# Standard Test Method for Shear Modulus at Room Temperature ${ }^{1}$ 


#### Abstract

This standard is issued under the fixed designation E143; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon $(\varepsilon)$ indicates an editorial change since the last revision or reapproval.


## 1. Scope*

1.1 This test method covers the determination of shear modulus of structural materials. This test method is limited to materials in which, and to stresses at which, creep is negligible compared to the strain produced immediately upon loading. Elastic properties such as shear modulus, Young's modulus, and Poisson's ratio are not determined routinely and are generally not specified in materials specifications. Precision and bias statements for these test methods are therefore not available-
1.2 For materials that follow nonlinear elastic stress-strain behavior, the value of tangent or chord shear modulus is useful for estimating the change in torsional strain to corresponding stress for a specified stress or stress-range, respectively. Such determinations are, however, outside the scope of this standard. (See for example Ref (1).) ${ }^{2}$
1.3 Units-The values stated in ineh-poundSI units are to be regarded as standard. The values given in parentheses are mathematieal conversions to SI units that are provided for information only and are not considered No other units of measurement are included in this standard.
1.4 This standard may involve hazardous materials, operations, and equipment. This standard-does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate sety safety, health, and henvironmental practices and determine the applicability of regulatory limitations prior to use.
1.5 This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.

## 2. Referenced Documents

2.1 ASTM Standards: ${ }^{3}$<br>E6 Terminology Relating to Methods of Mechanical Testing<br>E8/E8M Test Methods for Tension Testing of Metallic Materials<br>E111 Test Method for Young's Modulus, Tangent Modulus, and Chord Modulus<br>E1012 Practice for Verification of Testing Frame and Specimen Alignment Under Tensile and Compressive Axial Force Application<br>E2624 Practice for Torque Calibration of Testing Machines

## 3. Terminology

3.1 Definitions:

[^0]3.1.1 Definitions that appear in Terminology E6 apply to this test method, including accuracy, chord modulus, creep, eccentricity, Poisson's ratio, proportional limit, resolution, shear modulus, shear strain, stress-strain curve, stress-strain diagram, tangent modulus, testing machine, torsional stress, yield strength, and Young's modulus. Terms common to meehanieal testing.
3.1.1 angle of twist (torsion test) - the angle of relative rotation measured in a plane normal to the torsion speeimen's longitudinal axis over the gatige length.
3.1.2 shear modulus, $G,\left[\mathrm{FL}^{-2}\right], n-t h e ~ r a t i o ~ o f ~ s h e a r ~ s t r e s s ~ t o ~ c o r r e s p o n d i n g ~ s h e a r ~ s t r a i n ~ b e l o w ~ t h e ~ p r o p o r t i o n a l ~ l i m i t, ~ a l s o ~ c a l l e d ~$ torsional modulus and modulus of rigidity. (See-Fig. 1.)

### 3.1.2.1 Discussion-

The value of shear modulus maycan depend on the direction in which it is measured if the material is not isotropic. Wood, many plastics, and certain metals are markedly anisotropic. Deviations from isotropy should be suspected if the shear modulus, $G$, differs from that determined by substituting independently measured values of Young's modulus, $E$, and Poisson's ratio, $\mu_{2}$ in the relation

$$
\begin{equation*}
G=\frac{E}{2(1+\mu)} \tag{1}
\end{equation*}
$$

3.1.2.2 Discussion-

In general, it is advisable, in-When reporting values of shear modulus to state-modulus, the stress range over which it is meastred.measured should be stated.
3.1.3 torque, [FL],n-a moment (of forces) that produces or tends to produce rotation or torsion.
3.1.4 torsional stress $\left[\mathrm{FL}^{-2}\right], n$ - the shear stress in a body, in a plane normal to the axis or rotation, resulting from the application of torque.
3.2 Definitions of Terms Specific to This Standard:
3.2.1 angle of twist (torsion test)-the angle of relative rotation measured in a plane normal to the torsion specimen's longitudinal axis over the gauge length.

## 4. Summary of Test Method

4.1 The cylindrical or tubular test specimen is loaded either incrementally or continuously by applying an external torque so as to cause a uniform twist within the gauge length.
4.1.1 Changes in torque and the corresponding changes in angle of twist are determined either incrementally or continuously. The appropriate slope is then calculated from the shear stress-strain curve, which may be derived under conditions of either increasing or decreasing torque (increasing from pretorque to maximum torque or decreasing from maximum torque to pretorque).


FIG. 1 Shear Stress-Strain Diagram Showing a Straight Line, Corresponding to the Shear Modulus, Between R, a Pretorque Stress, and $P$, the Proportional Limit

## 5. Significance and Use

5.1 Shear modulus is a material property useful in calculating compliance of structural materials in torsion provided they follow Hooke's law, that is, the angle of twist is proportional to the applied torque. Examples of the use of shear modulus are in the design of rotating shafts and helical compression springs.

Nome 1-For materials that follow nenlinear elastic stress-strain behavior, the value of tangent or chord shear modulus is useful for estimating the ehange in torsional strain to correspending stress for a speciffed stress or stress-range, respectively. Sueh determinations are, however, outside the seope of this standard. (See for example Ref (1).) ${ }^{3}$
5.2 The procedural steps and precision of the apparatus and the test specimens should be appropriate to the shape and the material type, since the method applies to a wide variety of materials and sizes.
5.3 Precise determination of shear modulus depends on the numerous variables that may affect such determinations.
5.3.1 These factors include characteristics of the specimen such as residual stress, concentricity, wall thickness in the case of tubes, deviation from nominal value, previous strain history, and specimen dimension.
5.3.2 Testing conditions that influence the results include axial position of the specimen, temperature and temperature variations, and maintenance of the apparatus.
5.3.3 Interpretation of data also influences results.

## 6. General Considerations

6.1 Shear modulus for a specimen of circular cross-section is given by the equation ${ }^{4}$

where:
$G=$ shear modulus of the specimen,
$T=$ torque,
$L=$ gauge length,
$J=$ polar moment of inertia of the section about its center, and
$\theta=$ angle of twist, in radians.
6.1.1 For a solid cylinder:

| $J=\pi D^{4} / 32$ | (3) |
| ---: | :--- |
| $J=\frac{\pi D^{4}}{32}$ | (3) |

where:
$D=$ diameter.
6.1.2 For a tube:

$$
\begin{equation*}
J=\frac{\pi}{32}\left(D_{0}^{4}-D_{i}^{4}\right) \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
J=\frac{\pi}{32}\left(D_{\mathrm{o}}^{4}-D_{\mathrm{i}}^{4}\right) \tag{4}
\end{equation*}
$$

[^1]where:
$B_{\overline{0}}=$ eutside diameter, and
$\frac{D_{\mathrm{o}}}{D_{\mathrm{i}}} \equiv \frac{\text { outside diameter, and }}{\text { inside diameter }}$
$D_{\mathrm{i}}=$ inside diameter.

## 7. Apparatus

7.1 Testing Machine=The torsion testing maehine, which is to be used for applying the required torque to the speeimen, shall be ealibrated for the range of torques used in the determination. Correetions may be applied for demonstrated systematie errors. The torques should be chosen sueh as to bring the error $\Delta$ machine shall conform to the requirements of $G$ inPractice E2624shear modults, dut to errors in torque $\triangle . T$, well within the required aeeuracy (see 12.3.1).
7.2 Grips-The ends of the specimen shall be gripped firmly between the jaws of a testing machine that have been designed to produce a state of uniform twist within the gauge length. In the case of tubes, closely fitting rigid plugs, such as are shown in Fig. 11 (Metal Plugs for Testing Tubular Specimens) of Test Methods E8/E8M, may be inserted in the ends to permit tightening the grips without crushing the specimen. The grips shall be such that axial alignment can be obtained and maintained in order to prevent the application of bending moments. One grip shall be free to move axially to prevent the application of axial forces.
7.3 Twist Gages_Gauges_-_The angle of twist may be measured by two pairs of lightweight but rigid arms, each pair fastened diametrically to a ring attached at three points to the section at an end of the gauge length and at least one diameter removed from the grips. The relative rotational displacement of the two sections may be measured by mechanical, optical, or electrical means; for example, the displacement of a pointer on one arm relative to a scale on the other (2), or the reflection of a light beam from mirrors or prisms attached to the arms (3). Readings should be taken for both sets of arms and averaged to eliminate errors due to bending of the specimen (see 12.3.2).

## 8. Test Specimens

### 8.1 Selection and Preparation of Specimens:

8.1.1 Specimens shall be chosen from sound, clean material.

Note 1-Slight imperfections near the surface, such as fissures that would have negligible effect in determining Young's modulus, can cause appreciable errors in shear modulus.
8.1.2 In the case of machined specimens take care to prevent changing the properties of the material at the surface of the specimen.
8.1.3 Specimens in the form of solid cylinders should be straight and of uniform diameter for a length equal to the gauge length plus two to four diameters (see 12.2.1).
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8.2 Length-The gauge length should be at least four diameters. The length of the specimen shall be sufficient for a free length between grips equal to the gauge length plus two to four diameters, unless otherwise prescribed in the product specification. However, the ratio of free length to diameter shall not be so large that helical twisting of the axis of the specimen takes place before the determination is completed.

## 9. Procedure

9.1 Measurement of Specimens-Measure diameter to give an accurate determination of average-polar moment of inertia, $J$, for the gauge length. In addition, in the case of tubular specimens, determine the average wall thickness at each end to $\pm 0.0001 \mathrm{in}$. $\pm(0.0025 \mathrm{~mm})$.to $\pm 0.0025 \mathrm{~mm}$.
9.1.1 In the ease of thin-walled tubes, a survey of thiekness variation by more sensitive deviees, streh as a pnetmatie or eleetrie gage, may be needed to determine thieknesses with the required aeetracy.

Note 2-In the case of thin-walled tubes, a survey of thickness variation by more sensitive devices, such as pneumatic or electric gauges, can help achieve the required accuracy.
9.2 Alignment-Take care to ensure axial alignment of the specimen. Proeedtres for alignment are deseribed in detail in Practice E1012. Although E1012 is for a speeimen under uniaxial loading, it provides guidanee for machine setup and fixturing for other łoading regimes.

Note 3-Procedures for alignment are described in detail in Practice E1012. Although E1012 is for a specimen under uniaxial loading, it provides guidance for machine setup and fixturing for other loading regimes.
9.3 Torque and Angle of Twist-Twist: Make simultaneous measurements of torque and angle of twist and reeord the data.
9.3.1 Record simultaneous measurements of torque and angle of twist.
9.3.2 The torques used in determining the shear modulus shall be within the verified range of torques for the torsion testing machine, as defined in Practice E2624.
9.4 Speed of Testing-Maintain the speed of testing high enough to make creep negligible.
9.5 Temperature-Record the temperature. Avoid changes in temperature during the test.

## 10. Interpretation of Results

10.1 For the determination of shear modulus it is often helpful to use a variation of the strain deviation method (4-6), frequently used for determining Young's modulus. For this purpose, a graph (Fig. 2) may be plotted of torque versus twist deviation from the following equation:

$$
\begin{equation*}
\delta=L(0-T / K) \tag{5}
\end{equation*}
$$

where:
$\delta \quad=$ wist deviation,
$t=$ gattge length,
$\theta=$ angle of twist, in radians per unit length,
$T=$ torque, and
$K=$ a constant ehosen so that $0-T / K$ is nearly constant below the proportional limit.
The range for which data are used for obtaining the shear modulus may be determined by applying some suitable eriterion of departure from a straight line, for example, the least count of the twist gage, and examining the deviation graph with the aid of a sheet of transparent paper on which three paratlel lines are drawn with the spacing between them equivalent to the least count of the twist gage.
10.2 The shear modulus may be determined by means of the deviation graph by fitting graphieally a straight line to the appropriate points. From this line the deviation inerement corresponding to a given torque inerement can be read and substituted in the following equation (from Eq 2 and Eq 5):


[^0]:    ${ }^{1}$ This test method is under the jurisdiction of ASTM Committee E28 on Mechanical Testing and is the direct responsibility of Subcommittee E28.04 on Uniaxial Testing.
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    ${ }^{2}$ The boldface numbers in parentheses refer to a list of references at the end of this standard.
    ${ }^{3}$ For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For Annual Book of ASTM Standards volume information, refer to the standard's Document Summary page on the ASTM website.

[^1]:    ${ }^{4}$ See any standard text in Mechanics of Materials.

