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An American National Standard

Standard Practice for Life and Reliability Testing Based on the Exponential Distribution[1]

This standard is issued under the fixed designation E2696; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice presents standard sampling procedures and tables for life and reliability testing in procurement, supply, and maintenance quality control operations as well as in research and development activities.

1.2 This practice describes general procedures and definitions of terms used in life test sampling and describes specific procedures and applications of the life test sampling plans for determining conformance to established reliability requirements.

1.3 This practice is an adaptation of the Quality Control and Reliability Handbook H-108, "Sampling Procedures and Tables for Life and Reliability Testing (Based on Exponential Distribution)," U.S. Government Printing Office, April 29, 1960.

1.4 A system of units is not specified in this practice.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.6 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E456 Terminology Relating to Quality and Statistics
E2234 Practice for Sampling a Stream of Product by Attributes Indexed by AQL
E2555 Practice for Factors and Procedures for Applying the MIL-STD-105 Plans in Life and Reliability Inspection

3. Terminology

3.1 *Definitions:*

3.1.1 See Terminology E456 for a more extensive listing of terms in ASTM Committee E11 standards.

[1] This practice is under the jurisdiction of ASTM Committee E11 on Quality and Statistics and is the direct responsibility of Subcommittee E11.40 on Reliability.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

3.1.2 *consumer's risk, β, n*—probability that a lot having specified rejectable quality level will be accepted under a defined sampling plan. **E2555**

3.1.2.1 *Discussion*—
In this practice, the consumer's risk is the probability of accepting lots with mean time to failure θ_1.

3.1.2.2 *Discussion*—
For the procedures of 9.7 and 9.8, the consumer's risk may also be defined as the probability of accepting lots with unacceptable proportion of lot failing before specified time, p_1.

3.1.3 *life test, n*—process of placing one or more units of product under a specified set of test conditions and measuring the time until failure for each unit.

3.1.4 *mean time to ~~failure,~~ failure (MTTF), θ, n—in life testing*, the average length of life of items in a lot.

3.1.4.1 *Discussion*—
Also ~~known~~ referred to as mean life.

3.1.5 *number of failures, n*—number of failures that have occurred at the time the decision as to lot acceptability is reached.

3.1.5.1 *Discussion*—
The expected number of failures required for decision is the average of the number of failures required for decision when life tests are conducted on a large number of samples drawn at random from the same exponential distribution.

3.1.6 *producer's risk, α, n*—probability that a lot having specified acceptable quality level will be rejected under a defined sampling plan.

3.1.6.1 *Discussion*—
In this practice, the producer's risk is the probability of rejecting lots with mean time to failure θ_0.

3.1.6.2 *Discussion*—
For the procedures of 9.7 and 9.8, the producer's risk may also be defined as the probability of rejecting lots with acceptable proportion of lot failing before specified time, p_0.

3.1.7 *sequential life test, n*—life test sampling plan whereby neither the number of failures nor the time required to reach a decision are fixed in advance but instead decisions depend on the accumulated results of the life test.

3.1.8 *unit of product, n*—that which is inspected to determine its classification as defective or nondefective or to count the number of defects. **E2234**

3.1.9 *waiting time, n—in life testing*, the time elapsed from the start of testing until a decision is reached as to lot acceptability.

3.1.9.1 *Discussion*—
The expected waiting time required for decision is the average of the waiting times required for decision when life tests are conducted on a large number of samples drawn at random from the same exponential distribution.

4. Significance and Use

4.1 This practice was prepared to meet a growing need for the use of standard sampling procedures and tables for life and reliability testing in government procurement, supply, and maintenance quality control (QC) operations as well as in research and development activities where applicable.

4.2 A characteristic feature of most life tests is that the observations are ordered in time to failure. If, for example, 20 radio tubes are placed on life test, and t_i denotes the time when the *i*th tube fails, the data occur in such a way that $t_1 \leq t_2 \leq ... \leq t_n$. The same kind of ordered observations will occur whether the problem under consideration deals with the life of electric bulbs, the life of electronic components, the life of ball bearings, or the length of life of human beings after they are treated for a disease. The examples just given all involve ordering in time.

4.3 In destructive testing involving such situations as the current needed to blow a fuse, the voltage needed to break down a condenser, or the force needed to rupture a physical material, the test can often be arranged in such a way that every item in the sample is subjected to precisely the same stimulus (current, voltage, or stress). If this is done, then clearly the weakest item will be observed to fail first, the second weakest next, and so forth. While the random variable considered mostly in this guide is time to failure, it should be emphasized, however, that the methodology provided herein can be adapted to the testing situations mentioned above when the random variable is current, voltage, stress, and so forth.

4.4 Sections 6 and 7 describe general procedures and definitions of terms used in life test sampling. Sections 8, 9, and 10 describe specific procedures and applications of the life test sampling plans for determining conformance to established reliability requirements.

4.5 Whenever the methodology or choice of procedures in the practice requires clarification, the user is advised to consult a qualified mathematical statistician, and reference should be made to appropriate technical reports and other publications in the field.

5. Introduction

5.1 The theory underlying the development of the life test sampling plans of this section, including the operating characteristic curves, assumes that the measurements of the length of life are drawn from an exponential distribution. Statistical test procedures for determining the validity of the exponential distribution assumption have appeared in the technical statistical journals. Professor Benjamin Epstein published a comprehensive paper (in two parts) on this subject in the February and May 1960 issues of *Technometrics*.[3] Part I of the paper contains descriptions of the mathematical and graphical procedures as well as an extensive bibliography for reference purposes. Numerical examples illustrating the statistical procedures are included in Part II of the paper.

5.2 It is important to note that the life test sampling plans of this practice are not to be used indiscriminately simply because it is possible to obtain life test data. Only after the exponential assumption is deemed reasonable should the sampling plans be used.

5.3 Sections 6 and 7 describe general procedures and description of life test sampling plans. Section 8 describes specific procedures and applications of sampling plans when life tests are terminated upon the occurrence of a preassigned number of failures, and Section 9 provides sampling plans when life tests are terminated at a preassigned time. Section 10 describes sequential life test sampling plans. Section 8 covers: *(1)* acceptance procedures; *(2)* expected duration of life tests and cost considerations in selection of sample sizes; and *(3)* life test plans for certain specified values of α, β, and θ_1/θ_0. Section 9 covers: *(1)* acceptance procedures; *(2)* life test plans for certain specified values of α, β, θ_1/θ_0, and T/θ_0; and *(3)* life test plans based on proportion of lot failing before specified time. Section 10 covers: *(1)* acceptance procedures; *(2)* graphical acceptance procedures; and *(3)* expected number and waiting time required for decision.

5.4 Operating characteristic (OC) curves for the life test sampling plans of 8.1 – 8.5, 9.1 – 9.5, and Section 10 are shown in Fig. A1.1 for the corresponding sampling plans in these sections were matched with respect to their OC curves. The OC curves in Fig. A1.1 have been computed for the life test sampling plans of 8.1 – 8.5 but are equally applicable for the sampling plans of 9.1 – 9.5 and Section 10.

5.5 The procedures of this section are based on the premise that the life tests are monitored continuously. If the tests are monitored only periodically, the values obtained from the tables and curves are only approximations.

6. General Definitions of Life and Reliability Test Terms

6.1 *Discussion of Terms and Procedures:*

6.1.1 *Purpose*—This section provides definitions of terms required for the life test sampling plans and procedures of Sections 7 through 10.

6.1.2 *Life Test*—Life test is the process of placing the "unit of product" under a specified set of test conditions and measuring the time it takes until failure.

6.1.3 *Unit of Product*—The unit of product is the entity of product that may be placed on life test.

6.1.4 *Specifying Failure*—The state that constitutes a failure shall be specified in advance of the life test.

[3] Epstein, B., "Tests for the Validity of the Assumption that the Underlying Distribution of Life is Exponential," *Technometrics*, Vol 2, February and May 1960, pp. 83–101 and 167–183.

6.1.5 *Life Test Sampling Plan*—A life test sampling plan is a procedure that specifies the number of units of product from a lot that are to be tested and the criterion for determining acceptability of the lot.

6.1.6 *Life Test Terminated upon Occurrence of Preassigned Number of Failures*—Life test sampling plans whereby testing is terminated when a preassigned termination number of failures, r, occur are given in Section 8 of this practice.

6.1.7 *Life Test Terminated at Preassigned Time*—Life test sampling plans whereby testing is terminated when a preassigned termination time, T, is reached are given in Section 9 of this practice.

6.1.8 *Sequential Life Test*—Sequential life test is a life test sampling plan whereby neither the number of failures nor the time required to reach a decision are fixed in advance but, instead, decisions depend on the accumulated results of the life test. Information on the observed time to failure are accumulated over time and the results at any time determine the choice of one among three possible decisions: *(1)* the lot meets the acceptability criterion, *(2)* the lot does not meet the acceptability criterion, or *(3)* the evidence is insufficient for either decision *(1)* or *(2)* and the test must continue. Sequential life test sampling plans are given in Section 10 of this practice and have the advantage over the life test sampling plans mentioned in 6.1.6 and 6.1.7 in that, for the same OC curve, the expected waiting time and the expected number of failures required to reach a decision as to lot acceptability are less for the sequential life tests.

6.1.9 *Expected Number of Failures*—The number of failures required for decision is the number of failures that have occurred at the time the decision as to lot acceptability is reached. For the life test sampling plans mentioned in 6.1.6, this number of failures is known in advance of the life test; but, for the sampling plans mentioned in 6.1.7 and 6.1.8, this number cannot be predetermined. The expected number of failures required for decision is the average of the number of failures required for decision when life tests are conducted on a large number of samples drawn at random from the same exponential distribution. The expected number of failures can be predetermined for the sampling plans mentioned in 6.1.6 – 6.1.8.

6.1.10 *Expected Waiting Time*—The waiting time required for decision is the time elapsed from the start of the life test to the time decision is reached as to lot acceptability. The waiting time required for decision cannot be predetermined for any of the sampling plans mentioned in 6.1.6 – 6.1.8. The expected waiting time required for decision is the average of the waiting times required for decision when life tests are conducted on a large number of samples drawn at random from the same exponential distribution. The expected waiting time can be predetermined for the sampling plans mentioned in 6.1.6 – 6.1.8.

6.2 *Length of Life:*

6.2.1 *Length of Life*—The terms "length of life" and "time to failure" may be used interchangeably and shall denote the length of time it takes for a unit of product to fail after being placed on life test. The length of time may be expressed in any convenient time scale such as seconds, hours, days, and so forth.

6.2.2 *Mean Time to Failure*—The terms "mean time to failure" and "mean life" may be used interchangeably and shall denote the mean (or equivalently, the average) length of life of items in the lot. Mean life is denoted by θ.

6.2.3 *Acceptable Mean Life*—The acceptable mean life, θ_0, is the minimum mean time to failure that is considered satisfactory.

6.2.4 *Unacceptable Mean Life*—The unacceptable mean life, θ_1 ($\theta_1 < \theta_0$), is the mean time to failure such that lots having a mean life less than or equal to θ_1 are considered unsatisfactory. The interval between θ_0 and θ_1 is a zone of indifference in which there is a progressively greater degree of dissatisfaction as the mean life decreases from θ_0 to θ_1.

6.3 *Failure Rate:*

6.3.1 *Proportion of Lot Failing Before Specified Time*—The term "proportion of lot failing before specified time," p, denotes the fraction of the lot that fails before some specified time, T, that is:

$$p = 1 - \exp(-T/\theta) \tag{1}$$

6.3.2 *Failure Rate during Period of Time*—The "failure rate during period of time T," G, is given by:

$$G = \frac{1}{T}\{1 - \exp(T/\theta)\} = p/T \tag{2}$$

6.3.3 *Instantaneous Failure Rate*—The "instantaneous failure rate" or "hazard rate" is given by:

$$Z = 1/\theta \tag{3}$$

6.3.4 *Acceptable Proportion of Lot Failing Before Specified Time*—The "acceptable proportion of lot failing before specified time," p_0, is the maximum fraction of the lot that may fail before time, T, and still result in the lot being considered satisfactory.

6.3.5 *Unacceptable Proportion of Lot Failing Before Specified Time*—The "unacceptable proportion of lot failing before specified time," p_1, ($p_1 > p_0$), is the minimum fraction of the lot that may fail before time, T, and results in the lot being considered unsatisfactory. The interval between p_0 and p_1 is a zone of indifference in which there is a progressively greater degree of dissatisfaction as the fraction of the lot failing before time, T, increases from p_0 to p_1.

6.3.6 *Acceptable Failure Rate During Period of Time*—The "acceptable failure rate during period of time," G_0, is the maximum failure rate during the period of time that can be considered satisfactory.

6.3.7 *Unacceptable Failure Rate During Period of Time*—The "unacceptable failure rate during period of time," G_1, ($G_1 > G_0$), is the minimum failure rate during the period of time that results in the lot being considered unsatisfactory. The interval between G_0 and G_1 is a zone of indifference in which there is a progressively greater degree of dissatisfaction as the failure rate increases from G_0 to G_1.

6.3.8 *Life Test Sampling Plans Based on Failure Rates*—Life test sampling plans that are based on failure rates are given in 9.7 and 9.8.

6.4 *OC Curves and Sampling Risks:*

6.4.1 *OC Curve*—The OC curve of a life test sampling plan is the curve that shows the probability that a submitted lot with given mean life would meet the acceptability criterion on the basis of that sampling plan.

6.4.2 *Producer's Risk*—The producer's risk, α, is the probability of rejecting lots with mean life, θ_0. For the procedures of 9.7 and 9.8, the producer's risk may also be defined as the probability of rejecting lots with acceptable proportion of lot failing before specified time, p_0.

6.4.3 *Consumer's Risk*—The consumer's risk, β, is the probability of accepting lots with mean life, θ_1. For the procedures of 9.7 and 9.8, the consumer's risk may also be defined as the probability of accepting lots with p_1 as the unacceptable proportion of lot failing before specified time.

6.5 *Submittal of Product:*

6.5.1 *Lot*—The term "lot" shall mean either an "inspection lot," that is, a collection of units of product manufactured under essentially the same conditions from which a sample is drawn and tested to determine compliance with the acceptability criterion or, a "preproduction lot," that is, one or more units of product submitted before the initiation of production for test to determine compliance with the acceptability criterion.

6.6 *Sample Selection:*

6.6.1 *Drawing of Samples*—A sample is one or more units of product drawn at random from a lot.

6.6.2 *Testing without Replacement*—Life test sampling without replacement is a life test procedure whereby failed units are not replaced.

6.6.3 *Testing with Replacement*—Life test sampling with replacement is a life test procedure whereby the life test is continued with each failed unit of product replaced by a new one, drawn at random from the same lot, as soon as the failure occurred. In the case of complex unit of product, this may be interpreted to mean replacement of the component that caused the failure by a new component drawn at random from the same lot of components. When the "sample sizes" are the same in both instances, the expected waiting time required for decision when testing with replacement is less than when testing without replacement.

6.6.4 *Sample Size*—The sample size, n, for a life test is the number of units of product placed on test at the start of a life test. When testing with replacement, the total number of units of product placed on test will, in general, be greater than the original sample size. The sample sizes for the life test plans of Sections 8 to 10 depend on the relative cost of placing large numbers of units of product on test and the expected length of time the life tests must continue to determine acceptability of the lots. Increasing the sample size will, on one hand, cut the average time required to determine acceptability but, on the other hand, will increase the cost because of placing more units of product on test.

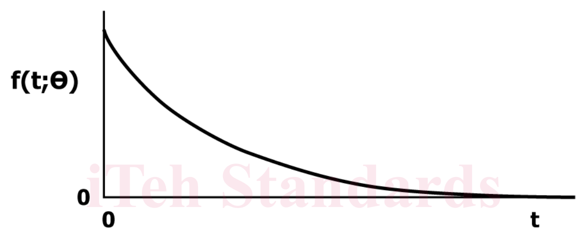
6.7 *Exponential Distribution:*

6.7.1 *Exponential Distribution with One Parameter*—The density function for the exponential distribution with one parameter is given by:

$$f(t;\theta) = 1/\theta \exp(-t/\theta) \qquad t \geq 0, \theta > 0 \tag{4}$$

$$= 0 \qquad t < 0$$

6.7.1.1 The function has the following general graphical form:

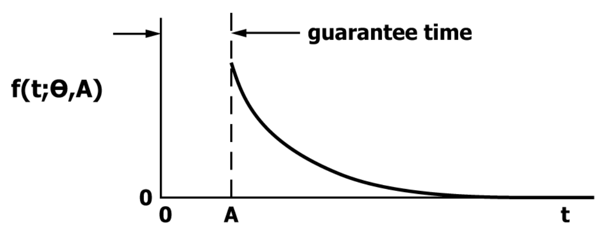


6.7.2 *Exponential Distribution with Two Parameters*—The density function for the exponential distribution with two parameters is given by:

$$f(t;\theta,A) = 1/\theta \exp[-(t-A)/\theta] \qquad t \geq A \geq 0 \tag{5}$$

$$= 0 \qquad \text{elsewhere}$$

6.7.2.1 The function has the following general graphical form:



6.7.2.2 The quantity, A, is called "guarantee time" and the one parameter case is a special case of the two-parameter distribution with a guarantee time of zero.

6.7.3 *Exponential Distribution when Number of Parameters Is Unspecified*—In this practice, whenever the term "exponential distribution" is mentioned without specific mention of the number of parameters, it shall be assumed to mean the exponential distribution with one parameter.

7. General Description of Life Test Sampling Plans

7.1 *Scope:*

7.1.1 *Purpose*—Sections 7 through 10 of this practice establish life test sampling plans for determining acceptability of a product when samples are drawn at random from an exponential distribution.

7.1.2 *Specifying Acceptable Mean Life*—Before the start of the life test, the particular value of the acceptable mean life, θ_0, shall be specified except when using the procedures of 9.7 and 9.8.

7.1.3 *Specifying Unacceptable Mean Life*—The particular value of the unacceptable mean life, θ_1, shall be specified in advance of the life test when using the life test procedures of 8.6 and 9.6.

7.1.4 *Specifying Acceptable Proportion of Lot Failing before Specified Time*—The particular value, p_0, of the acceptable proportion of lot failing before specified time to be used in the life test shall be specified in advance for the procedures of 9.7 and 9.8.

7.1.5 *Specifying Unacceptable Proportion of Lot Failing before Specified Time*—The particular value, p_1, of the unacceptable proportion of lot failing before specified time shall be specified in advance of the life test when using the procedures of 9.7 and 9.8.

7.2 *Sampling Risks:*

7.2.1 *Producer's Risk*—The producer's risk, α, is the probability of rejecting lots with mean life, θ_0. For the procedures of 9.7 and 9.8, the producer's risk may also be defined as the probability of rejecting lots with p_0 as the acceptable proportion of lot failing before specified time. Summarized in the following are the various numerical values of α and the master sampling tables in which they are given.

Procedures for	Producer's Risk	Table
8.1 – 8.5	0.01, 0.05, 0.10, 0.25, 0.50	Table A1.2
8.6	0.01, 0.05, 0.10, 0.25	Table A1.7
9.1 – 9.5	0.01, 0.05, 0.10, 0.25, 0.50	Tables A1.8-A1.12 and Tables A1.13-A1.17
9.6	0.01, 0.05, 0.10, 0.25	Table A1.18 and Table A1.19
9.7 and 9.8	0.01, 0.05, 0.10	Table A1.20
10	0.01, 0.05, 0.10, 0.25, 0.50	Tables A1.21-A1.25

7.2.2 *Specifying Producer's Risk*—The particular value of α to be used in the life test shall be selected from among those given in 7.2.1 and specified in advance of the life test.

7.2.3 *Consumer's Risk*—The consumer's risk, β, is the probability of accepting lots with mean life, θ_1. For the procedures of 9.7 and 9.8, the consumer's risk may also be defined as the probability of accepting lots with p_1 as the unacceptable proportion of lot failing before specified time. Summarized in the following are the various numerical values of β and the master sampling tables in which they are given.

Procedures for	Consumer's Risk	Table
8.1 – 8.5	0.10	Table A1.2
8.6	0.01, 0.05, 0.10, 0.25	Table A1.7
9.1 – 9.5	0.10	Tables A1.8-A1.12 and Tables A1.13-A1.17
9.6	0.01, 0.05, 0.10, 0.25	Table A1.18 and Table A1.19
9.7 and 9.8	0.01, 0.05, 0.10	Table A1.20
10	0.10	Tables A1.21-A1.25

7.2.3.1 The smaller the value of β, the greater is the protection against acceptance of lots with low mean life or high failure rate.

7.2.4 *Specifying Consumer's Risk*—The particular value of β to be used in the life test shall be selected from among those given in 7.2.3 and specified in advance of the life test.

7.3 *OC Curves:*

7.3.1 *OC Curve*—The OC curve of a life test sampling plan is the curve that shows the probability that a submitted lot with given mean life would meet the acceptability criterion on the basis of that sampling plan. The OC curves given in Fig. A1.1 are equally applicable for the sampling plans of 8.1 – 8.5, 9.1 – 9.5, and Section 10. Moreover, the OC curves are also equally applicable for both the sampling with and without replacement procedures. The abscissas of the OC curves are expressed as the ratio θ/θ_0 in Fig. A1.1 so that the same set of OC curves is applicable regardless of the value of the specified acceptable mean life θ_0.

7.3.2 *Sampling Plan Code Designation*—The life test sampling plans of 8.1 – 8.5, 9.1 – 9.5, and Section 10, along with their associated OC curves, are designated by code letters and numbers. The sample code is given in Table A1.1 and is determined by the values of α, β, and θ_1/θ_0. The OC curves of all sampling plans designated by the same code pass through the two points (1, 1-α) and (θ_1/θ_0, β = 0.10). Thus, all sampling plans that are designated by the same code offer essentially the same protection.

7.3.3 *Ratio θ_1/θ_0 as Measure of Protection Offered by Sampling Plan*—The consumer's risk β has been defined in 7.2.3 as the risk of accepting lots with mean life, θ_1. Because the OC curves are drawn with abscissa, θ_1/θ_0, the ratio, θ_1/θ_0, is also a measure of mean life that is accepted with probability, β. The ratio, θ_1/θ_0, shall be greater than zero but less than unity. If α, β, and θ_0 are kept constant, as θ_1/θ_0 increases, the protection offered by the sampling plan against accepting lots with low mean life also increases. Thus, Table A1.1 allows comparisons in the amount of protection offered by the various sampling plans, for in any column, the protection increases as θ_1/θ_0 increases.

7.4 *Specifying Acceptance Procedures*—To identify completely the sampling plan to be used, the following shall be specified for the sampling plans of:

8.1 – 8.5	α, r, θ_0 or sample plan code, θ_0
8.6	α, β, θ_0, θ_1
9.1 – 9.5	θ_0, r, α, n or sample plan code, n, θ_0
9.6	α, β, θ_0, θ_1, T
9.7 and 9.8	α, β, p_0, p_1, T or α, β, G_0, G_1, T
10	Sample plan code, θ_0

7.4.1 In addition, the use of life testing with or without replacement may be specified, except when using the sampling plans of 9.7 and 9.8.

8. Life Tests Terminated upon Occurrence of Preassigned Number of Failures

8.1 *Life Test Sampling Plans*—This part of the practice describes the procedures for use with life tests that are terminated upon the occurrence of a preassigned number of failures. Two procedures are given: *(1)* a procedure when testing without replacement and *(2)* another procedure when testing with replacement.

8.1.1 *Use of Life Test Sampling Plans*—To determine whether the lot meets the acceptability criterion with respect to average length of life, the applicable sampling plan shall be used in accordance with the provisions of Section 7 and those in this part of the practice.

8.1.2 *Drawing of Samples*—All samples shall be drawn in accordance with 6.6.

8.2 *Selecting the Life Test Sampling Plan:*

8.2.1 *Master Sampling Table*—The master sampling table for the life test sampling plans of this part of the practice is Table A1.2.

8.2.2 *Obtaining the Sampling Plan*—The life test sampling plan consists of a sample size, a termination number, and an associated acceptability constant. The sampling plan is obtained from Master Table A1.2.

8.2.2.1 *Sample Sizes*—For the procedures of 8.1 – 8.5, the acceptability constants and the OC curves do not depend on the number of units of product placed on test. The sample size, as mentioned in 6.6.4, depends on the relative cost of placing large numbers of units of product on test and the expected length of time the life test shall continue. The sample size may be selected by using the procedures of 8.5.

8.2.2.2 *Termination Number*—The termination number, r, may be selected from among those given in Table A1.2 and specified before the initiation of the life test. The choice of this number shall be dependent on the degree of protection desired against acceptance of material with unacceptable mean life. The larger the termination number, the larger is the ratio, θ_1/θ_0, and, as mentioned in 7.3.3, the greater is the assurance against accepting material with an unacceptable mean life.

8.2.2.3 *Acceptability Constant*—The acceptability constant, C, corresponding to the applicable termination number, r, and producer's risk, α, is obtained from the master table by multiplying the tabled entry by the acceptable mean life, θ_0.

8.3 *Lot Acceptability Procedures when Testing without Replacement:*

8.3.1 *Estimate of Mean Life*—The acceptability of a lot, when using a life test from this part of the practice, shall be judged by the quantity, $\hat{\theta}_{r,n}$.

8.3.2 *Computation*—The following quantity shall be computed from the test results:

$$\hat{\theta}_{r,n} = \frac{1}{r}\left[\sum_{i=1}^{r} x_{i,n} + (n-r)x_{r,n}\right] \quad (6)$$

where:

$\hat{\theta}_{r,n}$ = estimate of the lot mean life,
r = termination number,
n = sample size, and
$x_{i,n}$ = time when the ith failure occurs. $i = 1, 2, \ldots, r$.

8.3.3 *Acceptability Criterion*—Compare the quantity $\hat{\theta}_{r,n}$ with the acceptability constant C, mentioned in 8.2.2.3. If $\hat{\theta}_{r,n}$ is equal to or greater than C, the lot meets the acceptability criterion; if $\hat{\theta}_{r,n}$ is less than C, then the lot does not meet the acceptability criterion.

8.4 *Lot Acceptability Procedures when Testing With Replacement:*

8.4.1 *Estimate of Mean Life*—The acceptability of a lot, when using a life test from this part of the practice, shall be judged by the quantity $\hat{\theta}_{r,n}$.

8.4.2 *Computation*—The following quantity shall be computed from the test results:

$$\hat{\theta}_{r,n} = n\, x_{r,n}/r \quad (7)$$

where:

$\hat{\theta}_{r,n}$ = estimate of the lot mean life,
r = termination number,
n = original sample size, and
$x_{r,n}$ = time when the rth failure occurs.

8.4.3 *Acceptability Criterion*—Compare the quantity $\hat{\theta}_{r,n}$ with the acceptability constant, C, mentioned in 8.2.2.3. If $\hat{\theta}_{r,n}$ is equal to or greater than C, the lot meets the acceptability criterion; if $\hat{\theta}_{r,n}$ is less than C, then the lot does not meet the acceptability criterion.

8.4.3.1 *Example 1: Use of* Table A1.2—Find a life test plan that is to be stopped on the occurrence of the fifth failure and will accept a lot having an acceptable mean life of 1000 h with probability 0.90.

8.4.3.2 *Solution*—In the notation of this section, $\theta_0 = 1000$, $\alpha = 0.10$, and $r = 5$. In the testing without replacement case:

$$\hat{\theta}_{r,n} = \frac{1}{5}\left[x_{1,n} + x_{2,n} + x_{3,n} + x_{4,n} + x_{5,n} + (n-5)\,x_{5,n}\right] \quad (8)$$

(1) In the replacement case, $\hat{\theta}_{r,n} = nx_{5,n}/5$. The acceptability criterion is, accept the lot if:

$$\hat{\theta}_{5,n} \geq C \quad (9)$$

$$\geq \theta_0(C/\theta_0) = (1000)(0.487) = 487$$

(2) The quantity $C/\theta_0 = 0.487$ is obtained from Table A1.2. In words, place n items on test. Wait until the first five failures occur. Compute $\hat{\theta}_{5,n}$. Accept the lot if $\hat{\theta}_{5,n} \geq 487$; reject the lot otherwise.

(3) The code designation for the above life test sampling plan is obtained from Table A1.2 as C-5. From Fig. A1.1, the probability of accepting a lot with mean life of, say, 500 h may be obtained by finding the ordinate of the OC curve labeled C-5 at the point where the abscissa $\theta/\theta_0 = 500/1000 = 0.5$. The probability is seen to be equal to 0.47.

(4) In this example, if the termination number had been selected as 6 instead of 5, the probability of accepting a lot with mean life of 500 h is obtained from the OC curve labeled C-6. The probability is seen to equal 0.41. This illustrates the remark made in 8.2.2.2 that the larger the termination number, the higher the probability of rejecting lots with unacceptable mean life.

8.4.3.3 *Example 2: Calculations for Testing Without Replacement*—Suppose that in the life test of Example 1, 10 units of product had been placed on test. If the failed units were not replaced and the first 5 failure times were 50, 75, 125, 250, and 300, determine whether the lot met the acceptability criterion.

8.4.3.4 *Solution*—In this case:

$$\hat{\theta}_{5,10} = \frac{50+75+125+250+300+5(300)}{5} = 460 \quad (10)$$

(1) Since 460 < 487, the lot did not meet the acceptability criterion.

8.4.3.5 *Example 3: Calculations for Testing With Replacement*—Suppose that in the life test of Example 1, 10 units of product had been placed on test. If the failed units were replaced immediately and the first 5 failure times were 56, 128, 176, 276, and 442, determine whether the lot met the acceptability criterion.

8.4.3.6 *Solution*—In this case:

$$\hat{\theta}_{5,10} = \frac{10(442)}{5} = 884 \quad (11)$$

(1) Since 884 > 487, the lot met the acceptability criterion.

8.5 *Expected Waiting Time of Life Tests and Cost Considerations in Selection of Sample Sizes*—The operating characteristics of the life test sampling plans of 8.1 – 8.4 are independent of the number of units of product placed on test. Thus, all tests based on common values of the termination number, r, and producer's risk, α, are equally good, and the choice of the sample size, n, depends only on the relative cost of placing a large number of units of product on test and the expected waiting time required for decision. For fixed α and r, increasing n will, on one hand, cut the expected waiting time; but will, on the other hand, increase the cost because of placing more units of product on test. This part of the practice provides procedures for determining the optimum sample size based on considerations of cost.

8.5.1 *Expected Waiting Time*—The mean life of the lot and, as noted in 8.5, the size of the sample drawn from the lot affect the expected waiting time required to observe the rth failure in a sample of size n. The rth failure is expected to occur more quickly in samples drawn from lots with low values of mean life. The values of the expected waiting time divided by the mean life of the lot, when testing without replacement, are given in Table A1.3 and Table A1.4. Corresponding values for the testing with replacement situation are not tabled but may be calculated by dividing the termination number, r, by the sample size, n, that is:

$$\frac{\text{Expected Waiting Time}}{\text{Mean Life of a Lot}} = \frac{r}{n} \tag{12}$$

8.5.2 *Relative Saving in Time by Increasing Sample Size When Testing Without Replacement*—When testing without replacement, the expected waiting time required to observe the *r*th failure in a sample of size *n*, ($n \geq r$), may be obtained from Table A1.3 or Table A1.4 by multiplying the tabled entry by the mean life of the lot. By dividing the expected waiting time when *n* units of product are placed on test by that when only *r* units are placed on test, the mean life of the lot cancels out and the ratio

$$\frac{\text{Expected Waiting Time for } r \text{ Failures in Sample of } n}{\text{Expected Waiting Time for } r \text{ Failures in Sample of } r} \tag{13}$$

is a measure of the relative expected saving in time as a result of placing more units of product on test. A brief table of these ratios is given in Table A1.5.

8.5.3 *Relative Saving in Time by Increasing Sample Size When Testing With Replacement*—When testing with replacement, the expected waiting time required to observe the *r*th failure in a sample of size *n* is equal to the quantity $r\theta/ n$. By dividing the expected waiting time when *n* units of product are placed on test by that when only *r* units are placed on test, the mean life of the lot cancels out and the ratio

$$\text{Relative Saving} = r\theta/n\theta = r/n \tag{14}$$

is a measure of the relative expected saving in time as a result of using larger sample sizes.

8.5.4 *Relative Saving in Time by Testing with Replacement as Compared to Testing Without Replacement*—When testing with replacement, the expected waiting time required to observe the *r*th failure in a sample of size *n* ($n \geq r$) is equal to the quantity $r\theta/n$. When testing without replacement, this expected waiting time may be obtained from Table A1.3 or Table A1.4 by multiplying the tabled entry by the mean life of the lot θ. By dividing these two expected waiting times, the mean life of the lot cancels out and the ratio

$$\frac{\text{Expected Waiting Time for } r \text{ Failures in Sample of } n \text{ When Testing With Replacement}}{\text{Expected Waiting Time for } r \text{ Failures in Sample of } n \text{ When Testing Without Replacement}} \tag{15}$$

is a measure of the relative expected saving in time as a result of sampling with replacement. A brief table of these ratios is given in Table A1.6.

8.5.4.1 *Example 4: Saving in Time by Increasing Sample Size When Testing Without Replacement*—Compare the average length of time needed to observe the failure of the first two out of five units of product under test with the average length of time required to observe the failure of two out of two units when testing without replacement.

8.5.4.2 *Solution*—From Table A1.3, it is seen that for $r = 2$ and $n = 2$, the expected waiting time is 1.5000θ and that for $r = 2$ and $n = 5$, the expected waiting time is 0.4500θ. Thus, the relative saving in time by placing five units on test is 0.4500θ/1.5000θ = 0.300. This figure may also be obtained directly from Table A1.5. Hence, the average time required when five units are placed on test is 30 % of the average time required when only two units are used.

8.5.4.3 *Example 5: Saving in Time by Increasing Sample Size When Testing With Replacement*—Make the same comparison as in Example 4 (8.5.4.1) if the testing had been with replacement.

8.5.4.4 *Solution*—For $r = 2$ and $n = 2$, the expected waiting time is θ and that for $r = 2$ and $n = 5$ is $r\theta/n = 2\theta/5 = 0.4\theta$. Thus, the relative saving in time by placing five units on test is 0.4θ/θ = 0.4. Hence, the average time required when five units are placed on test is 40 % of the average time required when only two units are used.

8.5.4.5 *Example 6: Saving in Time by Testing With Replacement*—Compare the average length of time needed to observe the failure of the first five out of five units of product under test when testing with replacement with the average length of time needed when testing without replacement.

8.5.4.6 *Solution*—When testing with replacement, for $r = 5$ and $n = 5$, the expected waiting time is θ. When testing without replacement, Table A1.3 or Table A1.4 shows that the expected waiting time is 2.28330θ. Thus, the relative saving in time by testing with replacement is θ/2.28330θ = 0.438; or the average time required for a decision, by replacing failed units, is 43.8 % of the average time required when failed units are not replaced. This figure may also be obtained directly from Table A1.6.

8.5.5 *Cost Considerations in Choice of Sample Size*—Methods for finding the optimum sample size based on considerations of cost are given in this section.

8.5.5.1 *Cost When Testing Without Replacement*—The total expected cost of any of the life test plans of 8.2 when testing without replacement is given by:

$$c_1\theta_0 = \left(\frac{1}{n} + \frac{1}{n-1} + \ldots + \frac{1}{n-r-1}\right) + c_2 n \tag{16}$$

where:

c_1 = cost of waiting per unit time,
c_2 = cost of placing a unit of product on test,
θ_0 = acceptable mean life,
r = termination number, and
n = sample size.

8.5.5.2 *Optimum Sample Size When Testing Without Replacement*—The value of n, which minimizes the total cost, as determined by the method of 8.5.5.1, is the optimum sample size. A general method of obtaining the optimum n is to use Table A1.3 or Table A1.4. The smallest n is chosen such that the difference between the expected waiting time for the rth failure when that number of units of product are placed on test and that when n + 1 units are placed on test is less than the quantity $c_2/c_1\theta_0$.

(1) Example 7: Calculation of Costs—Consider the case in which $r = 10$, $\theta_0 = 1000$ h, $c_1 = \$1$ per hour, and $c_2 = \$100$ per unit of product tested. Using the total cost formula, determine the optimum sample size if failed units are not replaced.

(2) Solution—Using the formula of 8.5.5.1, the costs for various values of n are:

n	Expected Cost Because of Waiting	Cost of Units Tested	Total Cost
10	2929	1000	3929
11	2020	1100	3120
12	1603	1200	2803
13	1346	1300	2646
14	1168	1400	2568
15	1035	1500	2535
16	931	1600	2531
17	847	1700	2547

(a) The optimum sample size is thus n = 16.

(3) Example 8: Obtaining Optimum Sample Size With Expected Waiting Time—Use Table A1.3 to determine the optimum sample size for the problem of Example 7, 8.5.5.2*(1)*.

(4) Solution—The quantity $c_2/c_1\theta_0$ is equal to 0.1 and, from Table A1.3, the expected waiting times are:

n	Expected Waiting Time to Observe 10th Failure in n	in $n + 1$	Difference
10	2.9290	2.0199	0.9091
11	2.0199	1.6032	0.4167
12	1.6032	1.3468	0.2564
13	1.3468	1.1682	0.1786
14	1.1682	1.0349	0.1333
15	1.0349	0.9307	0.1042
16	0.9307	0.8467	0.0840[A]
17	0.8467	0.7773	0.0694

[A] The optimum sample size is n = 16, as was seen in Example 7 (8.5.5.2*(1)*), since that is the smallest sample size for which the difference in expected waiting times is less than $c_2/c_1\theta_0$ or 0.1.

8.5.5.3 *Cost When Testing With Replacement*—The total expected cost of any of the life test plans of 8.2, when testing with replacement, is given by:

$$c_1\theta_0\frac{r}{n} + c_2(n + r - 1) \tag{17}$$

where:

c_1 = cost of waiting per unit time,
c_2 = cost of placing a unit of product on test,
θ_0 = acceptable mean life,
r = termination number, and

n = sample size.

8.5.5.4 *Optimum Sample Size When Testing With Replacement*—The value of n, which minimizes the total cost, as determined by the method of 8.5.5.3, is the optimum sample size. In general, the optimum n for the case of testing with replacement is the integer nearest to:

$$\sqrt{\frac{c_1\theta_0 r}{c_2}+\frac{1}{4}} \tag{18}$$

(1) Example 9: Calculation of Cost—Consider the problem of Example 7 (8.5.5.2*(1)*), that is, r = 10, θ_0 = 1000, c_1 = \$1, and c_2 = \$100. Using the total cost formula, determine the optimum sample size if failed units were replaced.

(2) Solution—Using the formula of 8.5.5.3, the costs for various values of n, are:

n	Expected Cost Because of Waiting	Cost of Units Tested	Total Cost
9	1111	1800	2911
10	1000	1900	2900
11	909	2000	2909

(a) The optimum sample size is thus n = 10.

(3) Example 10: Obtaining Optimum Sample Size by Formula—Use the method of 8.5.5.4 to determine the optimum sample size for the problem of Example 9.

(4) Solution—The integer nearest to

$$\sqrt{\frac{1(1000)(10)}{100}+\frac{1}{4}} = 10.012 \tag{19}$$

is 10. This is the optimum sample size as was seen in Example 9 (8.5.5.4*(1)*).

8.6 *Life Test Sampling Plans for Certain Specified Values of α, β, and θ_1/θ_0*—A life test sampling plan may be designed so that its OC curve meets the following prescribed conditions: if $\theta = \theta_0$, then the probability of the lot meeting the acceptability criterion is less than or equal to β. This part of the practice, which may be considered an extension of 8.1 – 8.5, provides procedures for obtaining values of the termination number, r, and the acceptability constant, C, when certain selected values of α, β, and θ_1/θ_0 are specified. When other values of α, β, and θ_1/θ_0 than those provided in this part of the practice are needed, refer to 8.1 – 8.5 to determine whether one of the life test sampling plans given in those paragraphs are applicable.

8.6.1 *Life Test Sampling Plans*—From Table A1.7, values of the termination number, r, and the acceptability constant, C, may be obtained for values of α = 0.01, 0.05, 0.10, and 0.25; β = 0.01, 0.05, 0.10, and 0.25; and θ_1/θ_0 = 2/3, 1/2, 1/3, 1/5, and 1/10. The value of r is obtained directly from Table A1.7, but the acceptability constant, C, is obtained by multiplying the tabled entry by the acceptable mean life, θ_0 .

8.6.1.1 *Example 11*—Find a life test sampling plan that possesses the following OC curve: If the mean life is θ_0 = 900 h, the lot is accepted with probability 0.95; if the mean life is θ_1 = 300 h, it is accepted with probability approximately equal to 0.10.

8.6.1.2 *Solution*—In this example, θ_1/θ_0 = 1/3, α = 0.05, and β = 0.10. Looking in Table A1.7, the termination number r = 8 and the acceptability constant $C = \theta_0(C/\theta_0)$ = 900(0.498) = 448 are obtained. In word form, place eight or more units of product on test. Stop life testing after eight failures have occurred. If the estimate of lot mean life $\hat{\theta}_{8n}$ is greater than or equal to 448, the lot is acceptable; otherwise, the lot is not acceptable.

8.6.2 *Expansion of* Table A1.7 *for Values of θ_1/θ_0 Greater Than 2/3*—Approximate values of the termination number, r, and the acceptability constant, C, may be obtained to supplement those given in Table A1.7 for values of θ_1/θ_0 greater than 2/3 provided the same values of α and β as given in Table A1.7 are specified. Compute:

$$r = \left(\frac{K_\beta + (\theta_0/\theta_1)K_\alpha}{(\theta_0/\theta_1) - 1}\right)^2 \tag{20}$$

and

$$C = \theta_0\left(1 - \frac{K_\alpha}{\sqrt{r}}\right) \tag{21}$$

where values of K_α and K_β are tabulated in the following: