Designation: ~~D5270 – 96 (Reapproved 2014)~~ D5270/D5270M – 20

Standard ~~Test Method~~ Practice for (Analytical Procedures) Determining Transmissivity and Storage Coefficient of Bounded, Nonleaky, Confined Aquifers[1]

This standard is issued under the fixed designation ~~D5270;~~D5270/D5270M; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

~~1. Scope~~

~~1.1 This test method covers an analytical procedure for determining the transmissivity, storage coefficient, and possible location of boundaries for a confined aquifer with a linear boundary. This test method is used to analyze water-level or head data from one or more observation wells or piezometers during the pumping of water from a control well at a constant rate. This test method also applies to flowing artesian wells discharging at a constant rate. With appropriate changes in sign, this test method also can be used to analyze the effects of injecting water into a control well at a constant rate.~~

~~1.2 The analytical procedure in this test method is used in conjunction with the field procedure in Test Method D4050.~~

~~1.3 *Limitations*—The valid use of this test method is limited to determination of transmissivities and storage coefficients for aquifers in hydrogeologic settings with reasonable correspondence to the assumptions of the Theis nonequilibrium method (see Test Method D4106) (see 5.1), except that the aquifer is limited in areal extent by a linear boundary that fully penetrates the aquifer. The boundary is assumed to be either a constant-head boundary (equivalent to a stream or lake that hydraulically fully penetrates the aquifer) or a no-flow (impermeable) boundary (equivalent to a contact with a significantly less permeable rock unit). The Theis nonequilibrium method is described in Test Methods D4105 and D4106.~~

~~1.4 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.~~

~~1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*~~

~~2. Referenced Documents~~

~~2.1 *ASTM Standards:*~~[2]

~~D653 Terminology Relating to Soil, Rock, and Contained Fluids~~

~~D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction~~

~~D4043 Guide for Selection of Aquifer Test Method in Determining Hydraulic Properties by Well Techniques~~

~~D4050 Test Method for (Field Procedure) for Withdrawal and Injection Well Testing for Determining Hydraulic Properties of Aquifer Systems~~

~~D4105 Practice for (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Modified Theis Nonequilibrium Method~~

~~D4106 Practice for (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Theis Nonequilibrium Method~~

~~D6026 Practice for Using Significant Digits in Geotechnical Data~~

~~3. Terminology~~

~~3.1 *Definitions*—For definitions of general technical terms used within this practice, refer to Terminology D653.~~

~~3.2 *Definitions of Terms Specific to This Standard:*~~

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[1] This ~~test method~~ practice is under the jurisdiction of ASTM Committee D18 on Soil and Rock and is the direct responsibility of Subcommittee D18.21 on Groundwater and Vadose Zone Investigations.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

~~3.2.1 *constant-head boundary*—the conceptual representation of a natural feature such as a lake or river that effectively fully penetrates the aquifer and prevents water-level change in the aquifer at that location.~~

~~3.2.2 *equipotential line*—a line connecting points of equal hydraulic head. A set of such lines provides a contour map of a potentiometric surface.~~

~~3.2.3 *image well*—an imaginary well located opposite a control well such that a boundary is the perpendicular bisector of a straight line connecting the control and image wells; used to simulate the effect of a boundary on water-level changes.~~

~~3.2.4 *impermeable boundary*—the conceptual representation of a natural feature such as a fault or depositional contact that places a boundary of significantly less-permeable material laterally adjacent to an aquifer.~~

~~3.3 *Symbols and Dimensions:*~~
~~3.3.1 K_l [nd]—constant of proportionality, r_i/r_r.~~
~~3.3.2 Q [L^3T^{-1}]—discharge.~~
~~3.3.3 r [L]—radial distance from control well.~~
~~3.3.4 r_i [L]—distance from observation well to image well.~~
~~3.3.5 r_r [L]—distance from observation well to control well.~~
~~3.3.6 S [nd]—storage coefficient.~~
~~3.3.7 s [L]—drawdown.~~
~~3.3.8 s_i [L]—component of drawdown due to image well.~~
~~3.3.9 s_o [L]—drawdown at an observation well.~~
~~3.3.10 s_r [L]—component of drawdown due to control well.~~
~~3.3.11 T [L^2T^{-1}]—transmissivity.~~
~~3.3.12 t [T]—time since pumping or injection began.~~
~~3.3.13 t_o [T]—time at projection of zero drawdown.~~

~~4. Summary of Test Method~~

~~4.1 This test method prescribes two analytical procedures for analysis of a field test. This test method requires pumping water from, or injecting water into, a control well that is open to the entire thickness of a confined bounded aquifer at a constant rate and measuring the water-level response in one or more observation wells or piezometers. The water-level response in the aquifer is a function of the transmissivity and storage coefficient of the aquifer, and the location and nature of the aquifer boundary or boundaries. Drawdown or build up of the water level is analyzed as a departure from the type curve defined by the Theis nonequilibrium method (see Test Method D4106) or from straight-line segments defined by the modified Theis nonequilibrium method (see Test Method D4105).~~

~~4.2 A constant-head boundary such as a lake or stream that fully penetrates the aquifer prevents drawdown or build up of head at the boundary, as shown in Fig. 1. Likewise, an impermeable boundary provides increased drawdown or build up of head, as shown in Fig. 2. These effects are simulated by treating the aquifer as if it were infinite in extent and introducing an imaginary well or "image well" on the opposite side of the boundary a distance equal to the distance of the control well from the boundary. A line between the control well and the image well is perpendicular to the boundary. If the boundary is a constant-head boundary, the flux from the image well is opposite in sign from that of the control well; for example, the image of a discharging control well is an injection well, whereas the image of an injecting well is a discharging well. If the boundary is an impermeable boundary, the flux from the image well has the same sign as that from the control well. Therefore, the image of a discharging well across an impermeable boundary is a discharging well. Because the effects are symmetrical, only discharging control wells will be described in the remainder of this test method, but this test method is equally applicable, with the appropriate change in sign, to control wells into which water is injected.~~

~~4.3 *Solution*—The solution given by Theis (**3**)[3] can be expressed as follows:~~

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-y}}{y}\,dy \tag{1}$$

~~and:~~

$$u = \frac{r^2 S}{4Tt} \tag{2}$$

~~where:~~

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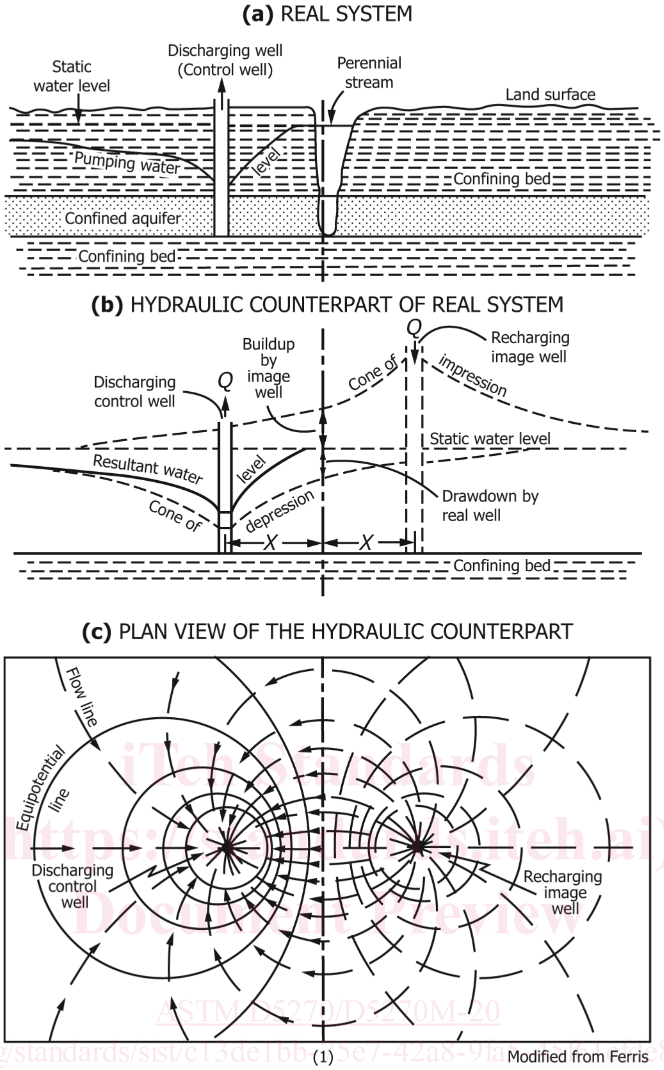
[3] The boldface numbers in parentheses refer to a list of references at the end of this standard.



NOTE 1—Modified from Ferris and others (1) and Heath (2).[3]

FIG. 1 Diagram Showing Constant-Head Boundary

$$\int_u^\infty \frac{e^{-y}}{y}\,dy = W(u) \tag{3}$$

$$= -0.577216 - \log_e u + u - \frac{u^2}{2!2} + \frac{u^3}{3!3} - \frac{u^4}{4!4} + \ldots$$

~~4.4 According to the principle of superposition, the drawdown at any point in the aquifer is the sum of the drawdown due to the real and image wells (3) and (4):~~

$$s_o = s_r \pm s_i \tag{4}$$

~~Equation (5) can be rewritten as follows:~~

$$s_o = \frac{Q}{4\pi T}\left[W(u_r) \pm W(u_i)\right] = \frac{Q}{4\pi T}\sum W(u) \tag{5}$$

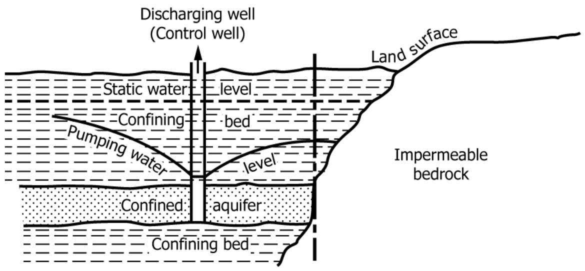
~~where:~~

$$u_r = \frac{r_r{}^2 S}{4Tt}, u_i = \frac{r_i{}^2 S}{4Tt} \tag{6}$$

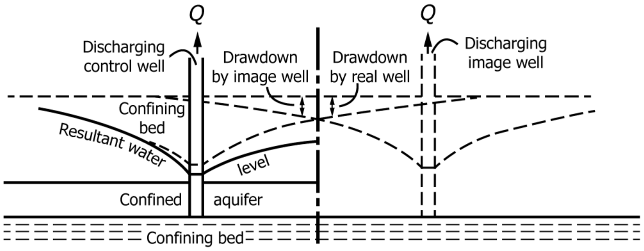
~~so that:~~

$$u_i = \left(\frac{r_i}{r_r}\right)^2 u_r, u_i = K_I{}^2 u_r \tag{7}$$

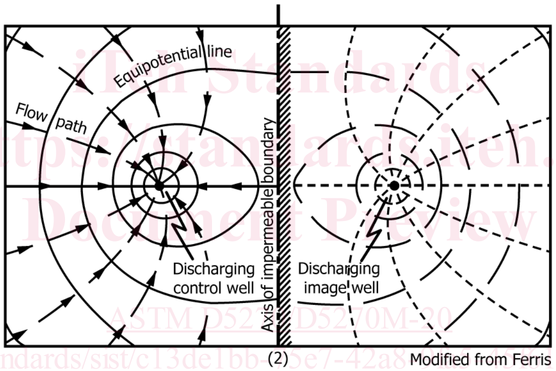
(a) REAL SYSTEM



(b) HYDRAULIC COUNTERPART OF REAL SYSTEM



(c) PLAN VIEW OF THE HYDRAULIC COUNTERPART



NOTE 1—Modified from Ferris and others (**1**) and Heath (**2**).

FIG. 2 Diagram Showing Impermeable Boundary

~~where:~~

$$K_I = \frac{r_i}{r_r} \tag{8}$$

~~NOTE 1—K_I is a constant of proportionality between the radii, not to be confused with hydraulic conductivity.~~

~~5. Significance and Use~~

~~5.1 *Assumptions:*~~

~~5.1.1 The well discharges at a constant rate.~~

~~5.1.2 Well is of infinitesimal diameter and is open through the full thickness of the aquifer.~~

~~5.1.3 The nonleaky confined aquifer is homogeneous, isotropic, and areally extensive except where limited by linear boundaries.~~

~~5.1.4 Discharge from the well is derived initially from storage in the aquifer; later, movement of water may be induced from a constant-head boundary into the aquifer.~~

~~5.1.5 The geometry of the assumed aquifer and well are shown in Fig. 1 or Fig. 2.~~

~~5.1.6 Boundaries are vertical planes, infinite in length that fully penetrate the aquifer. No water is yielded to the aquifer by impermeable boundaries, whereas recharging boundaries are in perfect hydraulic connection with the aquifer.~~

~~5.1.7 Observation wells represent the head in the aquifer; that is, the effects of wellbore storage in the observation wells are negligible.~~

~~5.2 *Implications of Assumptions:*~~

~~5.2.1 Implicit in the assumptions are the conditions of a fully-penetrating control well and observation wells of infinitesimal diameter in a confined aquifer. Under certain conditions, aquifer tests can be successfully analyzed when the control well is open to only part of the aquifer or contains a significant volume of water or when the test is conducted in an unconfined aquifer. These conditions are discussed in more detail in Test Method D4105.~~

~~5.2.2 In cases in which this test method is used to locate an unknown boundary, a minimum of three observation wells is needed. If only two observation wells are available, two possible locations of the boundary are defined, and if only one observation well is used, a circle describing all possible locations of the image well is defined.~~

~~5.2.3 The effects of a constant-head boundary are often indistinguishable from the effects of a leaky, confined aquifer. Therefore, care must be taken to ensure that a correct conceptual model of the system has been created prior to analyzing the test. See Guide D4043.~~

~~5.3 Practice D3740 provides evaluation factors for the activities in this standard.~~

~~NOTE 2—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself assure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.~~

~~6. Apparatus~~

~~6.1 Analysis of the data from the field procedure (see Test Method D4050) by this test method requires that the control well and observation wells meet the requirements specified in the following subsections.~~

~~6.2 *Construction of Control Well*—Install the control well in the aquifer and equip with a pump capable of discharging water from the well at a constant rate for the duration of the test. Preferably, the control well should be open throughout the full thickness of the aquifer. If the control well partially penetrates the aquifer, take special precautions in the placement or design of observation wells (see 5.2.1).~~

~~6.3 *Construction of Observation Wells and Piezometers*—Construct one or more observation wells or piezometers at specified distances from the control well.~~

~~6.4 *Location of Observation Wells and Piezometers* —Wells may be located at any distance from the control well within the area of influence of pumping. However, if vertical flow components are expected to be significant near the control well and if partially penetrating observation wells are to be used, the observation wells should be located at a distance beyond the effect of vertical flow components. If the aquifer is unconfined, constraints are imposed on the distance to partially penetrating observation wells and on the validity of early time measurements (see Test Method D4106).~~

~~NOTE 3—To ensure that the effects of the boundary may be observed during the tests, some of the wells should be located along lines parallel to the suspected boundary, no farther from the boundary than the control well.~~

~~7. Procedure~~

~~7.1 The general procedure consists of conducting the field procedure for withdrawal or injection wells tests (see Test Method D4050) and analyzing the field data, as addressed in this test method. Record information in accordance with Practice D6026.~~

~~7.2 Analysis of the field data consists of two steps: determination of the properties of the aquifer and the nature and distance to the image well from each observation well, and determination of the location of the boundary.~~

~~7.3 Two methods of analysis can be used to determine the aquifer properties and the nature and distance to the image well. One method is based on the Theis nonequilibrium method; the other method is based on the modified Theis nonequilibrium method.~~

~~7.3.1 *Theis Nonequilibrium Method*—Expressions in Eq 5-8 are used to generate a family of curves of $1/u_r$ versus $\sum W(u)$ for values of K_l for recharging and discharging image wells as shown in Fig. 3 (**4**).Table 1 gives values of $W(u)$ versus $1/u$. This table may be used to create a table of $\sum W(u)$ versus $1/u$ for each value of K_l by picking values for $W(u_r)$ and $W(u_i)$, and computing the $\sum W(u)$ for the each value of 1/u.~~

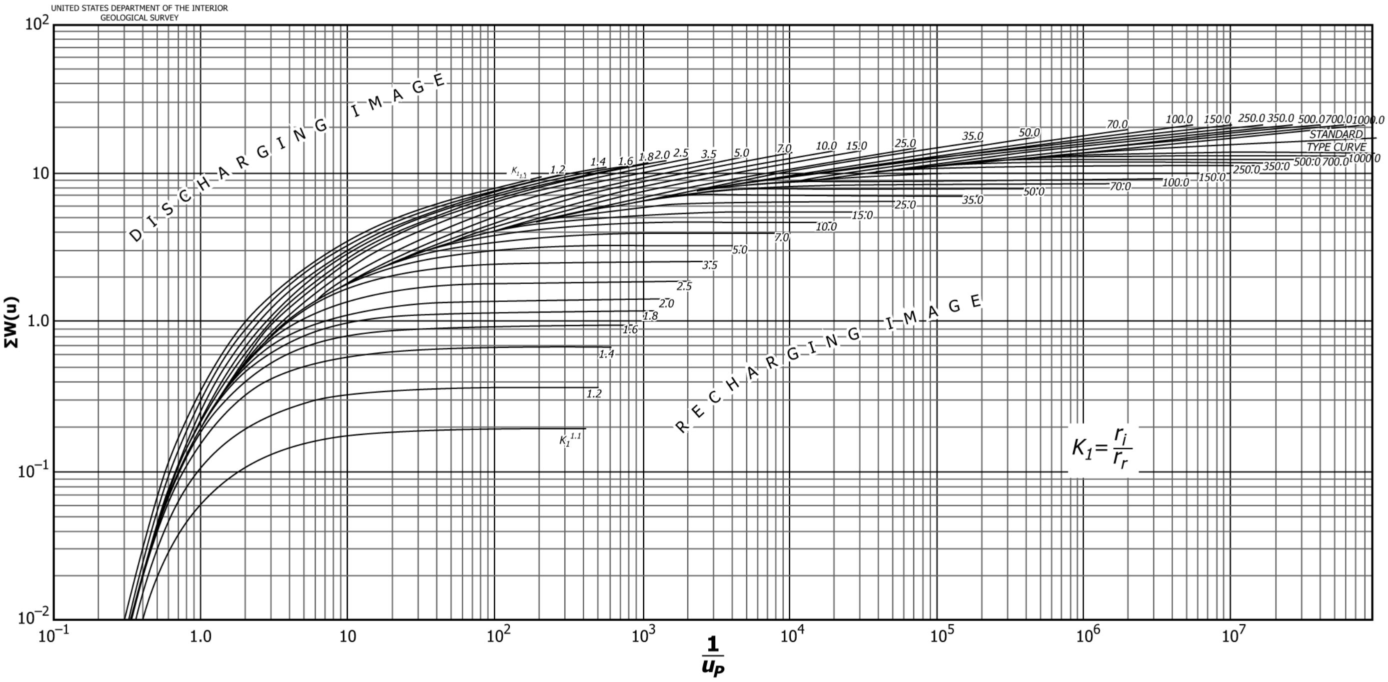
~~7.3.1.1 Transmissivity, storage coefficient, and the possible location of one or more boundaries are calculated from parameters determined from the match point and a curve selected from a family of type curves.~~

~~7.3.2 *Modified Theis Nonequilibrium Method*—The sum of the terms to the right of $\log_e u$ in Eq 3 is not significant when u becomes small, that is, equal to or less than 0.01.~~

~~NOTE 4—The limiting value for u of less than 0.01 may be excessively restrictive in some applications. The errors for small values of u, from Kruseman and DeRidder (**7**) are as follows:~~

~~Error less than, %:~~	~~1~~	~~2~~	5	~~10~~
~~For u smaller than:~~	~~0.03~~	~~0.05~~	~~0.1~~	~~0.15~~

~~7.3.2.1 The value of u decreases as time, t, increases and decreases as radial distance, r, decreases. Therefore, for large values of t and small values of r, the terms to the right of $\log_e u$ in Eq 3 may be neglected, as recognized by Theis (**3**). The modified Theis equation can then be written as follows:~~



NOTE 1—From Stallman (4).

FIG. 3 Family of Type Curves for the Solution of the Modified Theis Formula

TABLE 1 Values of Theis equation *W(u)* for values of *1/u* (6)

1/u	*1/u* × 10^{-1}	1	10	10^2	10^3	10^4	10^4	10^4
1.0	0.00000[A]	0.21938	1.82292	4.03793	6.33154	8.63322	10.93572	13.23830
1.2	0.00003	0.29255	1.98932	4.21859	6.51369	8.81553	11.11804	13.42062
1.5	0.00017	0.39841	2.19641	4.44007	6.73667	9.03866	11.34118	13.64376
2.0	0.00115	0.55977	2.46790	4.72610	7.02419	9.32632	11.62886	13.93144
2.5	0.00378	0.70238	2.68126	4.94824	7.24723	9.54945	11.85201	14.15459
3.0	0.00857	0.82889	2.85704	5.12990	7.42949	9.73177	12.03433	14.33691
3.5	0.01566	0.94208	3.00650	5.28357	7.58359	9.88592	12.18847	14.49106
4.0	0.02491	1.04428	3.13651	5.41675	7.71708	10.01944	12.32201	14.62459
5.0	0.04890	1.22265	3.35471	5.63939	7.94018	10.24258	12.54515	14.84773
6.0	0.07833	1.37451	3.53372	5.82138	8.12247	10.42490	12.72747	15.03006
7.0	0.11131	1.50661	3.68551	5.97529	8.27659	10.57905	12.88162	15.18421
8.0	0.14641	1.62342	3.81727	6.10865	8.41011	10.71258	13.01515	15.31774
9.0	0.18266	1.72811	3.93367	6.22629	8.52787	10.83036	13.13294	13.43551
1/u	*1/u* × 10^1	10^1	10^9	10^{10}	10^{11}	10^{12}	10^{13}	10^{14}
1.0	15.54087	17.84344	20.14604	22.44862	24.75121	27.05379	29.35638	31.65897
1.2	15.72320	18.02577	20.32835	22.63094	24.93353	27.23611	29.53870	31.84128
1.5	15.94634	18.24892	20.55150	22.85408	25.15668	27.45926	29.76184	32.06442
2.0	16.23401	18.53659	20.83919	23.14177	25.44435	27.74693	30.04953	32.35211
2.5	16.45715	18.75974	21.06233	23.36491	25.66750	27.97008	30.27267	32.57526
3.0	16.63948	18.94206	21.24464	23.54723	25.84982	28.15240	30.45499	32.75757
3.5	16.79362	19.09621	21.39880	23.70139	26.00397	28.30655	30.60915	32.91173
4.0	16.92715	19.22975	21.53233	23.83492	26.13750	28.44008	30.74268	33.04526
5.0	17.15030	19.45288	21.75548	24.05806	26.36054	23.66322	30.96582	33.26840
6.0	17.33263	19.63521	21.93779	24.24039	26.54297	28.84555	31.14813	33.45071
7.0	17.48677	19.78937	22.09195	24.39453	26.69711	28.99969	31.30229	33.60487
8.0	17.62030	19.92290	22.22548	24.52806	26.83064	29.13324	31.43582	33.73840
9.0	17.73808	20.04068	22.34326	24.64584	29.94843	29.25102	31.55360	33.85619

[A] Value shown as 0.00000 is nonzero but less than 0.000005.

$$s = \frac{Q}{4\pi T}\left(-0.577216 - \log_e\left(\frac{r^2S}{4Tt}\right)\right) \tag{9}$$

~~from which it has been shown by Lohman (5) that:~~

$$T = \frac{2.3Q}{4\pi\Delta s} \tag{10}$$

~~where:~~

~~Δs = the drawdown (measured or projected) over one log cycle of time.~~

~~8. Calculation and Interpretation of Results~~

~~8.1 Determine the aquifer properties and the nature and distance to the image well by either the Theis nonequilibrium method or the modified Theis method.~~

~~8.1.1 *Theis Nonequilibrium Method*—The graphical procedure for solution by the Theis nonequilibrium method is based on the relationship between $\sum W(u)$ and s, and between $1/u$ and t/r^2.~~

~~8.1.1.1 Plot the log of values of $\sum W(u)$ on the vertical coordinate and $1/u$ on the horizontal coordinate. Plot a family of curves for several values of K_l, for both recharging and discharging images. This plot (see Fig. 3) is referred to as a family of type curves. Plots of the family of type curves are contained in (**4**) and (**5**).~~

~~8.1.1.2 Plot values of the log of drawdown, s, on the vertical coordinate versus the log of t/r^2 on the horizontal coordinate. Use a different symbol for data from each observation well.~~

~~8.1.1.3 Overlay the data plot on the type curve plot and, while the coordinate axes are held parallel, shift the plot to align the data with the type curve. The data points for small values of t/r^2 should fall on or near the central (standard) type curve, and larger values of t/r^2 should fall on curves representing different values of K_l, ordinarily a different value of K_l for each observation well.~~

~~8.1.1.4 Select and record the values of $\sum W(u)$, $1/u$, s, and t/r^2 for a point (called the match point) common to both the type curve and the data plot. For convenience, the point may be selected where $\sum W(u)$ and $1/u$ are major axes, that is, 0.1, 1.0, 10.0, etc. Record a value of K_l for each observation well.~~

~~8.1.1.5 Using the match point coordinates, determine the transmissivity and storage coefficient from the following equations:~~

$$T = \frac{Q}{4\pi s} \sum W(u) \tag{11}$$

~~and:~~

$$S = 4T(t/r^2)u \tag{12}$$

~~8.1.1.6 For each observation well, determine the distance to the image well, r_i, using the following:~~

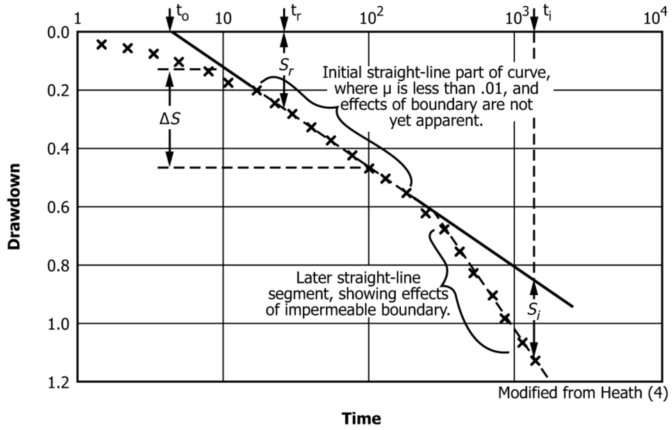
$$r_i = K_l r_r \tag{13}$$

~~8.1.2 *Modified Theis Method*—The graphical procedure for solution by the modified Theis nonequilibrium method is based on the relationship between $s$and $\log_{10} t$ using Eq 10.~~

~~8.1.2.1 Plot values of s for each observation well or piezometer on the vertical (arithmetic) coordinate and values of the log of t on the horizontal (logarithmic) coordinate. For values of t that are sufficiently large such that u is less than 0.01, the points should fall on a straight line. At larger values of t, the points will begin to diverge from the straight line due to the effects of the nearest boundary (see Fig. 4). A constant-head boundary will cause decreased drawdown, and measurements will fall above the projected straight line, whereas an impermeable boundary will cause increased drawdown and points will fall below the projected line. Note that an impermeable boundary doubles the slope of the drawdown plot.~~

~~8.1.2.2 Draw a straight line through the initial straight-line part of the data where $u < 0.01$ and the effects of boundary are not yet apparent. The drawdown over one log cycle of time (measured or projected) Δs, is used to calculate transmissivity from Eq 10. This method of calculating hydraulic properties is prescribed in more detail in Test Method D4105.~~

~~8.1.2.3 Determine the storage coefficient from the semilogarithmic plot of drawdown versus $\log_{10}$ time by a method proposed by Jacob (**8**), where:~~



Modified from Heath (4)

FIG. 4 Semilogarithmic Plot of Drawdown Versus Time Showing Effects of an Impermeable Boundary