



Designation: D4104/D4104M – 20

Standard Practice for (Analytical Procedures) Determining Transmissivity of Nonleaky Confined Aquifers by Overdamped Well Response to Instantaneous Change in Head (Slug Tests)¹

This standard is issued under the fixed designation D4104/D4104M; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This practice covers the determination of transmissivity from the measurement of force-free (overdamped) response of a well-aquifer system to a sudden change of water level in a well. Force-free response of water level in a well to a sudden change in water level is characterized by recovery to initial water level in an approximate exponential manner with negligible inertial effects.

1.2 The analytical procedure in this practice is used in conjunction with the field procedure in Test Method D4044/D4044M for collection of test data.

1.3 *Limitations*—Slug tests are considered to provide an estimate of transmissivity. Although the assumptions of this practice prescribe a fully penetrating well (a well open through the full thickness of the aquifer), the slug test is commonly conducted using a partially penetrating well. Such a practice may be acceptable for application under conditions in which the aquifer is stratified and horizontal hydraulic conductivity is much greater than vertical hydraulic conductivity. In such a case the test would be considered to be representative of the average hydraulic conductivity of the portion of the aquifer adjacent to the open interval of the well.

1.4 All observed and calculated values shall conform to the guidelines for significant digits and rounding established in Practice D6026.

1.4.1 The procedures used to specify how data are collected/recorded and calculated in the standard are regarded as the industry standard. In addition, they are representative of the significant digits that generally should be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any considerations for the user's objectives; and it is common practice to increase or reduce significant digits of reported data to be

commensurate with these considerations. It is beyond the scope of this practice to consider significant digits used in analysis methods for engineering data.

1.5 *Units*—The values stated in either SI units or inch-pound units are to be regarded separately as standard. The values stated in each system may not be exact equivalents; therefore, each system shall be used independently of the other. Combining values from the two systems may result in nonconformance with the standard. Reporting of results in units other than SI shall not be regarded as nonconformance with this standard.

1.6 This practice offers a set of instructions for performing one or more specific operations. This document cannot replace education or experience and should be used in conjunction with professional judgment. Not all aspects of the practice may be applicable in all circumstances. This ASTM standard is not intended to represent or replace the standard of care by which the adequacy of a given professional service must be judged, nor should this document be applied without the consideration of a project's many unique aspects. The word "Standard" in the title of this document means only that the document has been approved through the ASTM consensus process.

1.7 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.8 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

¹ This practice is under the jurisdiction of ASTM Committee D18 on Soil and Rock and is the direct responsibility of Subcommittee D18.21 on Groundwater and Vadose Zone Investigations.

Current edition approved June 1, 2020. Published June 2020. Originally approved in 1991. Last previous edition approved in 2017 as D4104 – 17. DOI: 10.1520/D4104_D4104M-20.

*A Summary of Changes section appears at the end of this standard

2. Referenced Documents

2.1 ASTM Standards:²

- D653** Terminology Relating to Soil, Rock, and Contained Fluids
- D3740** Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction
- D4043** Guide for Selection of Aquifer Test Method in Determining Hydraulic Properties by Well Techniques
- D4044/D4044M** Test Method for (Field Procedure) for Instantaneous Change in Head (Slug) Tests for Determining Hydraulic Properties of Aquifers
- D4750** Test Method for Determining Subsurface Liquid Levels in a Borehole or Monitoring Well (Observation Well) (Withdrawn 2010)³
- D5912** Practice for (Analytical Procedure) Determining Hydraulic Conductivity of an Unconfined Aquifer by Overdamped Well Response to Instantaneous Change in Head (Slug)
- D6026** Practice for Using Significant Digits in Geotechnical Data

3. Terminology

3.1 Definitions:

3.1.1 For definitions of common technical terms used in this standard, refer to Terminology **D653**.

3.2 The following terms used in this standard are contained in Terminology **D653** and provided here for the convenience of the user.

3.2.1 *control well*—well by which the aquifer is stressed, for example, by pumping, injection, or change of head.

3.2.2 *hydraulic conductivity*—(field aquifer tests), the volume of water at the existing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit area measured at right angles to the direction of flow.

3.2.3 *overdamped-well response*—characterized by the water level returning to the static level in an approximately exponential manner following a sudden change in water level. (See for comparison *underdamped-well response*.)

3.2.4 *slug*—a volume of water or solid object used to induce a sudden change of head in a well.

3.2.5 *head, static*—the height above a standard datum of the surface of a column of water (or other liquid) that can be supported by the static pressure at a given point.

3.3 Symbols:

- 3.3.1 J_0 [nd]—zero-order Bessel function of the first kind.
- 3.3.2 J_1 [nd]—first-order Bessel function of the first kind.
- 3.3.3 K [LT⁻¹]—hydraulic conductivity.
- 3.3.4 T [L²T⁻¹]—transmissivity.

² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

³ The last approved version of this historical standard is referenced on www.astm.org.

3.3.5 S [nd]—storage coefficient.

3.3.6 Y_0 [nd]—zero order Bessel function of the second kind.

3.3.7 Y_1 [nd]—first order Bessel function of the second kind.

3.3.8 r_c [L]—radius of control-well casing or open hole in interval where water level changes.

3.3.9 r_w [L]—radius of control well screen or open hole adjacent to water bearing unit.

3.3.10 u —variable of integration.

3.3.11 H [L]—change in head in control well.

3.3.12 H_o [L]—initial head rise (or decline) in control well.

3.3.13 t —time.

3.3.14 β — Tt/r_c^2 .

3.3.15 α — $r_w^2 S/r_c^2$.

4. Summary of Practice

4.1 This practice describes the analytical procedure for analyzing data collected during an instantaneous head (slug) test using an overdamped well. The field procedures in conducting a slug test are given in Test Method **D4044/D4044M**. The analytical procedure consists of analyzing the recovery of water level in the well following the change in water level induced in the well.

4.2 *Solution*—The solution given by Cooper et al (**1**)⁴ is as follows:

$$H = \frac{2H_o}{\pi} \int_0^\infty \left[\exp(-\beta u^2/\alpha) [J_0(ur/r_w) [uY_0(u) - 2\alpha Y_1(u)] - Y_0(ur/r_w) [uJ_0(u) - 2\alpha J_1(u)]] / \Delta(u) \right] du \quad (1)$$

where:

$$\alpha = r_w^2 S/r_c^2, \\ \beta = Tt/r_c^2,$$

and:

$$\Delta(u) = [uJ_0(u) - 2\alpha J_1(u)]^2 + [uY_0(u) - 2\alpha Y_1(u)]^2$$

NOTE 1—See Practice **D5912** and Hvorslev (**2**) Bouwer and Rice (**3**), and Bouwer (**4**).

5. Significance and Use

5.1 Assumptions of Solution of Cooper et al (**1**):

5.1.1 The head change in the control well is instantaneous at time $t = 0$.

5.1.2 Well is of finite diameter and fully penetrates the aquifer.

5.1.3 Flow in the nonleaky aquifer is radial.

NOTE 2—The exact conservation equation of Richards (**5**) with the volumetric water content can be simplified to take the form used in the solution of (**1**) with the storage coefficient, which implies several assumptions including that of constant total stresses (**6**).

5.2 Implications of Assumptions:

⁴ The boldface numbers in parentheses refer to a list of references at the end of the text.

5.2.1 The mathematical equations applied ignore inertial effects and assume the water level returns the static level in an approximate exponential manner. The geometric configuration of the well and aquifer are shown in Fig. 1.

5.2.2 Assumptions are applicable to artesian or confined conditions and fully penetrating wells. However, this practice is commonly applied to partially penetrating wells and in unconfined aquifers where it may provide estimates of hydraulic conductivity for the aquifer interval adjacent to the open interval of the well if the horizontal hydraulic conductivity is significantly greater than the vertical hydraulic conductivity.

NOTE 3—Slug and pumping tests implicitly assume a porous medium. Fractured rock and carbonate settings may not provide meaningful data and information.

5.2.3 As pointed out by Cooper et al (1) the determination of storage coefficient by this practice has questionable reliability because of the similar shape of the curves, whereas, the determination of transmissivity is not as sensitive to choosing the correct curve. However, the curve selected should not imply a storage coefficient unrealistically large or small.

NOTE 4—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself assure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.

NOTE 5—Some published literature (7, 6) have discussed the appropriateness of the slug test. These have not been universally accepted and the industry continues to use this practice.

6. Procedure

6.1 The overall procedure consists of conducting the slug test field procedure (see Test Method D4044/D4044M) and analysis of the field data, that is addressed in this practice.

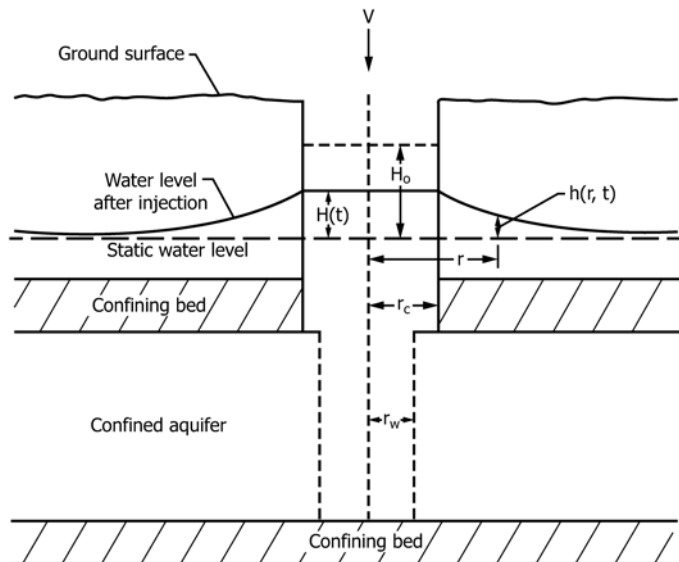


FIG. 1 Cross Section Through a Well in Which a Slug of Water is Suddenly Injected

6.2 The integral expression in the solution given in (Eq 1) cannot be evaluated analytically. A graphical solution for determination of transmissivity and coefficient of storage can be made using a set of type curves that can be drawn from the values in Table 1.

NOTE 6—Commercially available software can be used to assist in the calculations, plotting, and analyses for this practice. The user should verify the correctness of the formulas, calculations, plots, and data analyses.

TABLE 1 Values of H/H₀

From Cooper, Bredehoeft, and Papadopoulos (1)		10 ⁻¹	10 ⁻²	10 ⁻³	10 ⁻⁴	10 ⁻⁵
$\beta = Tt/r_c^2$	α					
10 ⁻³	1.00	0.9771	0.9920	0.9969	0.9985	0.9992
	2.15	0.9658	0.9876	0.9949	0.9974	0.9985
	4.64	0.9490	0.9807	0.9914	0.9954	0.9970
10 ⁻²	1.00	0.9238	0.9693	0.9853	0.9915	0.9942
	2.15	0.8860	0.9505	0.9744	0.9841	0.9883
	4.64	0.8293	0.9187	0.9545	0.9701	0.9781
10 ⁻¹	1.00	0.7460	0.8655	0.9183	0.9434	0.9572
	2.15	0.6289	0.7782	0.8538	0.8935	0.9167
	4.64	0.4782	0.6436	0.7436	0.8031	0.8410
10 ⁰	1.00	0.3117	0.4598	0.5729	0.6520	0.7080
	2.15	0.1665	0.2597	0.3543	0.4364	0.5038
	4.64	0.07415	0.1086	0.1554	0.2082	0.2620
10 ¹	7.00	0.04625	0.06204	0.08519	0.1161	0.1521
	1.00	0.03065	0.03780	0.04821	0.06355	0.08378
	1.40	0.02092	0.02414	0.02844	0.03492	0.04426
10 ²	2.15	0.01297	0.01414	0.01545	0.01723	0.01999
	3.00	0.009070	0.009615	0.01016	0.01083	0.01169
	4.64	0.005711	0.004919	0.006111	0.006319	0.006554
10 ²	7.00	0.003722	0.003809	0.003884	0.003962	0.004046
	1.00	0.002577	0.002618	0.002653	0.002688	0.002725
	2.15	0.001179	0.001187	0.001194	0.001201	0.001208
From Papadopoulos, Bredehoeft, and Cooper (8)		10 ⁻⁶	10 ⁻⁷	10 ⁻⁸	10 ⁻⁹	10 ⁻¹⁰
$\beta = Tt/r_c^2$	α					
10 ⁻³	1	0.9994	0.9996	0.9996	0.9997	0.9997
	2	0.9989	0.9992	0.9993	0.9994	0.9995
	4	0.9980	0.9985	0.9987	0.9989	0.9991
10 ⁻²	6	0.9972	0.9978	0.9982	0.9984	0.9986
	8	0.9964	0.9971	0.9976	0.9980	0.9982
	1	0.9956	0.9965	0.9971	0.9975	0.9978
10 ⁻¹	2	0.9919	0.9934	0.9944	0.9952	0.9958
	4	0.9848	0.9875	0.9894	0.9908	0.9919
	6	0.9782	0.9819	0.9846	0.9866	0.9881
10 ⁰	8	0.9718	0.9765	0.9799	0.9824	0.9844
	1	0.9655	0.9712	0.9753	0.9784	0.9807
	2	0.9361	0.9459	0.9532	0.9587	0.9631
10 ¹	4	0.8828	0.8995	0.9122	0.9220	0.9298
	6	0.8345	0.8569	0.8741	0.8875	0.8984
	8	0.7901	0.8173	0.8383	0.8550	0.8686
10 ²	1	0.7489	0.7801	0.8045	0.8240	0.8401
	2	0.5800	0.6235	0.6591	0.6889	0.7139
	3	0.4554	0.5033	0.5442	0.5792	0.6096
10 ³	4	0.3613	0.4093	0.4517	0.4891	0.5222
	5	0.2893	0.3351	0.3768	0.4146	0.4487
	6	0.2337	0.2759	0.3157	0.3525	0.3865
10 ⁴	7	0.1903	0.2285	0.2655	0.3007	0.3337
	8	0.1562	0.1903	0.2243	0.2573	0.2888
	9	0.1292	0.1594	0.1902	0.2208	0.2505
10 ⁵	1	0.1078	0.1343	0.1620	0.1900	0.2178
	2	0.02720	0.03343	0.04129	0.05071	0.06149
	3	0.01286	0.01448	0.01667	0.01956	0.02320
10 ⁶	4	0.008337	0.008898	0.009637	0.01062	0.01190
	5	0.006209	0.006470	0.006789	0.007192	0.007709
	6	0.004961	0.005111	0.005283	0.005487	0.005735
10 ⁷	8	0.003547	0.003617	0.003691	0.003773	0.003863
	1	0.002763	0.002803	0.002845	0.002890	0.002938
	2	0.001313	0.001322	0.001330	0.001339	0.001348