Designation: ~~E1169 – 20~~ E1169 – 21 An American National Standard

Standard Practice for Conducting Ruggedness Tests[1]

This standard is issued under the fixed designation E1169; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice covers conducting ruggedness tests. The purpose of a ruggedness test is to identify those factors that strongly influence the measurements provided by a specific test method and to estimate how closely those factors need to be controlled.

1.2 This practice restricts itself to experimental designs with two levels per factor. The designs require the simultaneous change of the levels of all of the factors, thus permitting the determination of the effects of each of the factors on the measured results.

1.3 The system of units for this practice is not specified. Dimensional quantities in the practice are presented only as illustrations of calculation methods. The examples are not binding on products or test methods treated.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.5 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E456 Terminology Relating to Quality and Statistics
E1325 Terminology Relating to Design of Experiments
E1488 Guide for Statistical Procedures to Use in Developing and Applying Test Methods
E2282 Guide for Defining the Test Result of a Test Method
F2082 Test Method for Determination of Transformation Temperature of Nickel-Titanium Shape Memory Alloys by Bend and Free Recovery

3. Terminology

3.1 *Definitions*—~~The terminology~~ Unless otherwise noted in this standard, all terms relating to quality and statistics are defined in Terminology E456 ~~applies to this practice unless modified herein.~~.

3.1.1 *factor, n*—independent variable in an experimental design. **E1325**

[1] This practice is under the jurisdiction of ASTM Committee E11 on Quality and Statistics and is the direct responsibility of Subcommittee E11.20 on Test Method Evaluation and Quality Control.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the ~~standard's~~standard's Document Summary page on the ASTM website.

3.1.1.1 *Discussion—*
For experimental purposes, factors must be temporarily controllable. In a ruggedness test, a factor is a test variable that may affect either the result obtained from the use of the test method or the variability of the result.

3.1.2 *fractional factorial design, n*—a factorial experiment in which only an adequately chosen fraction of the treatments required for the complete factorial experiment is selected to be run. **E1325**

3.1.3 *interaction, n*—differences in responses to a factor among levels (versions) of other factors in the experiment. **E1325**

3.1.3.1 *Discussion—*
Interaction is the condition where a factor effect changes with the level of other factors in the experiment design.

3.1.4 *level (of a factor), n*—a given value, a specification of procedure or a specific setting of a factor. **E1325**

3.1.5 *main effect, average effect, n*—a term describing a measure for the comparison of the responses at each level (version) of a factor averaged over all levels (versions) of other factors in the experiment. **E1325**

3.1.5.1 *Discussion—*
This is also known as a first-order effect. In a ruggedness test, the main effect is the change in the test result due to a change in the level of a factor. This is the difference of the average result at the high level of the factor minus the average result at the low level. There are only two levels in the ruggedness tests considered here.

3.1.6 *Plackett-Burman designs, n*—a set of screening designs using orthogonal arrays that permit evaluation of the linear effects of up to $n = t - 1$ factors in a study of t treatment combinations. **E1325**

3.1.7 *ruggedness, n*—insensitivity of a test method to departures from specified test or environmental conditions.

3.1.7.1 *Discussion—*
An evaluation of the "ruggedness" of a test method or an empirical model derived from an experiment is useful in determining whether the results or decisions will be relatively invariant over some range of environmental variability under which the test method or the model is likely to be applied.

3.1.8 *ruggedness test, n*—a planned experiment in which environmental factors or test conditions are deliberately varied in order to evaluate the effects of such variation.

3.1.8.1 *Discussion—*
Since there usually are many environmental factors that might be considered in a ruggedness test, it is customary to use a "screening" type of experiment design which concentrates on examining many first order effects. The validity of the estimates depends on the assumption that second order effects such as interactions and curvature are relatively negligible. Often in evaluating the ruggedness of a test method, if there is an indication that the results of a test method are highly dependent on the levels of the environmental factors, there is a sufficient indication that certain levels of environmental factors must be included in the specifications for the test method, or even that the test method itself will need further revision. This evaluation may include extra runs in a second experiment.

3.1.9 *screening design, n*—a balanced design, requiring relatively minimal amount of experimentation, to evaluate the lower order effects of a relatively large number of factors in terms of contributions to variability or in terms of estimates of parameters for a model. **E1325**

3.1.10 *test result, n*—the value of a characteristic obtained by carrying out a specified test method. **E2282**

3.1.11 *test unit, n*—the total quantity of material (containing one or more specimens) needed to obtain a test result as specified in the test method. **E2282**

3.2 *Definitions of Terms Specific to This Standard:*

~~3.2.1 *factor, n*—test variable that may affect either the result obtained from the use of a test method or the variability of that result.~~

~~3.2.1.1 *Discussion—*~~
~~For experimental purposes, factors must be temporarily controllable.~~

3.2.1 *foldover, n*—test runs, added to a two-level fractional factorial experiment, generated by duplicating the original design by switching levels of one or more factors in all runs, for the purpose of separating estimates of main effects from two factor interactions.

3.2.1.1 *Discussion*—
The most useful type of foldover is with signs of all factors switched. The foldover runs are combined with the initial test results. The combination allows main effects to be separated from interactions of other factors that are aliased in the original design.

~~3.2.3 *interaction, n*—condition where a factor effect changes with the level of other factors in the experiment design.~~

~~3.2.4 *main effect, n*—in a ruggedness test, the change in the test result due to a change in the level of a factor.~~

~~3.2.4.1 *Discussion*—~~
~~This is also known as a first-order effect.~~

3.2.2 *two-factor interaction effect, 2fi, n*—estimate of the condition where a factor effect changes with the level of another factor in the experiment design.

4. Significance and Use

4.1 A ruggedness test is a special application of a statistically designed experiment that makes changes in the test method variables, called factors, and then calculates the subsequent effect of those changes upon the test results. Factors are features of the test method or of the laboratory environment that are known to vary across laboratories and are subject to control by the test method.

4.1.1 Statistical design enables more efficient and cost-effective determination of the factor effects than would be achieved if separate experiments were carried out for each factor. The proposed designs are easy to use in developing the information needed for evaluating quantitative test methods.

4.2 In ruggedness testing, the two *levels* (settings) for each factor are chosen to use moderate separations between the high and low settings. In general, if there is an underlying difference between the levels, then the size of effects will increase with increased separation between the high and low settings of the factors. A *run* is an execution of the test method under prescribed settings of each of the factors under study. A ruggedness test consists of a set of runs.

4.3 A ruggedness test is usually conducted within a single laboratory on uniform material, so that the effects of changing only the factors are measured. The results may then be used to assist in determining the degree of control required of factors described in the test method.

4.4 Ruggedness testing should precede an interlaboratory (round robin) study to correct any deficiencies in the test method and may also be part of the validation phase of developing a standard test method as described in Guide E1488.

4.5 This standard discusses design and analysis of ruggedness testing in Section 5 and contains an example of a basic eight run design. Some caution must be used in interpretation of results, since interaction effects may be present. These effects are present when a factor effect changes with the level of other factors in the experimental design. If it is thought that there may be interaction between variables then additional testing of the basic design is necessary. This is discussed in Section 6. In addition, Annex A3 presents estimates of precision of factor effects when run settings are replicated. An example of a twelve run design is shown in Appendix X1. Annex A1 and Annex A2 provide supplemental information.

5. Basic Ruggedness Test Design and Analysis

5.1 *Design*—A series of fractional factorial designs are recommended for use with ruggedness tests for determining the factor effects on the test results. All designs considered here have only two settings (levels) for each factor, and are known as Plackett-Burman (PB) designs (**1~~, 2~~**).[3] These designs occur in multiples of four runs, such as 4, 8, 12, etc., and are listed in Annex A1, Each run conducts the test method at designated levels of the factors to produce a test result as the run response.

[3] The boldface numbers in parentheses refer to the list of references at the end of this standard.

5.1.1 Select *k* factors to investigate. Choose a PB design with at least *k*+1 runs. Assign each factor to a column in the design table. The unassigned columns in the design are denoted as "dummy" factors, and these may be used to estimate the experimental error (see 5.2.3.2).

5.1.2 Choose the factor levels for each factor such that the measured effects will be reasonably large relative to measurement error. It is suggested that the high and low levels be set at the extreme limits that could be expected to exist between different qualifying laboratories.

5.1.3 Factor levels may be either numerical or categorical. If categorical, only two categories are permitted in the design. If the lower level for a numerical factor is zero, then the factor is essentially categorical (that is, the factor is either present or not).

5.1.4 As an example for this section, Table 1 shows the PB eight run design for up to seven factors, with factors denoted by the letters A–G. Each row lists the factor levels for each of the eight runs as indicated by either (–1) or (1) for low or high levels, respectively. For factors with non-numerical scales (categorical), the designation "low" or "high" is arbitrary.

5.1.5 The design provides equal numbers of low and high level runs for every factor. In other words, the designs are *balanced*. Also, for any factor, while it is at its high level, all other factors will be run at equal numbers of high and low levels; similarly, while it is at its low level, all other factors will be run at equal numbers of high and low levels. In the terminology used by statisticians, the design is *orthogonal*.

5.1.6 The difference between the average response of runs at the high level and the average response of runs at the low level of a factor is the estimated "*main effect*" of that factor. This estimate is then used to quantify the factor's effect on the test result.

5.1.7 *Run Order*—The sequence of runs in Table 1 is not intended to be the actual sequence for carrying out the experiments. The order in which the runs of a ruggedness experiment are carried out should be randomized to reduce the probability of encountering any potential effects of unknown, time-related factors. The run order is to be listed in the second column of Table 1 for use by the experimenter. Alternatively, optimum run orders to control the number of required factor changes and the effect of linear time trends have been derived (**~~3~~2**). In some cases, it is not possible to change all factors in a completely random order. It is best if this limitation is understood before the start of the experiment. A statistician may be contacted for methods to deal with such situations.

5.1.8 The test results are entered in the last column of Table 1 for data analysis.

5.2 *Analysis*—The analysis of the experimental results consists of (*1*) calculating the main effects for each of the factors, including dummy factors, if any, (*2*) creating a half-normal plot to exhibit the magnitudes of the factor main effects, (*3*) assessing the statistical significance of each factor's main effect if an estimate of experimental error is available.

TABLE 1 PB Eight Run Design for Up to Seven Factors

NOTE 1—For four factors, use Columns A, B, C, and E; for five factors, use Columns A, B, C, D, and F; for six factors, use Columns A, B, C, D, F, and G.

PB Order	Run Order	A	B	C	D	E	F	G	Test Result
~~1~~		~~1~~	~~1~~	~~1~~	~~–1~~	~~1~~	~~–1~~	~~–1~~	
1		1	1	1	–1	1	–1	–1	
~~2~~		~~–1~~	~~1~~	~~1~~	~~1~~	~~–1~~	~~1~~	~~–1~~	
2		–1	1	1	1	–1	1	–1	
~~3~~		~~–1~~	~~–1~~	~~1~~	~~1~~	~~1~~	~~–1~~	~~1~~	
3		–1	–1	1	1	1	–1	1	
~~4~~		~~1~~	~~–1~~	~~–1~~	~~1~~	~~1~~	~~1~~	~~–1~~	
4		1	–1	–1	1	1	1	–1	
~~5~~		~~–1~~	~~1~~	~~–1~~	~~–1~~	~~1~~	~~1~~	~~1~~	
5		–1	1	–1	–1	1	1	1	
~~6~~		~~1~~	~~–1~~	~~1~~	~~–1~~	~~–1~~	~~1~~	~~1~~	
6		1	–1	1	–1	–1	1	1	
~~7~~		~~1~~	~~1~~	~~–1~~	~~1~~	~~–1~~	~~–1~~	~~1~~	
7		1	1	–1	1	–1	–1	1	
8		–1	–1	–1	–1	–1	–1	–1	
Ave+		—	—	—	—	—	—	—	—
Ave–		—	—	—	—	—	—	—	—
Effect		—	—	—	—	—	—	—	—

5.2.1 *Main Effect Estimation:*

5.2.1.1 For each factor column calculate the average of the test results corresponding to the + factor level and enter the result in the Ave+ row in the column. Calculate the average of the test results corresponding to the – factor level and enter the result in the Ave– row in the column.

5.2.1.2 Calculate the Main Effect = (Ave+ entry) minus (Ave– entry).

5.2.2 *Half-Normal Plot*—A half-normal plot is used to graphically identify ~~potentially statistically significant~~ active effects.

5.2.2.1 Construct a half-normal plot by plotting the absolute values of main effects on the *X*-axis, in order from smallest to largest, against the half-normal plotting values given in Annex A2 on the *Y*-axis. Effects for all columns in the design, including columns not used to assign levels to any real experiment factor, are plotted. The half-normal plotting values do not depend on the effect values. They depend only on the half-normal distribution and the number of effects plotted.

5.2.2.2 If none of the factors have a measurable main effect, the plotted points will form a straight line. Factors having true effects will lie to the right of the reference line formed by the smaller effects.

5.2.2.3 If an estimate of ~~experimental error~~ the standard deviation of test results (s_{etr}~~)~~), which may be either repeatability or laboratory precision, is available, a reference line in the half-normal plot can be provided with slope $1/s_{effect} = 1/\left[s_{tr} \times \sqrt{(4/N)}\right]$ ~~slope 1 /~~. Effects ~~s_e (see 5.2.3.2). Potentially significant effects are those~~ that fall farthest to the right of the ~~line.~~ line may be considered active effects.

5.2.2.4 If an estimate of error is not available, a reference line may be drawn by eye for the purpose of identifying potentially significant effects. Select points with the smallest effects that appear to fall on a straight line from the origin. Draw a line starting at the origin and passing close to the selected points.

NOTE 1—The slope of the line fit by eye does not provide a valid estimate of error when measurable effects appear to be present.

5.2.3 *Statistical Significance Testing*—If an estimate of precision is available or can be derived from the experiment (see Annex A3), statistical tests of factor effects can be determined using the Student's *t*-test. The *t*-test statistic for a factor is the main effect divided by its standard error s_e, which is the same for all factors in a balanced and orthogonal design. If the *t*-value is greater than the *t*-value corresponding to the 0.05 significance level, the factor is statistically significant at the ~~95 %~~ 95 % confidence level.

5.2.3.1 If fewer factors are used with the design than the maximum number, then the "dummy effects" estimated for the unused columns differ from zero only as a result of experimental error (or interactions of other factors). The root mean square of unused effects is an estimate of the standard error of an effect having degrees of freedom equal to the number of unused effects averaged (~~2~~1).

5.2.3.2 In this case, the line described in ~~5.2.2.2~~5.2.2.3 has slope ~~1~~ 1 ~~/ (MSE)~~ / $(MSE)^{1/2}$, where MSE denotes the mean square error of the dummy effects (see 5.1.1). For instance, if there are three dummy effects, ~~e1,~~ $e1$, ~~e2,~~ $e2$, and ~~e3,~~ $e3$, then MSE = ~~(e1~~ $(e1^2$ ~~+~~ + ~~e2~~ $e2^2$ ~~+~~ + ~~e3~~ $e3^2$~~)~~) ~~/ 3.~~ / 3.

5.2.4 *Ruggedness Test Conclusions:*

5.2.4.1 If no effects are identified as statistically significant and practically important, and if the experimenter is satisfied with the way that the experiment was carried out and with its statistical power, then there is reason to think that the method is rugged with regard to the factors tested.

5.2.4.2 If some effects are identified as statistically significant and practically important, then the method may have to be modified, or specifications may need to be added for the range of acceptable values of the identified factors. In cases where the factor effects may be statistically significant but not practically important the method can still be classified as "rugged."

5.2.5 *Supplemental Additions to the Basic Ruggedness Test Design:*

5.2.5.1 The basic design allows only for the estimation of main effects. When there is uncertainty whether the factor effects change with the levels of other factors in the experiment (**~~4~~3, ~~5~~4**), the main effects may be separated from the interaction effects by conducting additional runs, as discussed in Section 6.

5.2.5.2 Each of the runs may be replicated to obtain an estimate of experimental variability in addition to that supplied by dummy factors, and this may be conducted in three different ways, as discussed in Annex A3.

5.3 *Example:*

5.3.1 The example discussed here is part of a series of experiments that studied the effects of factors that influence determination of pH in dilute acid solutions (**~~4~~3, ~~5~~4**). The factors and their levels are shown in Table 2. Factors C, D, and G are numerical, and the rest are categorical. There are no dummy factors, so the design is said to be saturated (all columns assigned to factors).

5.3.2 The data and calculated main effects for the initial design are shown in Table 3. The results are recorded as 1000 pH.

5.3.3 In Table 3, the Ave+ value for factor A is the average of the four measurements at the "1" value for A (dilution of water): 3015, 2964, 2949 and 3055, the average of which is 2995.75. The Ave– value is the average of the four measurements at the "–1" value for A (no dilution of water): 3006, 2999, 3049 and 2904, the average of which is 2989.5. The main effect is the difference of these values 2995.75 – 2989.5 = 6.3. The other effect estimates are calculated analogously: B = 77.3, C = –0.8, D = 26.8, E = 28.3, F = –1.3, G = 40.8.

5.3.4 *Half-Normal Plot*—The half-normal plotting values are shown in Table 4. As was ~~stated~~suggested in ~~5.2.2~~5.2.2.4, a reference line which passes ~~through~~ close to the three smallest ~~values,~~values is added to the figure. From the half-normal plot in Fig. 1, we see that factors B, G, E, and D appear to be ~~significant.~~ active.

5.3.5 Although the data discussed here give evidence of ~~significant~~active effects, that will not always happen. When no effects appear ~~significant,~~active, the method shows no evidence of lack of ruggedness. When there are ~~significant~~active effects, it may be of value to do further experimentation to find ~~significant~~active two-factor interactions, as discussed in Section 6.

6. Separating Main Effects and Two-Factor Interactions with Added Foldover Runs

6.1 *Interactions*—If the effect of one factor depends on the level of another factor, then these two factors interact. As shown in Section 5, a main effect for a factor is estimated by the difference between the mean measurement of the four high level measurements and the mean of the four low level measurements. By contrast, the two-factor interaction between C and D is estimated as follows. For the high level of factor C compute the difference (high level D mean – low level D mean), where each mean is the average of two measurements. Calculate the corresponding difference at the low level. Half the difference of these two differences is the interaction between factors C and D, in the sense that if the factor D effect does not vary by levels of factor C, the two factor interaction calculated above should be close to 0. It turns out that the eight signs for Column C of Table 1, multiplied by the corresponding eight signs in Column D, give a column of signs that specifies this same CD interaction if we take the difference of the average of the measurements that correspond to "1" and the average of the measurements that correspond to "–1". In addition, it turns out that the negatives of these eight values is the same as the column A values. Thus, when we calculate the factor A effect we are also calculating the negative of the CD interaction, written as –CD. There is no way to know whether the main effect for A is really estimating that factor or the negative of the interaction between factors C and D.

6.1.1 Thus, the complication of the fractional factorial designs presented in Section 5 is that each main effect is confounded (aliased) with a group of two-factor interactions, as shown in Table 5. Note that factor A is confounded with three two-way interactions, one being –CD, which is discussed above. The other confounded interactions are –BF and –EG. Factors are said to be "aliased" when their columns of signs are the negatives or positives of each other.

TABLE 2 Example: Factors That Influence Determination of pH in Dilute Acid Solutions

Factor No.	Variable	Units	Level 1 (–)	Level 2 (+)
A	Dilution with water	yes or no	No	yes
B	Addition of potassium chloride	yes or no	No	yes
C	Equilibration time	minutes	5	10
D	Depth of electrode immersion	cm	1	3
E	Addition of sodium nitrate	yes or no	No	yes
F	Stirring	yes or no	No	yes
G	Temperature	°C	2	4

TABLE 3 Results and Effects for Initial Design

PB Order	A	B	C	D	E	F	G	Test Result
~~1~~	~~1~~	~~1~~	~~1~~	~~–1~~	~~1~~	~~–1~~	~~–1~~	~~3015~~
1	1	1	1	–1	1	–1	–1	3015
~~2~~	~~–1~~	~~1~~	~~1~~	~~1~~	~~–1~~	~~1~~	~~–1~~	~~3006~~
2	–1	1	1	1	–1	1	–1	3006
~~3~~	~~–1~~	~~–1~~	~~1~~	~~1~~	~~1~~	~~–1~~	~~1~~	~~2999~~
3	–1	–1	1	1	1	–1	1	2999
~~4~~	~~1~~	~~–1~~	~~–1~~	~~1~~	~~1~~	~~1~~	~~–1~~	~~2964~~
4	1	–1	–1	1	1	1	–1	2964
~~5~~	~~–1~~	~~1~~	~~–1~~	~~–1~~	~~1~~	~~1~~	~~1~~	~~3049~~
5	–1	1	–1	–1	1	1	1	3049
~~6~~	~~1~~	~~–1~~	~~1~~	~~–1~~	~~–1~~	~~1~~	~~1~~	~~2949~~
6	1	–1	1	–1	–1	1	1	2949
~~7~~	~~1~~	~~1~~	~~–1~~	~~1~~	~~–1~~	~~–1~~	~~1~~	~~3055~~
7	1	1	–1	1	–1	–1	1	3055
8	–1	–1	–1	–1	–1	–1	–1	2904
Ave+	2995.8	3031.3	2992.3	3006.0	3006.8	2992.0	3013.0	
Ave–	2989.5	2954.0	2993.0	2979.3	2978.5	2993.3	2972.3	
Main Effect	6.3	77.3	–0.8	26.8	28.3	–1.3	40.8	

TABLE 4 Estimated Effects and Half-Normal Plotting Values

Effect Order, *e*	Effect	Estimated Effect	Half-Normal Plotting Values
7	B	77.3	1.8
6	G	40.8	1.24
5	E	28.3	0.92
4	D	26.8	0.67
3	A	6.3	0.46
2	F	1.3	0.27
1	C	0.8	0.09

6.2 *Design*—To separate factor main effects from groups of two-factor interactions, the PB design is augmented with eight additional runs called a *foldover*. A set of foldover runs is generated from the original design by changing all the 1's to –1's and the –1's to ~~1's.Thus,~~ 1's. Thus, in the foldover, for each run, for each factor level of the initial design of Table 3, the opposite level in each factor is used. As will be seen below, combining the two sets of runs will allow us to estimate the main effects without confounding from the two-factor interactions. The set of foldover runs for the eight run PB design is shown in Table 6~~Table 6,~~ , together with the test results and calculated main effects for these eight runs.

6.3 *Analysis*—To combine the results of original design and foldover in Table 7, the main effects are estimated by averaging the main effect estimates from the two sets. The corresponding confounded interactions are estimated by taking half the difference of the main effect estimates.

6.4 *Half-Normal Plot*—Using data from the initial runs and the foldover together, the effects are ordered by absolute value and shown with the associated half-normal plot values in Table 8 and plotted in Fig. 2. The suffix –I, added to a factor label, indicates the two factor interactions that are confounded with the factor. The nine smallest estimates appear to lie approximately on a straight line, drawn in Fig. 2, ~~that~~following 5.2.2.4~~represents the standard error for the estimates. The line was drawn to pass through the nine smallest estimates approximately.~~ . From the distribution of points in the plot, factors B, G, E, and D–I appear to be ~~statistically significant. The significance of~~ active. Whether factor G–I is active is unclear.

6.5 In Table 5, it is shown that Interactions AF, CG, and DE are confounded with factor B. Thus, there is no way to know whether the apparent significance of factor B is due to a confounded interaction. As a general rule, factors interact only when they have large main effects in their own right. Hence, AF and CG are unlikely to be important, but a DE interaction could be contributing to the estimated B effect. Similarly, AC, BE, and FG are confounded with D; a BE interaction could be contributing.

6.6 When factors are separated from confounded interactions, it appears that factor D is not ~~significant,~~active, but the apparent significance of D in the initial portion of the experiment was due to confounded interactions. The most likely cause of the large D–I two-factor interaction is the BE interaction, since the main effects B and E are the largest, though only additional experimentation can confirm this.

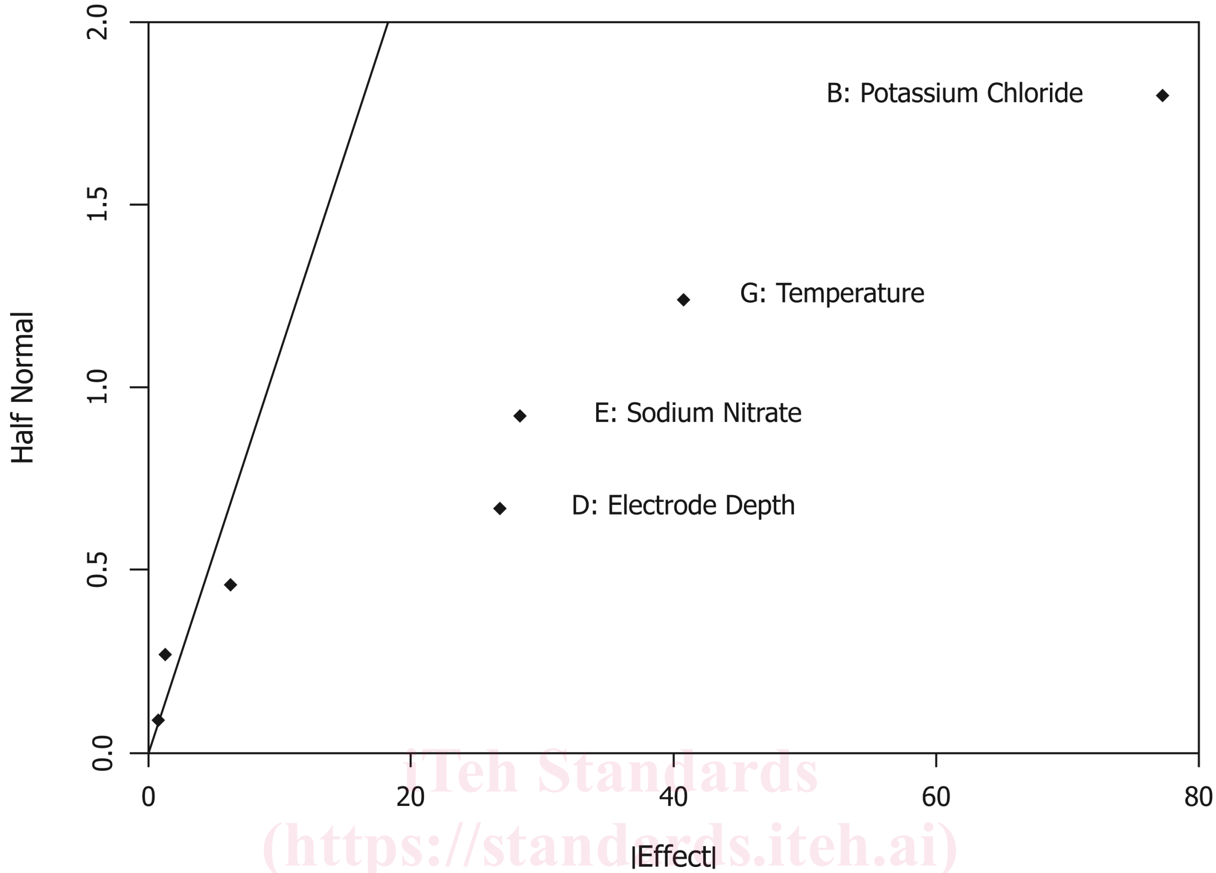

FIG. 1 Half-Normal Plot of pH Data

TABLE 5 Factorial Effect Aliases for Design in Table 1

[A] = A – BF – CD – EG
[B] = B – AF – CG – DE
[C] = C – AD – BG – EF
[D] = D – AC – BE – FG
[E] = E – AG – BD – CF
[F] = F – AB – CE – DG
[G] = G – AE – BC – DF

7. Keywords

7.1 foldover; fractional factorial design; half-normal plot; Plackett-Burman; ~~ruggedness;~~ ruggednessscreening design

TABLE 6 Results and Effects for Foldover Factor—Settings Are at the Opposite Level to the First Set (Table 3)

PB Order	A	B	C	D	E	F	G	Test Result
~~1~~	~~−1~~	~~−1~~	~~−1~~	~~1~~	~~−1~~	~~1~~	~~1~~	~~2931~~
1	−1	−1	−1	1	−1	1	1	2931
~~2~~	~~1~~	~~−1~~	~~−1~~	~~−1~~	~~1~~	~~−1~~	~~1~~	~~2978~~
2	1	−1	−1	−1	1	−1	1	2978
~~3~~	~~1~~	~~1~~	~~−1~~	~~−1~~	~~−1~~	~~1~~	~~−1~~	~~2967~~
3	1	1	−1	−1	−1	1	−1	2967
~~4~~	~~−1~~	~~1~~	~~1~~	~~−1~~	~~−1~~	~~−1~~	~~1~~	~~3030~~
4	−1	1	1	−1	−1	−1	1	3030
~~5~~	~~1~~	~~−1~~	~~1~~	~~1~~	~~−1~~	~~−1~~	~~−1~~	~~2874~~
5	1	−1	1	1	−1	−1	−1	2874
~~6~~	~~−1~~	~~1~~	~~−1~~	~~1~~	~~1~~	~~−1~~	~~−1~~	~~2979~~
6	−1	1	−1	1	1	−1	−1	2979
~~7~~	~~−1~~	~~−1~~	~~1~~	~~−1~~	~~1~~	~~1~~	~~−1~~	~~2911~~
7	−1	−1	1	−1	1	1	−1	2911
~~8~~	~~1~~	~~1~~	~~1~~	~~1~~	~~1~~	~~1~~	~~1~~	~~3040~~
8	1	1	1	1	1	1	1	3040
Ave+	2964.8	3004.0	2963.8	2956.0	2977.0	2962.3	2994.8	
Ave−	2962.8	2923.5	2963.8	2971.5	2950.5	2965.3	2932.8	
Main Effect	2.0	80.5	0.0	−15.5	26.5	−3.0	62.0	

TABLE 7 Calculation of Estimated Effects Using Data from Table 3 and Table 6

Factor	Table 3	Foldover (Table 6)	Average
A	6.3	2.0	4.1
B	77.3	80.5	78.9
C	−0.8	0.0	−0.4
D	26.8	−15.5	5.6
E	28.3	26.5	27.4
F	−1.3	−3.0	−2.1
G	40.8	62.0	51.4
			½ difference
A–I = −BF − CD − EG	6.3	2.0	−2.1
B–I = −AF − CG − DE	77.3	80.5	1.6
C–I = −AD − BG − EF	−0.8	0.0	0.38
D–I = −AC − BE − FG	26.8	−15.5	−21.1
E–I = −AG − BD − CF	28.3	26.5	−0.88
F–I = −AB − CE − DG	−1.3	−3.0	−0.88
G–I = −AE − BC − DF	40.8	62.0	10.6

TABLE 8 Ordered Effects and Half-Normal Plotting Positions

Factor	Effect	Abs (Effect)	Half-Normal Plotting Value
B	78.9	78.9	2.100
G	51.4	51.4	1.611
E	27.4	27.4	1.345
D–I	−21.1	21.1	1.150
G–I	10.6	10.6	0.992
~~D~~	~~−5.6~~	~~5.6~~	~~0.854~~
D	5.6	5.6	0.854
~~A~~	~~−4.1~~	~~4.1~~	~~0.732~~
A	4.1	4.1	0.732
~~A–I~~	~~−2.1~~	~~2.1~~	~~0.619~~
A–I	−2.1	2.1	0.619
~~F~~	~~−2.1~~	~~2.1~~	~~0.514~~
F	−2.1	2.1	0.514
~~B–I~~	~~−1.6~~	~~1.6~~	~~0.414~~
B–I	1.6	1.6	0.414
F–I	−0.88	0.88	0.319
E–I	−0.88	0.88	0.226
C–I	0.38	0.38	0.135
C	−0.38	0.38	0.045

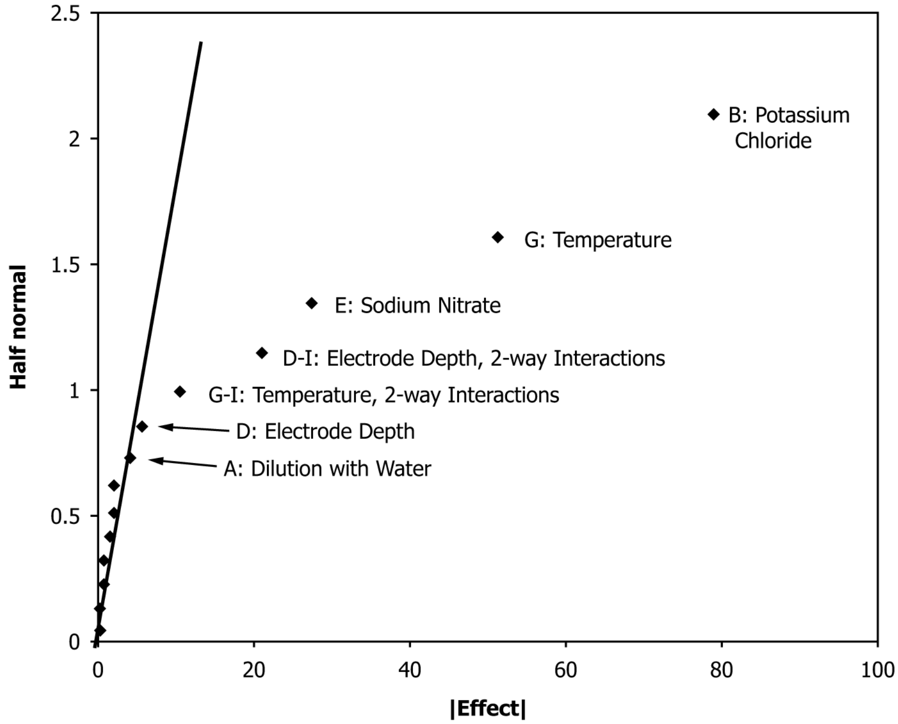

FIG. 2 Half-Normal Plot, Foldover pH Experiment

ANNEXES

(Mandatory Information)

A1. ADDITIONAL PLACKETT-BURMAN DESIGNS

A1.1 Plackett-Burman designs (**1**) are available for *N* values that are integer multiples of four. The following is a method for constructing the designs for *N* = 4, 8, 12, 16, 20, and 24. The first row of each of these designs is given below for the associated *N* value. Each row specifies which of the *N* – 1 factors will be set at the high level (1) or the low level (–1).

N = 4	1,1,–1
N = 8	1,1,1,–1,1,–1,–1
N = 12	1,1,–1,1,1,1,–1,–1,–1,1,–1
N = 16	1,1,1,1–1,1,–1,1,1,–1,–1,1,–1,–1,–1
N = 20	1,1,–1,–1,1,1,1,1,–1,1,–1,1,–1,–1,–1,–1,1,1,–1
N = 24	1,1,1,1,1–1,1,–1,1,1,–1,–1,1,1,–1,–1,1,–1,1,–1,–1,–1,–1

A1.2 For any selected *N* value, the corresponding set of *N* ~~– 1~~ – 1 (1) and (–1) signs is written down as the first row of the design. The second row of the design is obtained by copying the first row after shifting it one place to the right and putting the last sign of Row 1 in the first position of Row 2. This type of cyclic shifting should be done a total of *N* – 2 times, after which a final row of all minus signs is added. The result of this procedure for the *N* = 8 Plackett-Burman design is given in the first listed design of this practice.

A2. PLOTTING POSITIONS FOR HALF-NORMAL PLOTS

A2.1 Table A2.1 gives the coordinates on the vertical axis for half-normal plots with k effects. The numerical estimates of all effects, in order of smallest to largest, are the coordinates for the horizontal axis. Dummy effects, if present, are also plotted.

A2.2 k denotes the number of effects, and $\Phi(x)$ is the probability that the standard normal distribution gives a value less than x. $\Phi^{-1}(p)$ is the value x such that $\Phi(x) = p$. If the k effects are arranged in order of increasing absolute value, the pairs ($|effect_e|$, $\Phi^{-1}(0.5 + 0.5[e - 0.5] / k)$) produce the appropriate half-normal plot for (e = 1,2,3,…k). See Ref (**65**). In Table A2.1, $\Phi^{-1}(0.5 + 0.5[e - 0.5] / k)$ is denoted by $H(e,k)$.

TABLE A2.1 Half-Normal Plotting Values (*H*(*e,k*) by Number of Effects (*k*) and Ordered Effects (*e*)

Number of Effects, *k*	3	4	5	6	7	8	9	10	11
Ordered effects, smallest to largest, *e*									
1	0.210	0.157	0.126	0.105	0.090	0.078	0.070	0.063	0.057
2	0.674	0.489	0.385	0.319	0.272	0.237	0.210	0.189	0.172
3	1.383	0.887	0.674	0.549	0.464	0.402	0.355	0.319	0.289
4		1.534	1.036	0.812	0.674	0.579	0.508	0.454	0.410
5			1.645	1.150	0.921	0.776	0.674	0.598	0.538
6				1.732	1.242	1.010	0.862	0.755	0.674
7					1.803	1.318	1.085	0.935	0.825
8						1.863	1.383	1.150	0.998
9							1.915	1.440	1.207
10								1.960	1.489
11									2.000

Number of Effects, *k*	12	13	14	15	16	17	18	19	20	21	22	23
Ordered effects, smallest to largest, *e*												
1	0.052	0.048	0.045	0.042	0.039	0.037	0.035	0.033	0.031	0.030	0.028	0.027
2	0.157	0.145	0.135	0.126	0.118	0.111	0.105	0.099	0.094	0.090	0.086	0.082
3	0.264	0.243	0.226	0.210	0.197	0.185	0.175	0.166	0.157	0.150	0.143	0.137
4	0.374	0.344	0.319	0.297	0.278	0.261	0.246	0.233	0.221	0.210	0.201	0.192
5	0.489	0.448	0.414	0.385	0.360	0.338	0.319	0.301	0.286	0.272	0.259	0.248
6	0.610	0.558	0.514	0.477	0.445	0.417	0.393	0.371	0.352	0.334	0.319	0.304
7	0.742	0.674	0.619	0.573	0.533	0.499	0.469	0.443	0.419	0.398	0.379	0.362
8	0.887	0.801	0.732	0.674	0.626	0.585	0.549	0.517	0.489	0.464	0.441	0.421
9	1.054	0.942	0.854	0.784	0.725	0.674	0.631	0.594	0.561	0.531	0.505	0.481
10	1.258	1.105	0.992	0.903	0.831	0.770	0.719	0.674	0.636	0.601	0.571	0.543
11	1.534	1.304	1.150	1.036	0.947	0.874	0.812	0.760	0.714	0.674	0.639	0.608
12	2.037	1.574	1.345	1.192	1.078	0.987	0.913	0.851	0.798	0.751	0.711	0.674
13		2.070	1.611	1.383	1.230	1.115	1.025	0.950	0.887	0.833	0.786	0.745
14			2.100	1.645	1.418	1.265	1.150	1.059	0.984	0.921	0.866	0.819
15				2.128	1.676	1.450	1.298	1.183	1.092	1.016	0.952	0.897
16					2.154	1.705	1.480	1.328	1.213	1.122	1.046	0.982
17						2.178	1.732	1.508	1.356	1.242	1.150	1.074
18							2.200	1.757	1.534	1.383	1.269	1.177
19								2.222	1.780	1.559	1.408	1.294
20									2.241	1.803	1.582	1.432
21										2.260	1.824	1.604
22											2.278	1.844
23												2.295