INTERNATIONAL

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Standard Test Method for Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio by Sonic Resonance[1]

This standard is issued under the fixed designation E1875; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This test method covers the determination of the dynamic elastic properties of elastic materials. Specimens of these materials possess specific mechanical resonant frequencies that are determined by the modulus of elasticity, mass, and geometry of the test specimen. Therefore, the dynamic elastic properties of a material can be computed if the geometry, mass, and mechanical resonant frequencies of a suitable test specimen of that material can be measured. The dynamic Young's modulus is determined using the fundamental flexural resonant frequency. The dynamic shear modulus, or modulus of rigidity, is found using the fundamental torsional resonant frequency. Dynamic Young's modulus and dynamic shear modulus are used to compute Poisson's ratio.

1.2 This test method is specifically appropriate for materials that are elastic, homogeneous, and isotropic (**1**).[2]

1.3 Materials of a composite character (particulate, whisker, or fiber reinforced) may be tested by this test method with the understanding that the character (volume fraction, size, morphology, distribution, orientation, elastic properties, and interfacial bonding) of the reinforcement in the test specimen will have a direct effect on the elastic properties. These reinforcement effects shall be considered in interpreting the test results for composites.

1.4 This test method shall not be used for determination of Poisson's ratio of anisotropic materials.

NOTE 1—For anisotropic materials, Poisson's ratio can have different values in different directions. Due to the lack of symmetry in anisotropic materials, the elasticity tensor cannot be reduced to only two independent numbers, and the simplified relation between E, G, and μ is not valid.

1.5 This test method should not be used for specimens that have cracks or voids that are major discontinuities in the specimen.

1.6 The test method should not be used when materials cannot be fabricated in a uniform rectangular or circular cross section.

1.7 An elevated-temperature furnace and cryogenic chamber are described for measuring the dynamic elastic moduli as a function of temperature from –195 °C to 1200 °C.

1.8 This test method may be modified for use in quality control. A range of acceptable resonant frequencies is determined for a specimen with a particular geometry and mass. Any specimen with a frequency response falling outside this frequency range is rejected. The actual modulus of each specimen need not be determined as long as the limits of the selected frequency range are known to include the resonant frequency that the specimen must possess if its geometry and mass are within specified tolerances.

1.9 There are material-specific ASTM standards that cover the determination of resonant frequencies and elastic properties of specific materials by sonic resonance or by impulse excitation of vibration. Test Methods C215, C623, C747, C848, C1198, C1259, and C1548 differ from this test method in several areas (for example; specimen size, dimensional tolerances, specimen preparation). The testing of these materials shall be done in compliance with these material specific standards. Where possible, the procedures, specimen specifications, and calculations are consistent with these test methods.

1.10 A separate standard, Test Method E1876, governs determination of dynamic elastic moduli by impulse excitation instead of sonic resonance.

1.11 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.12 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.13 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the*

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[1] This test method is under the jurisdiction of ASTM Committee E28 on Mechanical Testing and is the direct responsibility of Subcommittee E28.04 on Uniaxial Testing.

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[2] The boldface numbers in parentheses refer to a list of references at the end of this standard.

*A Summary of Changes section appears at the end of this standard

Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.

2. Referenced Documents

2.1 *ASTM Standards:*[3]

C215 Test Method for Fundamental Transverse, Longitudinal, and Torsional Resonant Frequencies of Concrete Specimens

C623 Test Method for Young's Modulus, Shear Modulus, and Poisson's Ratio for Glass and Glass-Ceramics by Resonance

C747 Test Method for Moduli of Elasticity and Fundamental Frequencies of Carbon and Graphite Materials by Sonic Resonance

C848 Test Method for Young's Modulus, Shear Modulus, and Poisson's Ratio For Ceramic Whitewares by Resonance

C1198 Test Method for Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio for Advanced Ceramics by Sonic Resonance

C1259 Test Method for Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio for Advanced Ceramics by Impulse Excitation of Vibration

C1548 Test Method for Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio of Refractory Materials by Impulse Excitation of Vibration

E6 Terminology Relating to Methods of Mechanical Testing

E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods

E691 Practice for Conducting an Interlaboratory Study to Determine the Precision of a Test Method

E1876 Test Method for Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio by Impulse Excitation of Vibration

3. Terminology

3.1 *Definitions:*Terms common to mechanical testing that appear in Terminology E6 and are listed in this section apply to this test method. In addition, the terms indicated temperature and specified temperature are used as defined in Terminology E6.

3.1.1 *dynamic mechanical measurement, n*—a technique in which either the modulus or damping, or both, of a substance under oscillatory applied force or displacement is measured as a function of temperature, frequency, or time, or a combination thereof.

3.1.2 *dynamic Young's modulus,* E_d [FL^{-2}], *n*—the value of the Young's modulus determined using an oscillatory applied force or displacement and in conformance with this test method.

3.1.3 *dynamic shear modulus,* G_d [FL^{-2}], *n*—the value of the shear modulus determined using an oscillatory applied force or displacement and in conformance with this test method.

3.1.4 *elastic limit* [FL^{-2}], *n*—the greatest stress that a material is capable of sustaining without permanent strain remaining upon complete release of the stress.

3.1.4.1 *Discussion*—Due to practical considerations in determining the elastic limit, measurements of strain using a small force, rather than zero force, are usually taken as the initial and final reference.

3.1.5 *modulus of elasticity* [FL^{-2}], *n*—the ratio of stress to corresponding strain below the proportional limit.

3.1.5.1 *Discussion*—The stress-strain relationships of many materials do not conform to Hooke's law throughout the elastic range, but deviate therefrom even at stresses well below the elastic limit. For such materials, the slope of either the tangent to the stress-strain curve at the origin or at a low stress, the secant drawn from the origin to any specified point on the stress-strain curve, or the chord connecting any two specified points on the stress-strain curve is usually taken to be the "modulus of elasticity." In these cases, the modulus should be designated as the "tangent modulus," the "secant modulus," or the "chord modulus," and the point or points on the stress-strain curve described. Thus, for materials where the stress-strain relationship is curvilinear rather than linear, one of the four following terms may be used:

(a) initial tangent modulus [FL^{-2}], *n*—the slope of the stress-strain curve at the origin.

(b) tangent modulus [FL^{-2}], *n*—the slope of the stress-strain curve at any specified stress or strain.

(c) secant modulus [FL^{-2}], *n*—the slope of the secant drawn from the origin to any specified point on the stress-strain curve.

(d) chord modulus [FL^{-2}], *n*—the slope of the chord drawn between any two specified points on the stress-strain curve below the elastic limit of the material.

3.1.5.2 *Discussion*—Modulus of elasticity, like stress, is expressed in force per unit of area (pounds per square inch, etc.).

3.1.6 *Poisson's ratio,* μ, *n*—the negative of the ratio of transverse strain to the corresponding axial strain resulting from an axial stress below the proportional limit of the material.

3.1.6.1 *Discussion*—Poisson's ratio can be negative for some materials, for example, a tensile transverse strain will result from a tensile axial strain.

3.1.6.2 *Discussion*—Poisson's ratio will have more than one value if the material is not isotropic.

3.1.7 *proportional limit* [FL^{-2}], *n*—the greatest stress that a material is capable of sustaining without deviation from proportionality of stress to strain (Hooke's law).

3.1.7.1 *Discussion*—Many experiments have shown that values observed for the proportional limit vary greatly with the sensitivity and accuracy of the testing equipment, eccentricity of loading, the scale to which the stress-strain diagram is plotted, and other factors. When determination of proportional

[3] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

limit is required, the procedure and the sensitivity of the test equipment should be specified.

3.1.8 *shear modulus, G* [FL^{-2}], *n*—the ratio of shear stress to corresponding shear strain below the proportional limit, also called *torsional modulus* and *modulus of rigidity*.

3.1.8.1 *Discussion*—The value of the shear modulus can depend on the direction in which it is measured if the material is not isotropic. Wood, many plastics and certain metals are markedly anisotropic. Deviations from isotropy should be suspected if the shear modulus differs from that determined by substituting independently measured values of Young's modulus, *E*, and Poisson's ratio, μ, in the relation:

$$G = \frac{E}{2(1+\mu)}$$

3.1.8.2 *Discussion*—When reporting values of shear modulus, the range of stress over which it is measured should be stated.

3.1.9 *Young's modulus, E* [FL^{-2}], *n*—the ratio of tensile or compressive stress to corresponding strain below the proportional limit of the material.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *anti-nodes, n*—two or more locations in an unconstrained slender rectangular or cylindrical specimen in resonance that have local maximum displacements.

3.2.1.1 *Discussion*—For the fundamental flexural resonance, the anti-nodes are located at the two ends and the center of the specimen.

3.2.2 *elastic, adj*—the property of a material such that an application of stress within the elastic limit of that material making up the body being stressed will cause an instantaneous and uniform deformation that will be eliminated upon removal of the stress, with the body returning instantly to its original size and shape without energy loss.

3.2.2.1 *Discussion*—Most elastic materials conform to this definition well enough to make this resonance test valid.

3.2.3 *flexural vibrations, n*—oscillations that occur in a slender rectangular or cylindrical specimen in a vertical plane normal to the length dimension.

3.2.4 *homogeneous, adj*—the condition of a specimen such that the composition and density are uniform, such that any smaller specimen taken from the original is representative of the whole.

3.2.4.1 *Discussion*—Practically, as long as the geometrical dimensions of the test specimen are large with respect to the size of individual grains, crystals, or components, the body can be considered homogeneous.

3.2.5 *isotropic, adj*—the condition of a specimen such that the values of the elastic properties are the same in all directions in the material.

3.2.5.1 *Discussion*—Materials are considered isotropic on a macroscopic scale, if they are homogeneous and there is a random distribution and orientation of phases, crystallites, and components.

3.2.6 *nodes, n*—one or more locations of a slender rectangular or cylindrical specimen in resonance that have a constant zero displacement.

3.2.6.1 *Discussion*—For the fundamental flexural resonance, the nodes are located at 0.224 *L* from each end, where *L* is the length of the specimen.

3.2.7 *resonance, n*—state of slender rectangular or cylindrical specimen driven into one of the modes of vibration described in 3.2.3 or 3.2.9 when the imposed frequency is such that the resultant displacements for a given amount of driving force are at a maximum.

3.2.7.1 *Discussion*—The resonant frequencies are natural vibration frequencies that are determined by the modulus of elasticity, mass, and dimensions of the test specimen.

3.2.8 *slender rectangular or cylindrical specimen, n*—in dynamic mechanical measurement, a specimen whose ratio of length to minimum cross-sectional dimension is at least five and should be in the range from 20 to 25.

3.2.9 *torsional vibrations, n*—oscillations that occur in each cross-sectional plane of a slender rectangular or cylindrical specimen, such that the plane twists around the length dimension axis.

4. Summary of Test Method

4.1 This test method measures the resonant frequencies of test specimens of suitable geometry by exciting them at continuously variable frequencies. Mechanical excitation of the specimen is provided through the use of a driving transducer that transforms a cyclic electrical signal into a cyclic mechanical force on the specimen. A detecting transducer senses the resulting mechanical vibrations of the specimen and transforms them into an electrical signal. The amplitude and frequency of the signal are measured by an oscilloscope or other means to detect resonance. The resonant frequencies, dimensions, and mass of the specimen are used to calculate dynamic Young's modulus and dynamic shear modulus.

5. Significance and Use

5.1 This test method has advantages in certain respects over the use of static loading systems for measuring moduli.

5.1.1 This test method is nondestructive in nature. Only minute stresses are applied to the specimen, thus minimizing the possibility of fracture.

5.1.2 The period of time during which measurement stress is applied and removed is of the order of hundreds of microseconds. With this test method it is feasible to perform measurements at elevated temperatures, where delayed elastic and creep effects would invalidate modulus of elasticity measurements calculated from static loading.

5.2 This test method is suitable for detecting whether a material meets the specifications, if cognizance is given to one important fact in materials are often sensitive to thermal history. Therefore, the thermal history of a test specimen must be considered in comparing experimental values of moduli to reference or standard values. Specimen descriptions should include any specific thermal treatments that the specimens have received.

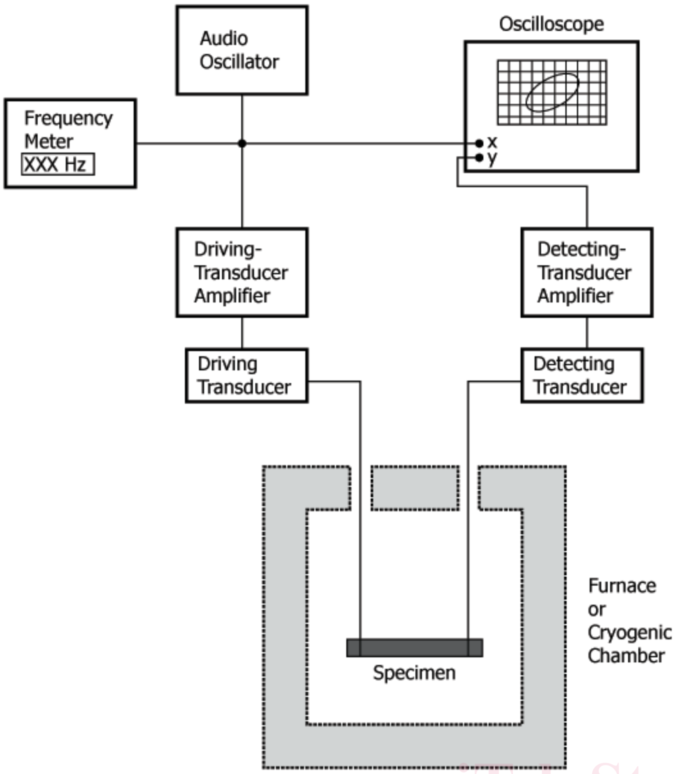


FIG. 1 Block Diagram of a Typical Test Apparatus

6. Apparatus

6.1 The test apparatus is shown in Fig. 1. It consists of a variable-frequency audio oscillator, used to generate a sinusoidal voltage, and a power amplifier and suitable driving transducer to convert the electrical signal to a mechanical driving vibration. A frequency meter, which should be digital, monitors the audio oscillator output to provide an accurate frequency determination. A suitable suspension-coupling system supports the test specimen. A dectecting transducer senses the mechanical vibration in the specimen and to convert it into an electrical signal that is passed through an amplifier and displayed on an indicating meter. The meter may be a voltmeter, microammeter, or oscilloscope. An oscilloscope should be used, because it enables the operator to positively identify resonances, including higher order harmonics, by Lissajous figure analysis. If a Lissajous figure is desired, the output of the audio oscillator should be displayed on the horizontal axis of the oscilloscope. If temperature-dependent data are desired, a suitable furnace or cryogenic chamber shall be used. Details of the equipment are as follows:

6.2 *Audio Oscillator,* having a continuously variable frequency output from about 100 Hz to at least 30 kHz. Frequency drift shall not exceed 1 Hz/min for any given setting.

6.3 *Driving-transducer Amplifier,* having a power output sufficient to ensure that the driving transducer can excite any specimen the mass of which falls within a specified range.

6.4 *Transducers*—Two transducers shall be used.

6.4.1 The driving transducer may be a speaker of the tweeter type, or a magnetic cutting head, or other similar device depending on the type of coupling chosen for use between the driving transducer and the specimen.

6.4.2 The detecting transducer may be a crystal or magnetic reluctance type of phonograph cartridge or a capacitive pickup. An electromagnetic coupling system with an attached metal foil may also be used, with due consideration for effects of the foil on the natural vibration of the test specimen. The frequency response of the detecting transducer across the frequency range of interest shall have at least a 6.5 kHz bandwidth before –3 dB power loss occurs.

6.5 *Detecting-transducer Amplifier,* in the detector circuit shall be impedance matched with the type of detecting transducer selected and shall serve as a preamplifier for the ocsilloscope.

6.6 *Oscilloscope,* any model suitable for general laboratory work.

6.7 *Frequency Meter,* should be digital, and shall be able to measure frequencies to within ±1 Hz.

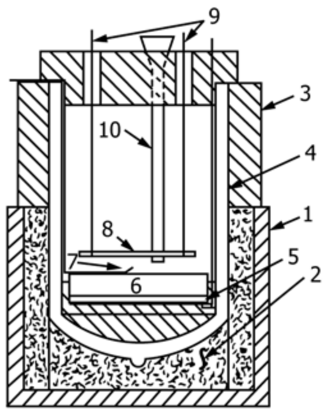
6.8 *Furnace*—For data at elevated temperature, a furnace shall be used that is capable of controlled heating and cooling. It shall have a specimen zone large enough for the specimen to be uniform in temperature within ±5 °C along its length through the range of specified temperatures.

6.8.1 An independent thermocouple should be placed in close proximity to (within 5 mm), but not touching, the center of the specimen to accurately measure temperature during heating and cooling. Ensure that the indicated temperature from the remote thermocouple and the specimen temperature do not differ.

6.9 *Cryogenic Chamber*—For data at cryogenic temperatures, the cryogenic chamber shall be capable of controlled heating and cooling, frost-free, and uniform in temperature within ±5 °C over the length of the specimen at any specified temperature. A suitable cryogenic chamber is shown in Fig. 2 (**2**).

6.9.1 An independent thermocouple should be placed in close proximity to (within 5 mm), but not touching, the center of the specimen to accurately measure temperature during heating and cooling. Ensure that the indicated temperature from the remote thermocouple and the specimen temperature do not differ.

6.10 *Specimen Suspension*—The method of specimen suspension shall be adequate for the specified temperatures and allow the specimen to vibrate without significant restriction. Thread suspension should be used for cryogenic and elevated-temperature testing. (See Fig. 1 and Fig. 3.) Common cotton thread, silica-glass fiber thread, oxidation-resistant nickel (or platinum) alloy wire, or platinum wire may be used. The specimen should be initially suspended at distances of approximately 0.1 *L* from each end. The specimen should not be suspended at its fundamental flexural node locations (0.224 *L* from each end). The suspension point distances may be adjusted experimentally to maximize the vibrational deflection and resulting signal. For torsional vibration, the axes of suspension shall be off-center from the longitudinal axis of the specimen (shown in Fig. 3).



NOTE 1—Legend:
1 = Cylindrical glass jar
2 = Glass wool
3 = Plastic foam
4 = Vacuum jar
5 = Heater disk
6 = Copper plate
7 = Thermocouple
8 = Specimen
9 = Suspension wires
10 = Fill port for liquid

FIG. 2 Detail Drawing of a Typical Cryogenic Chamber

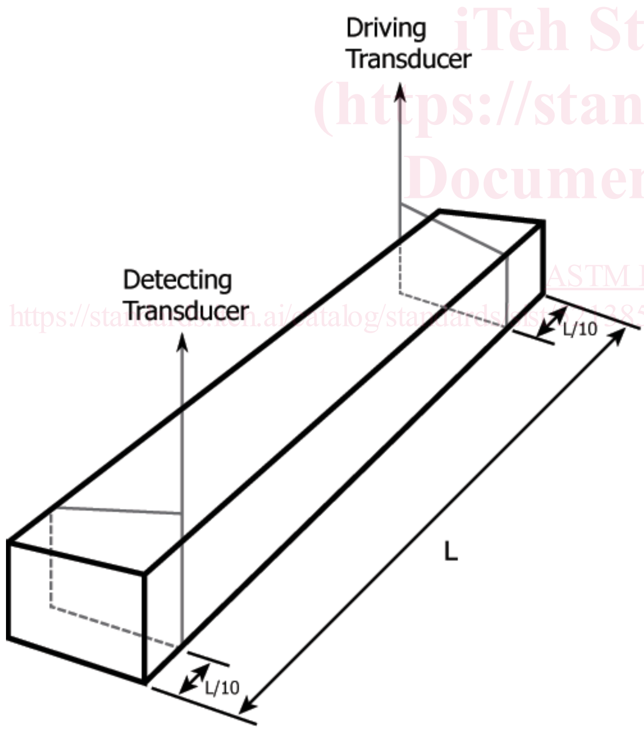


FIG. 3 Specimen Positioned for Measurement of Flexural and Torsional Resonant Frequencies Using Thread or Wire Suspension

NOTE 2—If metal wire suspension is used in the furnace, coupling characteristics will be improved if, outside the temperature zone, the wire is coupled to cotton thread, and the thread is coupled to the transducer.

6.11 *Specimen Supports*—If the specimen is supported on direct contact supports, the supports shall permit the specimen

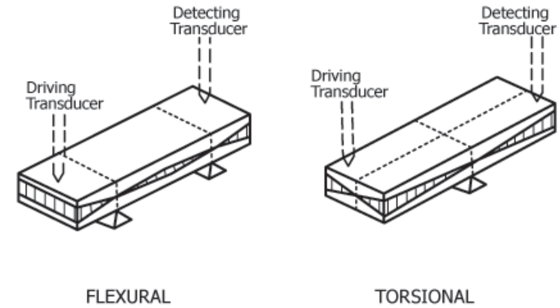


FIG. 4 Specimen Positioned for Measurement of Flexural and Torsional Resonant Frequencies Using Direct Support and Direct Contact Transducers

to oscillate without significant restriction in the desired mode. In flexural modes, the specimen should be supported at its transverse fundamental node locations (0.224 *L* from each end). In torsional modes the specimen should be supported at its center point. The supports should have minimal area in contact with the specimen and shall be cork, rubber, or similar material. In order to properly identify resonant frequencies, the transducers should be movable along the total specimen length and width. See Fig. 4. The transducer contact pressure should be consistent with good response and minimal interference with the free vibration of the specimen.

7. Test Specimen

7.1 Prepare the specimens so that they are either rectangular or circular in cross section. Either geometry may be used to measure both dynamic Young's modulus and dynamic shear modulus.

NOTE 3—Experimental difficulties in obtaining torsional resonant frequencies for a cylindrical specimen usually preclude its use in determining dynamic shear modulus, although the equations for computing dynamic shear modulus with a cylindrical specimen are both simpler and more accurate than those used with a rectangular specimen.

7.2 Select the size so that, for an estimated dynamic modulus of elasticity, the resonant frequencies measured will fall within the range of frequency response of the transducers used. A slender specimen with a ratio of length to minimum cross-sectional dimension greater than 5 and approximately 25 should be used for ease in calculation. For dynamic shear modulus measurements of rectangular specimens, a ratio of width to thickness of five should be used to minimize experimental difficulties.

NOTE 4—Resonant frequencies for a given specimen are functions of the specimen dimensions as well as its mass and moduli.

7.2.1 These specimen sizes should produce a fundamental flexural resonant frequency in the range from 1000 Hz to 10 000 Hz and a fundamental torsional resonant frequency in the range from 10 000 Hz to 30 000 Hz. Specimens shall have a minimum mass of 5 g to avoid coupling effects; any size of specimen that has a suitable length-to-cross section ratio in terms of frequency response and meets the minimum mass may be used.

NOTE 5—Maximum specimen size and mass are determined primarily by the power of the test system and physical space capabilities.

TABLE 1 Effects of Variable Error on Dynamic Modulus of Elasticity Calculation

Variable	Measurement Uncertaintly	Variable Exponent in Dynamic Modulus Equation	Calculation Uncertainty
Frequency (*f*)	0.1 %	f^2	0.2 %
Length (*L*)	0.1 %	L^3	0.3 %
Mass (*m*)	0.1 %	m	0.1 %
Width (*b*)	0.1 %	b^{-1}	0.1 %
Thickness (*t*)	0.1 %	t^{-3}	0.3 %
Diameter (*D*)	0.1 %	D^{-4}	0.4 %

7.3 All surfaces on the rectangular specimen shall be flat. Opposite surfaces across the length, thickness, and width shall be parallel to within 0.1 %. The cylindrical specimen shall be round and constant in diameter to within 0.1 %.

7.4 Measure specimen mass to within 0.1 %.

7.5 Measure specimen length to within 0.1 %. Measure the thickness and width of the rectangular specimen to within 0.1 % at three locations and determine an average. Measure the diameter of the cylindrical specimen to within 0.1 % at three locations and determine an average.

7.5.1 Take special care when measuring thicknesses less than 3 mm.

NOTE 6—Table 1 illustrates how uncertainties in the measured parameters influence the calculated modulus of elasticity. It shows that calculations are most sensitive to uncertainty in the measurement of the thickness.

7.6 If the material is anisotropic, the axes of a specimen for determining the dynamic shear modulus should coincide with the axes of anisotropy.

8. Procedure

8.1 *Procedure A—Room-Temperature Testing:*

8.1.1 Switch on all electrical equipment and allow to stabilize in accordance with the manufacturer's recommendations. Suspend or support the specimen properly (see Fig. 3 and Fig. 4). Activate the equipment so that power adequate to excite the specimen is delivered to the driving transducer. Set the gain of the detector circuit high enough to detect vibration in the specimen and to display it on the oscilloscope screen with sufficient amplitude to measure accurately the frequency at which the signal amplitude is maximized. Adjust the oscilloscope so that a sharply defined horizontal baseline exists when the specimen is not excited. Scan frequencies with the audio oscillator until specimen resonance is indicated by a sinusoidal pattern of maximum amplitude on the oscilloscope or by a single closed loop Lissajous pattern. The frequency scan should start at a low frequency and then increase.

8.1.2 Find the fundamental flexural resonant frequency. To ensure that the frequency is fundamental and not an overtone, either the node/anti-node locations or one or more overtones should be identified (see Note 7).

NOTE 7—The proper identification of the fundamental flexural mode is important, because spurious frequencies inherent in the system can interfere, especially when greater excitation power and detection sensitivity are required for work with a specimen that has a poor response. The location of the nodes for the fundamental resonant frequency and the first four overtones are indicated in Fig. 5. One method to locate the nodes on

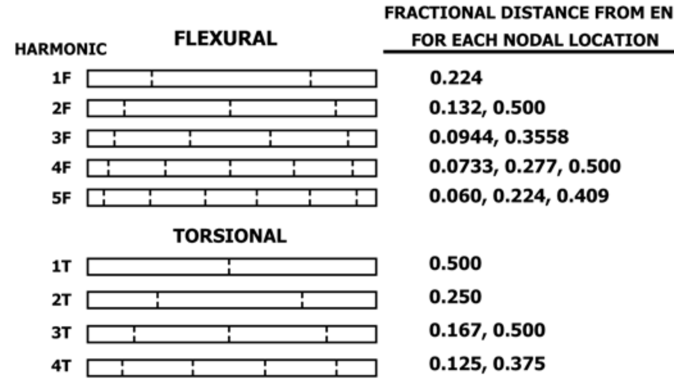


FIG. 5 Dynamic Modulus Resonant Modes and Nodal Locations Tracking Guide Template

the specimen is to move the detector along the length of the specimen; a node is indicated when the output amplitude goes to zero. An anti-node is indicated when the output amplitude reaches a local maximum. Another node location method (used often with string suspensions) is to lay a thin rod across the specimen at a presumed node or anti-node location. If the output amplitude is not affected, then the rod is on a node; if the output amplitude goes to zero, then the location is an anti-node. When several flexural resonant frequencies have been identified, the lowest frequency can be verified as the fundamental, if the numerical ratios of the first three overtone frequencies to the lowest frequency are 2.7, 5.4, and 8.9. Note that these ratios are for a Bernoulli-Euler (simple) beam under ideal conditions. Typically the ratios will be slightly lower.

8.1.3 If a determination of the dynamic shear modulus is made, offset the coupling to the transducers so that the torsional mode of vibration can be induced and detected. (See Fig. 3 and Fig. 4.)

8.1.4 Find (see Note 8) the fundamental torsional resonant frequency.

NOTE 8—Identification of the fundamental torsional mode is based on the same approaches used in identifying the flexural modes: node identification or frequency ratios, or both. Fig. 5 locates the node positions for torsional vibrations. The ratios of the first three torsional overtones to the fundamental torsional frequency are 2, 3, and 4.

8.1.5 The dimensions and mass of the specimen may be measured before or after the test.

8.2 *Procedure B—Elevated-Temperature Testing:*

8.2.1 Potential changes in the mass and dimensions of the specimen should be considered in planning the range of specified temperatures and in interpreting test results as a function of temperature.

NOTE 9—Elevated temperatures can alter the specimen either reversibly or permanently (for example, phase change, devitrification, or microcracking.)

8.2.2 Determine the mass, dimensions, and resonant frequencies at room temperature in air as outlined in 7.4, 7.5, and 8.1.

8.2.3 Place the specimen in the furnace and adjust the driver-detector system so that all the frequencies to be measured can be detected without further adjustment. Determine the resonant frequencies at room temperature in the furnace cavity with the furnace doors closed, etc., as will be the case at elevated temperatures.

8.2.4 Heat the furnace at a controlled rate that does not exceed 150 °C/h. Take data at 25 °C intervals or at 15 min intervals as dictated by heating rate and specimen composition. Data may also be taken on cooling. Follow the change in resonant frequencies with time and temperature closely to avoid losing the identity of each frequency.

NOTE 10—The overtones in flexure and the fundamental in torsion can be difficult to differentiate if not followed closely; spurious frequencies inherent in the system can also appear at temperatures above 600 °C using certain types of suspensions, particularly wire.

8.2.5 Dimensions and mass of the specimen should be measured both before and after the test to check for permanent thermal effects. Measurements should be made to the requirements described in 7.4 and 7.5.

8.3 *Procedure C—Cryogenic Testing:*

8.3.1 Determine the mass, dimensions, and resonant frequencies in air at room temperature, as outlined in 7.4, 7.5, and 8.1.

8.3.2 Measure the resonant frequencies at room temperature in the cryogenic chamber.

8.3.3 Remove water vapor from the chamber by flushing with dry nitrogen gas prior to chilling so that frost deposits on the specimen do not cause anomalous results.

8.3.4 Cool the chamber to the minimum specified temperature. Monitor frequencies as the chamber is cooled. Allow the specimen to stabilize at the minimum specified temperature for at least 15 min. Heating rate should not exceed 50 °C/h, and data may be taken at intervals of 10 min or 15 °C or as desired.

8.3.5 Dimensions and mass of the specimen should be measured both before and after the test to check for permanent thermal effects. Measurements should be made to the requirements described in 7.4 and 7.5.

9. Calculation

9.1 *Dynamic Young's Modulus* (**1**, **3**)*:*

9.1.1 For the fundamental flexural mode of a rectangular specimen calculate dynamic Young's modulus as follows (**3**):

$$E_d = 0.9465\left(\frac{mf_f^2}{b}\right)\left(\frac{L^3}{t^3}\right)T_1 \quad (1)$$

where:

E_d = dynamic Young's modulus, Pa,
m = mass of the rectangular specimen, g, (see Note 11),
b = width of the rectangular specimen, mm, (see Note 11),
L = length of the rectangular specimen, mm, (see Note 11),
t = thickness of the rectangular specimen, mm, (see Note 11),
f_f = fundamental flexural resonant frequency of rectangular specimen in flexure, Hz, and
T_1 = correction factor for fundamental flexural mode to account for finite thickness of rectangular specimen, Poisson's ratio, and so forth.

and:

$$T_1 = 1 + 6.585\,(1 + 0.0752\,\mu + 0.8109\,\mu^2)(t/L)^2 - 0.868\,(t/L)^4 - \left[\frac{8.340\,(1 + 0.2023\,\mu + 2.173\,\mu^2)(t/L)^4}{1.000 + 6.338\,(1 + 0.1408\,\mu + 1.536\,\mu^2)(t/L)^2}\right] \quad (2)$$

where:

μ = Poisson's ratio.

NOTE 11—In the dynamic modulus of elasticity equations the mass and length terms are given in units of grams and millimetres. However, the defined equations can also be used with mass and length terms in units of kilograms and metres with no changes in terms or exponents.

9.1.1.1 If $L/t \geq 20$, T_1 may be simplified to:

$$T_1 = \left[1.000 + 6.585\,(t/L)^2\right] \quad (3)$$

and E_d may be calculated directly.

9.1.1.2 If $L/t < 20$ and Poisson's ratio is known, calculate T_1 from Eq 2 and then calculate E_d.

9.1.1.3 If $L/t < 20$ and Poisson's ratio is not known, use a three step iterative process after assuming an initial value of Poisson's ratio, based on experimental dynamic Young's modulus and dynamic shear modulus.

Step 1–Determine the fundamental flexural and torsional resonant frequency of the rectangular test specimen. Using Eq 7 and Eq 8, calculate the dynamic shear modulus of the test specimen from the fundamental torsional resonant frequency, dimensions, and mass of the specimen.

Step 2–Using Eq 1 and Eq 2, calculate the dynamic Young's modulus of the rectangular test specimen from the fundamental flexural resonant frequency, the dimensions, and mass of the specimen and the initial/iterative Poisson's ratio. Exercise care to use consistent units for all the parameters throughout the computations.

Step 3–Substitute the dynamic shear modulus and dynamic Young's modulus values calculated in *Step 1* and *Step 2* into Eq 11 for Poisson's ratio and calculate a new value for Poisson's ratio for another iteration starting at *Step 1*.

Repeat *Steps 1*, *2*, and *3* until the difference between the last iterative value and the final computed value of the Poisson's ratio is less than 2 % and the values for the moduli are self-consistent.

9.1.2 For the fundamental in flexural mode of a cylindrucal specimen, calculate the dynamic Young's modulus as follows (**3**)

$$E_d = 1.6067\,(m\,f_f^2)\left(\frac{L^3}{D^4}\right)T_1' \quad (4)$$

where:

D = diameter of specimen, mm, (see Note 11), and
T_1' = correction factor for fundamental flexural mode to account for finite diameter of specimen, Poisson's ratio, and so forth.

and

$$T_1' = 1 + 4.939(1 + 0.0752\,\mu + 0.8109\,\mu^2)\,(D/L)^2 - 0.4883\,(D/L)^4 - \left[\frac{4.691(1 + 0.2023\,\mu + 2.173\,\mu^2)(D/L)^4}{1.000 + 4.754(1 + 0.1408\,\mu + 1.536\,\mu^2)(D/L)^2}\right] \quad (5)$$

9.1.2.1 If $L/D \geq 20$, the T_1' may be simplified to the following:

$$T_1' = \left[1.000 + 4.939\,(D/L)^2\right] \quad (6)$$