



Standard Practice for Evaluating Equivalence of Two Testing Processes¹

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1. Scope

1.1 This practice provides statistical methodology for conducting equivalence studies on numerical data from two sources of test results to determine if their true means, variances, or other parameters differ by no more than predetermined limits.

1.2 Applications include (1) equivalence studies for bias against an accepted reference value, (2) determining means equivalence of two test methods, test apparatus, instruments, reagent sources, or operators within a laboratory or equivalence of two laboratories in a method transfer, and (3) determining non-inferiority of a modified test procedure versus a current test procedure with respect to a performance characteristic.

1.3 The guidance in this standard applies to experiments conducted either on a single material at a given level of the test result or on multiple materials covering a selected range of test results.

1.4 Guidance is given for determining the amount of data required for an equivalence study. The control of risks associated with the equivalence decision is discussed.

1.5 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.6 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.7 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

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2. Referenced Documents

2.1 ASTM Standards:²

E122 Practice for Calculating Sample Size to Estimate, With Specified Precision, the Average for a Characteristic of a Lot or Process

E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods

E456 Terminology Relating to Quality and Statistics

E2282 Guide for Defining the Test Result of a Test Method

E2586 Practice for Calculating and Using Basic Statistics

E3080 Practice for Regression Analysis with a Single Predictor Variable

2.2 USP Standard:³

USP <1223> Validation of Alternative Microbiological Methods

3. Terminology

3.1 *Definitions*—See Terminology E456 for a more extensive listing of statistical terms.

3.1.1 *accepted reference value, n*—a value that serves as an agreed-upon reference for comparison, and which is derived as: (1) a theoretical or established value, based on scientific principles, (2) an assigned or certified value, based on experimental work of some national or international organization, or (3) a consensus or certified value, based on collaborative experimental work under the auspices of a scientific or engineering group. **E177**

3.1.2 *bias, n*—the difference between the expectation of the test results and an accepted reference value. **E177**

3.1.3 *confidence interval, n*—an interval estimate [L, U] with the statistics L and U as limits for the parameter θ and with confidence level $1 - \alpha$, where $\Pr(L \leq \theta \leq U) \geq 1 - \alpha$. **E2586**

3.1.3.1 *Discussion*—The confidence level, $1 - \alpha$, reflects the proportion of cases that the confidence interval [L, U] would contain or cover the true parameter value in a series of repeated random samples under identical conditions. Once L and U are

² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

³ Available from U.S. Pharmacopeial Convention (USP), 12601 Twinbrook Pkwy., Rockville, MD 20852-1790, http://www.usp.org.

given values, the resulting confidence interval either does or does not contain it. In this sense “confidence” applies not to the particular interval but only to the long run proportion of cases when repeating the procedure many times.

3.1.4 *confidence level, n*—the value, $1 - \alpha$, of the probability associated with a confidence interval, often expressed as a percentage. **E2586**

3.1.4.1 *Discussion*— α is generally a small number. Confidence level is often 95 % or 99 %.

3.1.5 *confidence limit, n*—each of the limits, L and U, of a confidence interval, or the limit of a one-sided confidence interval. **E2586**

3.1.6 *degrees of freedom, n*—the number of independent data points minus the number of parameters that have to be estimated before calculating the variance. **E2586**

3.1.7 *equivalence, n*—condition that two population parameters differ by no more than predetermined limits.

3.1.8 *intermediate precision conditions, n*—conditions under which test results are obtained with the same test method using test units or test specimens taken at random from a single quantity of material that is as nearly homogeneous as possible, and with changing conditions such as operator, measuring equipment, location within the laboratory, and time. **E177**

3.1.9 *mean, n—of a population, μ , average or expected value of a characteristic in a population; of a sample, \bar{X} sum of the observed values in the sample divided by the sample size.* **E2586**

3.1.10 *percentile, n*—quantile of a sample or a population, for which the fraction less than or equal to the value is expressed as a percentage. **E2586**

3.1.11 *population, n*—the totality of items or units of material under consideration. **E2586**

3.1.12 *population parameter, n*—summary measure of the values of some characteristic of a population. **E2586**

3.1.13 *precision, n*—the closeness of agreement between independent test results obtained under stipulated conditions. **E177**

3.1.14 *quantile, n*—value such that a fraction f of the sample or population is less than or equal to that value. **E2586**

3.1.15 *repeatability, n—precision of test results from tests conducted within the shortest practical time period on identical material by the same test method in a single laboratory with all known sources of variability conditions controlled at the same levels (see repeatability conditions).* **E177**

3.1.16 *repeatability conditions, n*—conditions where independent test results are obtained with the same method on identical test items in the same laboratory by the same operator using the same equipment within short intervals of time. **E177**

3.1.17 *repeatability standard deviation (s_r), n*—the standard deviation of test results obtained under repeatability conditions. **E177**

3.1.18 *sample, n*—a group of observations or test results, taken from a larger collection of observations or test results,

which serves to provide information that may be used as a basis for making a decision concerning the larger collection. **E2586**

3.1.19 *sample size, n, n*—number of observed values in the sample. **E2586**

3.1.20 *sample statistic, n*—summary measure of the observed values of a sample. **E2586**

3.1.21 *standard deviation—of a population, σ , the square root of the average or expected value of the squared deviation of a variable from its mean; of a sample, s , the square root of the sum of the squared deviations of the observed values in the sample from their mean divided by the sample size minus 1.* **E2586**

3.1.22 *test result, n*—the value of a characteristic obtained by carrying out a specified test method. **E2282**

3.1.23 *test unit, n*—the total quantity of material (containing one or more test specimens) needed to obtain a test result as specified in the test method. See test result. **E2282**

3.1.24 *variance, σ^2, s^2, n* —square of the standard deviation of the population or sample. **E2586**

3.2 Definitions of Terms Specific to This Standard:

3.2.1 *bias equivalence, n*—equivalence of a population mean with an accepted reference value.

3.2.2 *equivalence limit, E, n—in equivalence testing, a limit on the difference between two population parameters.*

3.2.2.1 *Discussion*—In certain applications, this may be termed *practical limit* or *practical difference*.

3.2.3 *equivalence test, n*—a statistical test conducted within predetermined risks to confirm equivalence of two population parameters.

3.2.4 *means equivalence, n*—equivalence of two population means.

3.2.5 *non-inferiority, n*—condition that the difference in means or variances of test results between a modified testing process and a current testing process with respect to a performance characteristic is no greater than a predetermined limit in the direction of inferiority of the modified process to the current process.

3.2.5.1 *Discussion*—Other terms used for *non-inferior* are “equivalent or better” or “at least equivalent as.”

3.2.6 *paired samples design, n—in means equivalence testing, single samples are taken from the two populations at a number of sampling points.*

3.2.7 *power, n—in equivalence testing, the probability of accepting equivalence, given the true difference between two population means.*

3.2.7.1 *Discussion*—In the case of testing for bias equivalence the power is the probability of accepting equivalence, given the true difference between a population mean and an accepted reference value.

3.2.8 *range equivalence, n*—equivalence of two population means over a range of test result values.

3.2.9 *slope equivalence, n*—equivalence of the slope of a linear statistical relationship with the value one (1).

3.2.10 *two independent samples design, n—in means equivalence testing*, replicate test results are determined independently from two populations at a single sampling time for each population.

3.2.10.1 *Discussion*—This design is termed a completely randomized design for a general number of sampled populations.

3.2.11 *two one-sided tests (TOST) procedure, n*—a statistical procedure used for testing the equivalence of the parameters from two distributions (see equivalence).

3.3 *Symbols:*

- B = bias (9.1.1)
- b_0 = intercept estimate (8.3.1.4)
- b_1 = slope estimate (8.3.1.3)
- d_j = difference between a pair of test results at sampling point j (7.1.1)
- \bar{d} = average difference (7.1.1)
- D = difference in sample means (6.1.2) (A1.1.2)
- E = equivalence limit (5.2.1)
- E_1 = lower equivalence limit (5.2.1)
- E_2 = upper equivalence limit (5.2.1)
- e_i = residual estimate (8.6.2)
- f = degrees of freedom for s (9.1.1) (A1.1.2)
- $F_{1-\alpha}$ = $(1 - \alpha)^{\text{th}}$ percentile of the F distribution (10.3.1)
- f_i = degrees of freedom for s_i (6.1.1)
- f_p = degrees of freedom for s_p (6.1.2)
- $\mathcal{F}(\bullet)$ = the cumulative F distribution function (A1.7.3)
- H_0 : = null hypothesis (A1.1.1)
- H_a : = alternate hypothesis (A1.1.1)
- n = sample size (number of test results) from a population (5.4) (6.1.3) (7.1.1) (9.1.1)
- n_i = sample size from i^{th} population (6.1.1)
- n_1 = sample size from population 1 (6.1.2)
- n_2 = sample size from population 2 (6.1.2)
- R = ratio of two sample variances (5.5.2.1)
- r = sample correlation coefficient (8.5.1)
- \mathcal{R} = ratio of two population variances (A1.7.3)
- S_{XX} = sum of squared deviations of X from their mean (8.3.1.2)
- S_{XY} = sum of products of deviations of X and Y from their means (8.3.1.2)
- S_{YY} = sum of squared deviations of Y from their mean (8.3.1.2)
- s = sample standard deviation (9.1.1)
- s_B = sample standard deviation for bias (9.1.2)
- s_d = standard deviation of the difference between two test results (7.1.1)
- s_D = sample standard deviation for mean difference (6.1.3) (A1.1.2)
- s_i = sample standard deviation for i^{th} population (6.1.1)
- s_i^2 = sample variance for i^{th} population (6.1.1)
- s_1^2 = sample variance for population 1 (6.1.2)
- s_1^2 = variance of test results from the current process (10.3.1)
- s_2^2 = sample variance for population 2 (6.1.2)
- s_2^2 = variance of test results from the modified process (10.3.1)
- s_p = pooled sample standard deviation (6.1.2)
- s_r = repeatability sample standard deviation (6.2) (7.2)

- t = Student's t statistic (6.1.4) (7.1.3) (9.1.3)
- $t_{1-\alpha, f}$ = $(1 - \alpha)^{\text{th}}$ percentile of the Student's t distribution with f degrees of freedom (A1.1.2)
- X_{ij} = j^{th} test result from the i^{th} population (6.1)
- UCL_R = upper confidence limit for \mathcal{R} (10.3.1)
- \bar{X} = test result average (9.1.1)
- \bar{X}_i = test result average for the i^{th} population (6.1.1)
- \bar{X}_1 = test result average for population 1 (6.1.3)
- \bar{X}_2 = test result average for population 2 (6.1.3)
- $Z_{1-\alpha}$ = $(1 - \alpha)^{\text{th}}$ percentile of the standard normal distribution (A1.7.1)
- α = (alpha) consumer's risk (5.2.2) (6.2) (7.2)
- β = (beta) producer's risk (5.4.1)
- β_0 = (beta) intercept parameter (8.1)
- β_1 = (beta) slope parameter (8.1)
- Δ = (capital delta) true mean difference between populations (5.4.1)
- δ = (delta) measurement error of X (A2.1.1)
- ε = (epsilon) measurement error of Y (A2.1.1)
- η = (eta) true mean of Y (A2.1.1)
- θ = (theta) angle of the straight line to the horizontal axis (8.3.2.1)
- $\hat{\theta}$ = estimate of θ (8.3.2.1)
- κ^2 = (kappa squared) information size (A2.4)
- λ = (lambda) ratio of measurement error variances of Y over X (A2.1.1.1)
- μ = (mu) population mean (A1.5.1)
- μ_i = (mu) i^{th} population mean (A1.1.1)
- ν = (nu) approximate degrees of freedom for s_D (A1.1.4)
- ξ = (xi) true mean of X (A2.1.1)
- σ_d = (sigma) standard deviation of the true difference between two populations (7.2)
- σ_e^2 = (sigma) measurement error variances of Y (8.2) (A2.1.1)
- σ_δ^2 = (sigma) measurement error variances of X (8.2) (A2.1.1)
- τ = (tau) perpendicular distance from line to origin (A2.1.4)
- $\Phi(\bullet)$ = (capital phi) standard normal cumulative distribution function (A1.7.1)
- φ = (phi) half width of confidence interval for θ (8.3.2.2)
- v = (upsilon) probability associated with informative confidence interval (A2.4.1.1)
- ω = (omega) width of the equivalence interval for θ (8.3.3)

3.4 *Acronyms:*

- 3.4.1 *ARV, n*—accepted reference value (5.5.1.1) (9.1) (A1.5.1)
- 3.4.2 *CRM, n*—certified reference material (5.5.1.1) (9.1)
- 3.4.3 *ILS, n*—interlaboratory study (6.2)
- 3.4.4 *IUT, n*—intercept-union test (8.7) (A1.4)
- 3.4.5 *LCL, n*—lower confidence limit (6.1.4) (7.2.3)
- 3.4.6 *TOST, n*—two one-sided tests (5.5.1) (Section 6) (Section 7) (Section 9) (Annex A1)
- 3.4.7 *UCL, n*—upper confidence limit (6.1.4) (7.2.3)

4. Significance and Use

4.1 Laboratories conducting routine testing have a continuing need to make improvements in their testing processes. In these situations it must be demonstrated that any changes will neither cause an undesirable shift in the test results from the current testing process nor substantially affect a *performance characteristic* of the test method. This standard provides guidance on experiments and statistical methods needed to demonstrate that the test results from a modified testing process are equivalent to those from the current testing process, where *equivalence* is defined as agreement within a prescribed limit, termed an *equivalence limit*.

4.1.1 The equivalence limit, which represents a worst-case difference or ratio, is determined prior to the equivalence test and its value is usually set by consensus among subject-matter experts.

4.1.2 Examples of modifications to the testing process include, but are not limited, to the following:

- (1) Changes to operating levels in the steps of the test method procedure,
- (2) Installation of new instruments, apparatus, or sources of reagents and test materials,
- (3) Evaluation of new personnel performing the testing, and
- (4) Transfer of testing to a new location.

4.1.3 Examples of performance characteristics directly applicable to the test method include bias, precision, sensitivity, specificity, linearity, and range. Additional characteristics are test cost and elapsed time needed to conduct the test procedure.

4.2 Equivalence studies are performed by a designed experiment that generates test results from the modified and current testing procedures on the same types of materials that are routinely tested. The design of the experiment depends on the type of equivalence needed as discussed below. Experiment design and execution for various objectives is discussed in Section 5.

4.2.1 *Means equivalence* is concerned with a potential shift in the mean test result in either direction due to a modification in the testing process. Test results are generated under repeatability conditions by the modified and current testing processes on the same material, and the difference in their mean test results is evaluated.

4.2.1.1 In situations where testing cannot be conducted under repeatability conditions, such as using in-line instrumentation, test results may be generated in pairs of test results from the modified and current testing processes, and the mean differences among paired test results are evaluated.

4.2.2 *Slope equivalence* evaluates the slope of the linear statistical relationship between the test results from the two testing procedures. If the slope is equivalent to the value one (1), then the two testing processes meet slope equivalence.

4.2.3 *Range equivalence* evaluates the differences in means over a selected wider range of test results and the experiment uses materials that cover that range. The combination of slope equivalence and means equivalence defines range equivalence.

4.2.4 *Non-inferiority* is concerned with a difference only in the direction of an inferior outcome in a performance characteristic of the modified testing procedure versus the current

testing procedure. Non-inferiority may involve the comparisons of means, standard deviations, or other statistical parameters.

4.2.4.1 Non-inferiority studies may involve trade-offs in performance characteristics between the modified and current procedures. For example, the modified process may be slightly inferior to the established process with respect to assay sensitivity or precision but may have off-setting advantages such as faster delivery of test results or lower testing costs.

4.3 *Risk Management*—Guidance is provided for determining the amount of data required to control the risks of making the wrong decision in accepting or rejecting equivalence (see 5.4 and Section A1.2).

4.3.1 The *consumer's risk* is the risk of falsely declaring equivalence. The probability associated with this risk is directly controlled to a low level so that accepting equivalence gives a high degree of assurance that the true difference is less than the equivalence limit.

4.3.2 The *producer's risk* is the risk of falsely rejecting equivalence. The probability associated with this risk is controlled by the amount of data generated by the experiment. If valid improvements are rejected by equivalence testing, this can lead to opportunity losses to the company and its laboratories (the producers) or cause unnecessary additional effort in improving the testing process.

5. Planning and Executing the Equivalence Study

5.1 This section discusses the stages of conducting an equivalence study: (1) determining the information needed, (2) setting up and conducting the study design, and (3) performing the statistical analysis of the resulting data. The study is usually conducted either in a single laboratory or, in the case of a method transfer, in both the originating and receiving laboratories. Using multiple laboratories will almost always increase the inherent variability of the data in the study, which will increase the cost of performing the study due to the need for more data.

5.2 *Prior information* required for the study design includes the equivalence limit, the consumer's risk, and an estimate of the test method precision.

5.2.1 For means equivalence tests there are two equivalence limits, $-E$ and E , because the need to detect a potential shift in either direction. Limits may be non-symmetrical around zero, such as $-E_1$ and E_2 , and this will usually be the case for slope equivalence. For non-inferiority tests only one of these limits is tested.

5.2.2 The consumer's risk may be determined by an industry norm or a regulatory requirement. A probability value often used is $\alpha = 0.05$, which is a 5 % risk to the user of the test results that the study falsely declares equivalence due to the modification of the testing process.

5.2.3 A prior estimate of the test method precision is essential for determining the number of test results required in the equivalence study design for adequate producer's risk control. This estimate can be available from method development work, from an interlaboratory study (ILS), or from other

sources. The precision estimate should take into account the test conditions of the ILS, such as *repeatability* or *intermediate precision* conditions.

5.2.4 For slope equivalence an additional piece of required information is the ratio λ of the measurement variability of the modified and current test methods, expressed as variances. These estimates are usually available from experience or from method development work, but see 5.3.2.1.

5.3 The *design type* determines how the data are collected and how much data are needed to control the producer’s risk, or the risk of a wrong decision. For generating test result data from the modified and current testing processes, three basic designs are discussed in this practice, the Two Independent Samples Design, the Paired Samples Design, and the Single Sample Design.

5.3.1 The Two Independent Samples Design is used for means equivalence and non-inferiority testing. In this design, sets of independent test results are usually generated in a single laboratory on a quantity of a single homogeneous material by both testing procedures under repeatability conditions. For method transfers each laboratory generates independent test results using the same testing procedure on the same material under repeatability conditions at each laboratory. If this is not possible due to constraints on time or facilities, then the test results can be conducted under intermediate precision conditions, but then a statistician is recommended for the design and analysis of the test.

5.3.2 The Paired Samples Design is used for slope equivalence and may also be used for means equivalence. In this design, pairs of single test results from each testing procedure are generated on the same material over different time periods, or on various materials that are sampled either from a manufacturing process over time or from a set of materials that cover a predetermined range.

5.3.2.1 If information on measurement error is not available for slope equivalence studies, the experiment design can be modified to run duplicate test results by each testing process on

each of the n materials to provide these precision estimates needed for estimation of their ratio.

5.3.3 The Single Sample Design used for bias equivalence. In this design, test results are generated by the current testing process on a certified reference material.

5.4 *Sample size* in the design context refers to the number n of test results required by each testing process to manage the producer’s risk. It is possible to use different sample sizes for the modified and current test processes, but this can lead to poor control of the consumer’s risk (see A1.1.4).

5.4.1 The number of test results, symbol n , from each of the two testing processes controls the producer’s risk β of falsely rejecting means equivalence at a given true mean difference, Δ . The producer’s risk may be alternatively stated in terms of the *power*, defined as the probability $1 - \beta$ of correctly accepting equivalence at a given value of Δ .

5.4.1.1 For symmetric equivalence limits in means equivalence studies the power profile plots the probability $1 - \beta$ against the absolute value of Δ , due to the symmetry of the equivalence limits. This calculation can be performed using a spreadsheet computer package (see A1.7.1 and Appendix X1).

5.4.1.2 An example of a set of power profiles in means equivalence studies is shown in Fig. 1. The probability scale for power on the vertical axis varies from 0 to 1. The horizontal axis is the true absolute difference Δ . The power profile, a reversed S-shaped curve, should be close to a power probability of 1 at zero absolute difference and will decline to the consumer risk probability at an absolute difference of E . Power for absolute differences greater than E are less than the consumer risk and decline asymptotically to zero as the absolute difference increases.

5.4.1.3 In Fig. 1, power profiles are shown for three different sample sizes for testing means equivalence. Increasing the sample size moves the power curve to the right, giving a greater chance of accepting equivalence for a given true difference Δ . Equations for power profiles are shown in Section A1.6 and a spreadsheet example in Appendix X1.

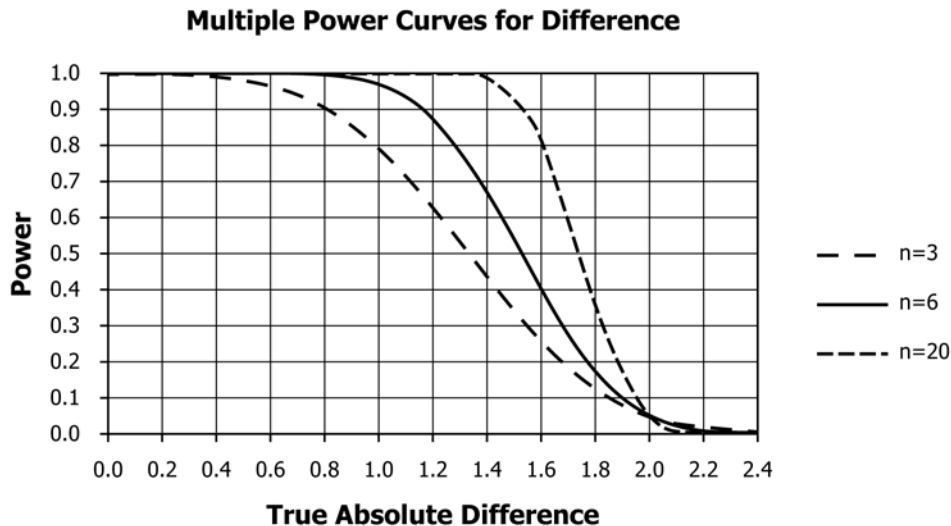


FIG. 1 Multiple Power Curves for Lab Transfer Example

5.4.2 Power curves for bias equivalence and non-inferiority are constructed by different formulas but have the same shape and interpretation as those for means equivalence.

5.4.2.1 For non-inferiority testing, the power profile plots the probability $1 - \beta$ against the true mean difference Δ (see A1.7.2) or against the true variance ratio \mathcal{R} for variances (see A1.7.3).

5.4.3 Power curves are evaluated by entering different values of n and evaluating the curve shape. A practical solution is to choose n such that the power is above a 0.9 probability out to about one-half to two-thirds of the distance from zero to E , thus giving a high probability that equivalence will be demonstrated for a range of true absolute differences that are deemed of little or no scientific import in the test result.

5.4.4 Annex A2 provides criteria for determining the number of samples required to meet power requirements for slope equivalence.

5.5 The statistical analysis for accepting or rejecting equivalence of means and variances for a single material is similar for all cases and depends on the outcome of one-sided statistical hypothesis tests. These calculations are given in detail with examples in Sections 6, 7, 9, and 10, with statistical theory given in Annex A1. The statistical analysis for slope and range equivalence is given in Section 8, with statistical theory given in Annex A2.

5.5.1 The data analysis for means equivalence uses a statistical methodology termed the two one-sided tests (TOST) procedure. The null hypothesis (see A1.1.1) is that the average difference between two sets of data exceeds an equivalence limit in one of the directions from zero, and this is tested in both directions. If the hypothesis is rejected in both directions then the alternate hypothesis that the mean difference is less than the equivalence limit is accepted and the two sources of data are deemed means equivalent.

NOTE 1—Historically, this procedure originated in the pharmaceutical industry for use in bioequivalence trials (1, 2),⁴ and was denoted as the Two One-Sided Tests Procedure, which has since been adopted for use in testing and measurement applications (3, 4).

5.5.1.1 For bias equivalence, the statistical test is based on only a single set of data conducted on a certified reference material (CRM) because its accepted reference value (ARV) is considered to be a known mean with zero variability for the purpose of the equivalence study.

5.5.2 The data analysis for non-inferiority testing of population means uses a single one-sided test in the direction of an inferior outcome with respect to a performance characteristic determined by the test results. When the performance characteristic is defined as “higher is better,” such as method sensitivity, the statistical test supports non-inferiority when $LCL > -E$. Conversely, when the performance characteristic is defined as “lower is better,” such as incidence of misclassifications, the statistical test supports non-inferiority when $UCL < E$.

5.5.2.1 For the non-inferiority testing of precision, the variances of the two data sets are used, and “lower is better” for

⁴ The boldface numbers in parentheses refer to a list of references at the end of this standard.

this parameter, so the test for non-inferiority applies. Because variances are a scale parameter, the single non-inferiority test is based the ratio R of the two sample variances, and the non-inferiority limit E is also in the form of a ratio.

6. The TOST Procedure for Statistical Analysis of Means Equivalence — Two Independent Samples Design

6.1 *Statistical Analysis*—Let the sample data be denoted as X_{ij} = the j^{th} test result from the i^{th} population. The equivalence limit E , consumer’s risk α , and sample sizes have been previously determined.

6.1.1 Calculate averages, variances, and standard deviations, and degrees of freedom for each sample:

$$\bar{X}_i = \frac{\sum_{j=1}^{n_i} X_{ij}}{n_i}, \quad i = 1, 2 \quad (1)$$

$$s_i^2 = \frac{\sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2}{(n_i - 1)}, \quad i = 1, 2 \quad (2)$$

$$s_i = \sqrt{s_i^2}, \quad i = 1, 2 \quad (3)$$

$$f_i = n_i - 1, \quad i = 1, 2 \quad (4)$$

6.1.2 Calculate the pooled standard deviation and degrees of freedom:

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 + n_2 - 2)}} \quad (5)$$

It is assumed that the sample variances come from populations having equal variances; and, if this appears not to be the case, then use the procedure in A1.1.4.

If $n_1 = n_2 = n$, then:

$$s_p^2 = \frac{(s_1^2 + s_2^2)}{2} \quad (6)$$

$$f_p = (n_1 + n_2 - 2)$$

6.1.3 Calculate the difference between means and its standard error:

$$D = \bar{X}_2 - \bar{X}_1 \quad (7)$$

$$s_D = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \quad (8)$$

If $n_1 = n_2 = n$, then:

$$s_D = s_p \sqrt{\frac{2}{n}}$$

6.1.4 *Statistical Test for Equivalence*—Compute the upper (UCL) and lower (LCL) confidence limits for the 100 $(1 - 2\alpha)$ % two-sided confidence interval on the true difference. If the confidence interval is completely contained within the equivalence limits $(0 \pm E)$, equivalently if $LCL > -E$ and $UCL < E$, then accept equivalence. Otherwise, reject equivalence.

$$UCL = D + t s_D \quad (9)$$

$$LCL = D - t s_D \quad (10)$$

where t is the upper 100 $(1 - \alpha)$ % percentile of the Student’s t distribution with $(n_1 + n_2 - 2)$ degrees of freedom.

6.2 *Example for Means Equivalence*—The example shown is data from a transfer of an ASTM test method from R&D Lab 1 to Plant Lab 2 (Table 1). An equivalence of limit of 2 units was proposed with a consumer risk of 5 %. An interlaboratory study (ILS) on this test method had given an estimate of $s_r = 0.5$ units for the repeatability standard deviation. Thus $E = 2$ units, $\alpha = 0.05$, and estimated $\sigma = 0.5$ units are inputs for this study (the actual units are unspecified for this example).

6.2.1 *Sample Size Determination*—Power profiles for $n = 3, 6,$ and 20 were generated for a set of absolute difference values ranging 0.00 (0.20) 2.40 units as shown in Fig. 1. All three curves intersect at the point $(2, 0.05)$ as determined by the consumer’s risk at the equivalence limit.

6.2.1.1 A sample size of $n = 6$ replicate assays per laboratory yielded a satisfactory power curve, in that the probability of accepting equivalence (power) was greater than a 0.9 probability (or a 90 % power) for a difference of about 1.2 units or less. Therefore, there would be less than an estimated 10 % risk to the producer that such a difference would fail to support equivalence in the actual study.

6.2.1.2 A comparison of the three power curves indicates that the $n = 3$ design would be underpowered, as the power falls below 0.9 at 0.8 units. The $n = 20$ design gives somewhat more power than the $n = 6$ design but is more costly to conduct and may not be worth the extra expenditure.

6.2.2 Averages, variances, standard deviations, and degrees of freedom for the two laboratories are:

$$\begin{aligned} \bar{X}_1 &= (96.9 + 97.9 + 98.5 + 97.5 + 97.7 + 97.2)/6 \\ &= 97.62 \text{ mg/g} \\ \bar{X}_2 &= (97.8 + 97.6 + 98.1 + 98.6 + 98.6 + 98.9)/6 \\ &= 98.27 \text{ mg/g} \end{aligned}$$

$$\begin{aligned} s_1^2 &= [(96.9 - 97.62)^2 + \dots + (97.2 - 97.62)^2]/(6 - 1) \\ &= 0.31367 \end{aligned}$$

$$\begin{aligned} s_2^2 &= [(97.8 - 98.27)^2 + \dots + (98.9 - 98.27)^2]/(6 - 1) \\ &= 0.26267 \end{aligned}$$

$$s_1 = \sqrt{0.31367} = 0.560$$

$$s_2 = \sqrt{0.26267} = 0.513$$

$$f_1 = n_1 - 1 = 6 - 1 = 5$$

The estimates of standard deviation are in good agreement with the ILS estimate of 0.5 mg/g.

6.2.3 The pooled standard deviation is:

$$s_p = \sqrt{\frac{(6 - 1)0.31367 + (6 - 1)0.26267}{(6 + 6 - 2)}} = \sqrt{\frac{2.8817}{10}} = 0.537 \text{ mg/g}$$

with 10 degrees of freedom.

6.2.4 The difference of means is $D = 98.27 - 97.62 = 0.65$ mg/g. The plant laboratory average is 0.65 mg/g higher than the development laboratory average. The standard error of the difference of means is $s_D = 0.537 \sqrt{2/6} = 0.310$ mg/g with 10 degrees of freedom (same as that for s_p).

6.2.5 The 95th percentile of Student’s t with 10 degrees of freedom is 1.812. Upper and lower confidence limits for the difference of means are:

$$UCL = 0.65 + (1.812)(0.310) = 1.21$$

$$LCL = 0.65 - (1.812)(0.310) = 0.09$$

The 90 % two-sided confidence interval on the true difference is 0.09 to 1.21 mg/g and is completely contained within the equivalence interval of -2 to 2 mg/g. Since $0.09 > -2$ and $1.21 < 2$, equivalence is accepted.

7. The TOST Procedure for Statistical Analysis of Means Equivalence — Paired Samples Design

7.1 *Statistical Analysis*—Let the sample data be denoted as X_{ij} = the test result from the i^{th} population and the j^{th} pair, where $i = 1$ or $2, j = 1, \dots, n$. Each pair represents a test result from each population at a given sampling point. The equivalence limit E , consumer’s risk α , and sample size (number of pairs, symbol n) have been previously determined (see Section 5).

7.1.1 Calculate the n differences, symbol d_j , between the two test results within each pair, the average of the differences, symbol \bar{d} , and the standard deviation of the differences, symbol s_d , with its degrees of freedom, symbol f .

$$d_j = X_{1j} - X_{2j}, j = 1, \dots, n \quad (11)$$

$$\bar{d} = \frac{\sum_{j=1}^n d_j}{n} = D \quad (12)$$

$$s_d = \sqrt{\frac{\sum_{j=1}^n (d_j - \bar{d})^2}{(n - 1)}} \quad (13)$$

$$f = n - 1 \quad (14)$$

7.1.2 Calculate the standard error of the mean difference, symbol s_D .

$$s_D = \frac{s_d}{\sqrt{n}} \quad (15)$$

7.1.3 *Statistical Test for Equivalence*—Compute the upper (UCL) and lower (LCL) confidence limits for the $100(1 - 2\alpha)$ % two-sided confidence interval on the true difference. If the confidence interval is completely contained within the equivalence limits ($0 \pm E$), or equivalently if $LCL > -E$ and $UCL < E$, then accept equivalence. Otherwise, reject equivalence.

$$UCL = D + t s_D \quad (16)$$

$$LCL = D - t s_D \quad (17)$$

where t is the upper $100(1 - \alpha)$ % percentile of the Student’s t distribution with $(n - 1)$ degrees of freedom.

7.2 *Example for Means Equivalence*—Total organic carbon in purified water was measured by an on-line analyzer, wherein a water sample was taken directly into the analyzer from the pipeline through a sampling port and the test result was determined by a series of operations within the instrument. A new analyzer was to be qualified by running a TOC analysis at the same time as the current analyzer utilizing a parallel sampling port on the pipeline. The sampling time was the pairing factor, and the data from the two instruments constituted a pair of single test results measured at a particular sampling time. Sampling was to be conducted at a frequency of four hours between sampling periods.

An equivalence limit of 2 parts per billion (ppb), or 4 % of the nominal process average of 50 ppb, was proposed with a

TABLE 1 Data for Equivalence Test Between Two Laboratories

	Test Results					
Laboratory 1	96.9	97.9	98.5	97.5	97.7	97.2
Laboratory 2	97.8	97.6	98.1	98.6	98.6	98.9

consumer risk of 5 %. A repeatability estimate of $s_r = 0.7$ ppb, based on previous validation work, gave an estimate for $\sigma_d = 0.7\sqrt{2}$ or approximately 1 ppb. Thus $E = 2$ ppb, $\alpha = 0.05$, and $\sigma_d = 1$ ppb were inputs for this study.

7.2.1 Sample Size Determination—Because the paired samples design uses the differences of the test results within sampling periods for data analysis, the sample size equals the number of pairs for purposes of calculating the power curve. In this example, the cost of obtaining test results was not a major consideration once the new analyzer was installed in the system. Comparative power profiles for $n = 10, 20$, and 50 sample pairs are shown in Fig. 2. The sample size of 20 pairs yielded a satisfactory power curve, in that the probability of accepting equivalence was greater than a 0.9 (or a 90 % power) for a true difference of about 1.25 ppb. Therefore, there would be less than an estimated 10 % risk to the producer that such a difference would fail to support equivalence in the actual study.

7.2.2 Test results for the two instruments at each of the 20 sampling times are listed in Table 2. The current analyzer was designated as Instrument A, and the new analyzer was designated as Instrument B. The differences d_j at each sampling time period were calculated and listed in Table 2 as differences in the test results of Instrument B minus Instrument A. The averages and standard deviations of the test results for each analyzer and their differences are also listed in Table 2.

7.2.2.1 The average difference \bar{d} was 0.46 ppb and the standard deviation of the differences s_d was 1.05 ppb with $f = 19$ degrees of freedom. The standard error of the average difference was:

$$s_{\bar{d}} = \frac{1.05}{\sqrt{20}} = 0.235 \text{ ppb}$$

7.2.2.2 Note that the standard deviations of test results for each analyzer over time were about 6 ppb due to process fluctuations in a range of 37–59 ppb. The source of variation due to pairs (sampling times from the process) is eliminated in the variation of the differences by pairing the test results.

TABLE 2 Data for Paired Samples Equivalence Test

Sampling Time	TOC in Water, ppb		
	Inst A	Inst B	Diff
1	46.4	48.8	2.4
2	44.2	43.5	-0.7
3	52.4	53.0	0.6
4	37.6	37.3	-0.3
5	49.3	49.1	-0.2
6	45.0	44.5	-0.5
7	51.4	51.3	-0.1
8	57.6	56.8	-0.8
9	43.4	44.9	1.5
10	45.2	44.1	-1.1
11	59.0	58.5	-0.5
12	43.1	44.1	1.0
13	39.3	40.9	1.6
14	48.2	48.4	0.2
15	48.7	49.0	0.3
16	44.4	46.1	1.7
17	52.7	53.2	0.5
18	43.3	44.6	1.3
19	54.4	56.7	2.3
20	58.4	58.4	0.0
Average	48.20	48.66	0.46
Std Dev	6.13	5.99	1.05

7.2.3 The 95th percentile of Student’s t with 19 degrees of freedom was 1.729. Upper and lower confidence limits for the difference of means were:

$$UCL = D + t s_{\bar{d}} = 0.46 + (1.729)(0.235) = 0.87 \text{ ppb}$$

$$LCL = D - t s_{\bar{d}} = 0.46 - (1.729)(0.235) = 0.05 \text{ ppb}$$

The 90 % two-sided confidence interval on the true difference is 0.05 to 0.87 ppb and is completely contained within the equivalence interval of -2 to 2 ppb. Since $0.05 > -2$ and $0.87 < 2$, equivalence of the two analyzers is accepted.

8. Procedure for Equivalence of Test Results Over a Range of Values

8.1 Range equivalence is the condition that means equivalence of two testing processes holds over a predetermined range of a material’s characteristic being measured. The

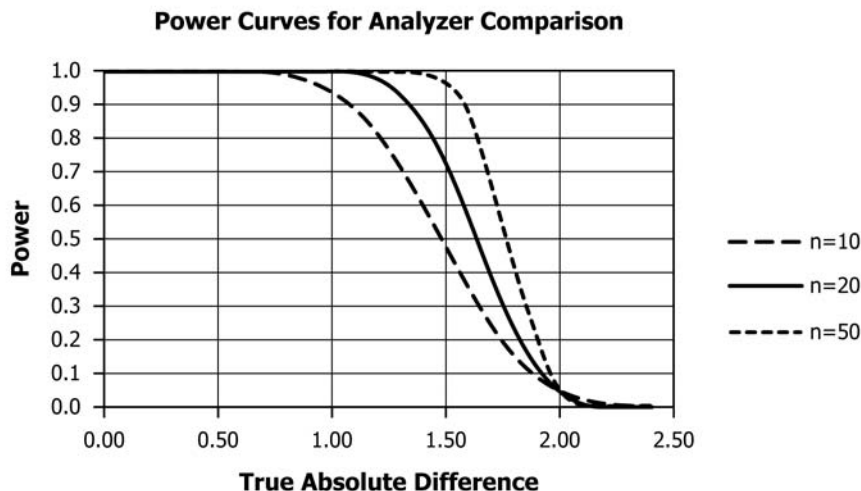


FIG. 2 Power Curves for Total Organic Carbon Analyzers Comparison

experiment consists of obtaining pairs of test results by each process on a number of different material samples. The approach taken for evaluating range equivalence is through a linear statistical function, $Y = \beta_0 + \beta_1 X$, describing a straight-line statistical relationship between the test results from two testing processes denoted as X and Y . The Y intercept β_0 is the value of Y when $X = 0$, and the slope β_1 is the amount of change in Y units for a unit change in X . The criterion for range equivalence is twofold: (1) that the intercept $\beta_0 = 0$, and (2) that the slope $\beta_1 = 1$, each within predetermined limits. This states that, within limits, the relationship $Y = X$ holds over the range of the data.

8.1.1 In many cases, the data range is far removed from zero, so that the intercept parameter is not precisely estimated and can even be a negative test result value. For that reason, the equivalence procedure for $\beta_0 = 0$ should be replaced by a test that the means of the X and Y test results are equivalent, indicating that the center of the straight-line relationship is locally close to the $Y = X$ line. This means-equivalence procedure is covered in Section 7. Note that the estimated line must pass through the point determined from the X and Y averages of the data.

8.1.2 The equivalence testing procedure for $\beta_1 = 1$ is termed *slope equivalence* and this topic will be covered in the remainder of this section.

8.1.3 Statistical tests for means equivalence and slope equivalence must each meet equivalence to assert range equivalence, because together they constitute an intersection-union test (see A1.4.1). If each of the component statistical tests is an α -level test, then the range equivalence test is also an α -level test.

8.2 *Slope Equivalence*—The Y intercept and the slope for the linear statistical function in 8.1 are estimated from the data by a process known as least-squares, which minimizes the sum of the squared deviations of the data points from the estimated line. Unlike the similar simple linear regression model that predicts Y from X , where the X values are known constants and only Y is subject to measurement variation (see Practice E3080), both X and Y are subject to measurement errors, with their variances denoted as σ_x^2 , σ_y^2 respectively (see A2.1). This fact requires a different criterion for the least-squares procedure, which is known as *errors-in variables* regression. Instead of minimizing the point-to-line differences in the Y direction, the direction of that distance is dependent on the precision ratio of Y with respect to X , denoted as $\lambda = \sigma_y^2/\sigma_x^2$.

8.2.1 The measurement error variances can be estimated from experience with current procedure use and method development data for the modified procedure. Alternatively, the comparison experiment can conduct duplicate test results from both procedures for estimating these variances, as noted in the referenced article (5).

8.2.2 In many situations it can be assumed that the two test methods have similar measurement error, thus dealing with the case that $\lambda = 1$. Then the least squares procedure minimizes the squared differences in the perpendicular direction from the points to the line. This is termed *orthogonal least squares*, and this procedure will be described in this section. Annex A2

discusses methodology for the situation where $\lambda \neq 1$.

8.3 *Statistical Analysis for Slope Equivalence*—For n pairs of test results, let the i^{th} pair consist of test results Y_i from the modified testing process and X_i from the current testing process, where $i = 1, \dots, n$. Slope equivalence is accepted if the slope β_1 from the estimated line of 8.1 is equivalent to the value 1, representing a 45° line slope relationship between X and Y .

8.3.1 Calculations for the estimated *orthogonal least squares* regression line are as follows.

8.3.1.1 Calculate the averages of X and Y :

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n} \quad (18)$$

$$\bar{Y} = \frac{\sum_{i=1}^n Y_i}{n} \quad (19)$$

8.3.1.2 Calculate the sums of squared deviations of X and Y from their averages, respectively S_{XX} and S_{YY} , and the sum of cross products of the X and Y deviations from their averages, S_{XY} :

$$S_{XX} = \sum_{i=1}^n (X_i - \bar{X})^2 \quad (20)$$

$$S_{YY} = \sum_{i=1}^n (Y_i - \bar{Y})^2 \quad (21)$$

$$S_{XY} = \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) \quad (22)$$

8.3.1.3 Calculate b_1 , the estimate of the slope β_1 :

$$b_1 = \frac{S_{YY} - S_{XX} + \sqrt{(S_{YY} - S_{XX})^2 + 4S_{XY}^2}}{2S_{XY}} \quad (23)$$

8.3.1.4 Calculate b_0 , the estimate of the Y intercept β_0 :

$$b_0 = \bar{Y} - b_1 \bar{X} \quad (24)$$

8.3.2 *Confidence Intervals*—Before consideration of confidence intervals on the slope, define the angle θ (theta) that the line makes with the horizontal (X) axis. The confidence limits will be symmetrical around the estimate of θ and thus will be more suitable for use in slope equivalence. Confidence intervals on the slope will not be symmetrical around the estimate of β_1 .

8.3.2.1 Calculate $\hat{\theta}$ the estimate of θ :

$$\hat{\theta} = \arctan(b_1) \quad (25)$$

The arctangent function is available on most spreadsheet or statistical software programs. For a brief summary of transformation to polar coordinates, see A2.1.4.

8.3.2.2 Calculate the half width φ (phi) of the two-sided 90 % confidence interval for θ :

$$\varphi = 0.5 \arcsin \left[t_{n-2,0.95} \frac{2}{\sqrt{n-2}} \sqrt{\frac{S_{YY}S_{XX} - S_{XY}^2}{(S_{YY} - S_{XX})^2 + 4S_{XY}^2}} \right] \quad (26)$$

where $t_{n-2,0.95}$ is the upper 95th quantile of the Student's t distribution with $n - 2$ degrees of freedom.

The arcsine function is available on most spreadsheet or statistical software programs.

8.3.2.3 Calculate the upper (UCL) and lower (LCL) confidence limits for the 100(1 - 2α) % two-sided confidence interval on θ:

$$LCL(\theta) = \hat{\theta} - \varphi \tag{27}$$

$$UCL(\theta) = \hat{\theta} + \varphi \tag{28}$$

8.3.2.4 The lower and upper two-sided 90 % confidence limits on β are the tangents of the confidence limits for θ:

$$LCL(\beta_1) = \tan(\hat{\theta} - \varphi) \tag{29}$$

$$UCL(\beta_1) = \tan(\hat{\theta} + \varphi) \tag{30}$$

The confidence limits for β₁ will not be symmetrical around the slope estimate b₁.

8.3.3 *Setting Equivalence Limits*—The equivalence limits for θ are symmetrical around π/4 = 0.7854 radians. The consideration of limits will usually be started in terms of the slope and converted to limits for θ using Table 3. For example, bioequivalence limits for pharmaceuticals are often 80 to 125 % percent of target. Thus, for example, slope equivalence limits of 0.80 and 1.25 around a target slope of 1.00, translates to equivalence limits on θ of 0.7854 ± 0.1107 radians, or to lower and upper equivalence limits 0.6747 and 0.8961 radians around the target of π/4 radians. The equivalence limit width ω is used in A2.4.1.

8.3.4 *Statistical Test for Equivalence*—Compute the upper (UCL) and lower (LCL) confidence limits for the 100(1 - 2α) % two-sided confidence interval on θ. If the confidence interval (θ̂ ± φ) is completely contained within the equivalence limits (π/4 ± E), or equivalently if LCL > -E and UCL < E, then accept slope equivalence. Otherwise, reject slope equivalence. This is a TOST procedure.

8.3.4.1 The angular values can also be back-transformed to slope values using β₁ = tan(θ), and a similar comparison of the confidence limits for the slope versus the slope equivalence limits can be made, which might be more intuitive to the user.

8.4 *Example*—A new in-line instrument (Inst B) for measuring total organic carbon (TOC) in purified water is compared with an existing instrument (Inst A) in 7.2 for means equivalence. This is a paired comparison design and the TOC test results by each instrument for 20 water samples are listed in Table 2. The data covered a TOC range of 35 to 60 parts per billion (ppb) and the data supported means equivalence (see 7.2.3) using an equivalence limit of E = 2 ppm. This example is now used for the linear statistical relationship model

approach to establish slope equivalence for the new instrument as compared with the current instrument.

8.4.1 Measurement error was assumed to be approximately the same for both instruments, thus for the slope equivalence test, λ = 1, supporting the use of the estimation of the linear statistical relationship by orthogonal least squares. The equivalence limits on the slope were set at E₁ = 0.8 and E₂ = 1.25. (In this application, slope units are a ratio of the same X and Y test result units, hence dimensionless.)

8.4.2 The TOC data from Table 2 are again listed for each instrument in Table 4 for 20 water sampling times. The current Instrument A is designated as the X variable and the new Instrument B is designated as the Y variable. The averages (Eq 18, Eq 19) are X̄ = 48.20 ppb and Ȳ = 48.66 ppb, and these values represent the centroid of the X, Y data set. The sums of squares and cross products (Eq 20-22) gave S_{XX} = 714.62, S_{YY} = 681.37, and S_{XY} = 687.53. The slope estimate (Eq 23) is b₁ = 0.9761. The Y intercept estimate (Eq 24) is 1.61 ppb.

8.4.3 To obtain the 90 % two-sided confidence interval for the slope, the conversion to the angular scale was made, resulting in θ̂ = arctan(b₁) = arctan (0.9761) = 0.7733 radians (Eq 25), which was close to the 45 degree line having arctan(1.0) = π/4 or 0.7854 radians. The confidence half-interval for φ (phi) for θ is calculated (Eq 26) as:

$$\begin{aligned} \varphi &= 0.5 \arcsin \left[t \frac{2}{\sqrt{n-2}} \sqrt{\frac{S_{YY}S_{XX} - S_{XY}^2}{(S_{YY} - S_{XX})^2 + 4S_{XY}^2}} \right] \\ &= 0.5 \arcsin \left[1.7341 \frac{2}{\sqrt{20-2}} \sqrt{\frac{(681.37)(714.62) - 687.53^2}{(681.37 - 714.62)^2 + 4(687.53)^2}} \right] \\ &= 0.5 \arcsin(0.0709) = 0.0355 \text{ radians} \end{aligned}$$

using t = 1.7341 based on 18 df. Then the 90 % confidence limits for θ (Eq 27, Eq 28) are (0.7378, 0.8088) and the 90 % confidence limits for β (Eq 29, Eq 30) are (0.9091, 1.0479). The confidence interval for β falls within the equivalence range of (0.8, 1.25) thus accepting slope equivalence.

8.5 *Sample Size Determination for Slope Equivalence*—Power is the probability of accepting equivalence for a given experiment, and power curves graphically illustrate the power versus the difference of the true slope from its target. As discussed in A2.2, the power curves for slope equivalence are dependent on the sample size (number of data pairs) and the sample correlation coefficient r. The correlation coefficient is a dimensionless statistic intended to measure the strength of a linear relationship between two variables. The value of the correlation coefficient ranges between -1 and +1, but for this

TABLE 3 Equivalence Limits for Slope and Angle

Equivalence Limits				Equivalence Intervals	
For Slope β ₁		For Angle θ		Interval for θ	Width ω
Lower	Upper	Lower	Upper	π/4 ± φ	2φ
0.75	1.33	0.6435	0.9273	0.7854 ± 0.1419	0.2838
0.80	1.25	0.6747	0.8961	0.7854 ± 0.1107	0.2214
0.85	1.18	0.7045	0.8663	0.7854 ± 0.0809	0.1618
0.90	1.11	0.7328	0.8380	0.7854 ± 0.0526	0.1052
0.95	1.05	0.7598	0.8110	0.7854 ± 0.0256	0.0512

TABLE 4 Data and Calculations for Straight Line Regression on TOC Example

Sample	InstA	InstB	Deviations from Avg		Residual	Statistic	Results
Point, <i>i</i>	X_i	Y_i	$X_i - \bar{X}$	$Y_i - \bar{Y}$	e_i		
1	46.4	48.8	-1.80	0.14	1.36	S_{XX}	714.62
2	44.2	43.5	-4.00	-5.16	-0.90	S_{YY}	618.37
3	52.4	53.0	4.20	4.34	0.17	S_{XY}	687.53
4	37.6	37.3	-10.60	-11.36	-0.73	Slope, b_1	0.9761
5	49.3	49.1	1.10	0.44	-0.45	Intercept, b_0	1.61
6	45.0	44.5	-3.20	-4.16	-0.74	Angle, θ	0.7733
7	51.4	51.3	3.20	2.64	-0.35	Half interval, φ	0.0355
8	57.6	56.8	9.40	8.14	-0.74	90 % Conf. Int. for θ :	
9	43.4	44.9	-4.80	-3.76	0.66	LCL, θ	0.7378
10	45.2	44.1	-3.00	-4.56	-1.17	UCL, θ	0.8088
11	59.0	58.5	10.80	9.84	-0.50	90 % Conf. Int. for β_1 :	
12	43.1	44.1	-5.10	-4.56	0.30	LCL, β_1	0.9091
13	39.3	40.9	-8.90	-7.76	0.66	UCL, β_1	1.0479
14	48.2	48.4	0.00	-0.26	-0.19	Corr. Coef., r	0.9853
15	48.7	49.0	0.50	0.34	-0.11	Equiv. Limit for β_1 :	
16	44.4	46.1	-3.80	-2.56	0.82	Lower, E_1	0.80
17	52.7	53.2	4.50	4.54	0.11	Upper, E_2	1.25
18	43.3	44.6	-4.90	-4.06	0.52		
19	54.4	56.7	6.20	8.04	1.42		
20	58.4	58.4	10.20	9.74	-0.15		
Average	48.20	48.66	0.00	0.00	0.00		
Variance	37.6116	35.8615			0.5702		

application the value of r should be close to 1, say $r > 0.9$, in order to have a good chance of meeting slope equivalence.

8.5.1 The estimated correlation coefficient, r , from a set of paired data (X_i, Y_i) is calculated from the three statistics, S_{XX} , S_{YY} , and S_{XY} :

$$r = \frac{S_{XY}}{S_{XX}S_{YY}} \quad (31)$$

For the TOC example:

$$r = \frac{687.53}{\sqrt{(714.62)(681.37)}} = 0.9853$$

8.5.2 Power curves for the TOC example with $r=0.985$ are shown in Fig. 3 for sample sizes of 10, 20, and 40 pairs of data,

and the calculations for these are given in A2.2.4. These curves are shown for the true slope (β_1) greater than 1 only, but the pattern will be similar for slopes less than 1. For $n = 10$ data pairs the power is poor, and the power curve is almost linear as the true slope runs from 1 to its upper equivalence limit of 1.25. The power for the example, $n = 20$, is above 80 % for true slopes out to about half-way towards the equivalence limit. Doubling the experiment to $n = 40$ gives enhanced power and sharper definition. All power curves intersect at 1.25, the upper equivalence limit for slope, at a power of $\alpha = 0.05$.

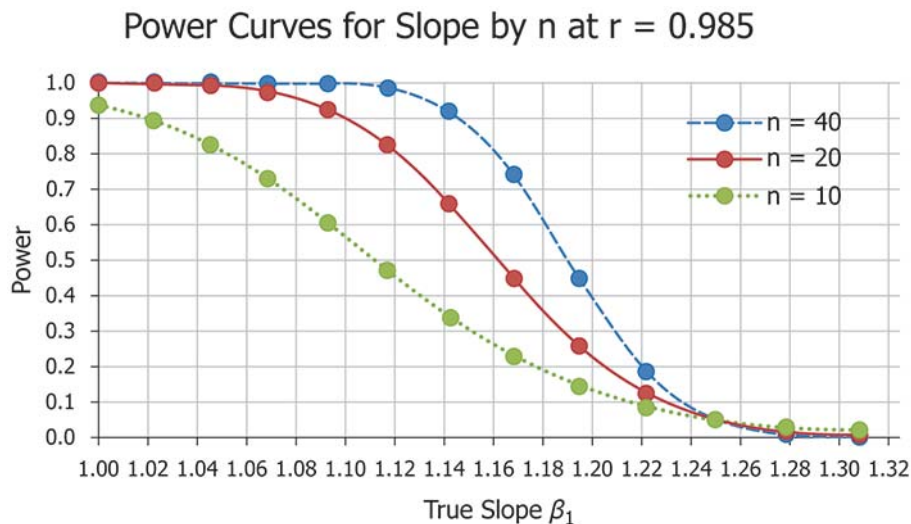


FIG. 3 Power Curves for Slope Equivalence versus n