of elapsed time since maximum initial water-level displacement of paper of the same scale as the type curve.