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Standard Guide for General Reliability[1]

This standard is issued under the fixed designation E3159; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This guide covers fundamental concepts, applications, and mathematical relationships associated with reliability as used in industrial areas and as applied to simple components, processes, and systems or complex final products.

1.2 The system of units for this guide is not specified. Quantities in the guide are presented only as illustrations of the method or of a calculation. Any examples used are not binding on any particular product or industry.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.4 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E456 Terminology Relating to Quality and Statistics

E2334 Practice for Setting an Upper Confidence Bound for a Fraction or Number of Non-Conforming items, or a Rate of Occurrence for Non-Conformities, Using Attribute Data, When There is a Zero Response in the Sample

E2555 Practice for Factors and Procedures for Applying the MIL-STD-105 Plans in Life and Reliability Inspection

E2696 Practice for Life and Reliability Testing Based on the Exponential Distribution

2.2 *ISO Standards:*[3]

ISO 3534-1 Statistics–Vocabulary and Symbols, Part 1: Probability and General Statistical Terms

ISO Guide 73 Risk Management Vocabulary

3. Terminology

3.1 *Definitions:*

3.1.1 Unless otherwise noted, terms relating to quality and statistics are as defined in Terminology E456. Other general statistical terms and terms related to risk are defined in ISO 3534-1 and ISO Guide 73.

3.1.2 B_p *life, n—for continuous variables*, the life at which there is a probability, *p*, (expressed as a percentage) of failure at or less than this value.

3.1.2.1 *Discussion*—Example: The B_{10} life is a value of life, *t*, such that cumulative distribution function, $F(t) = 0.1$ or 10 %.

3.1.3 *failure mode, n*—the way in which a device, process or system has failed.

3.1.3.1 *Discussion*—Under some set of conditions, any device, process or system may be vulnerable to several failure modes. For example, a tire may fail in the course of time due to a puncture by a sharp object, from the tire simply wearing out, or from a tire manufacturing anomaly. Each of these describe different failure modes. These three failure modes are said to be competing with respect to the failure event.

3.1.4 *hazard rate, n*—differential fraction of items failing at time *t* among those surviving up to time *t*, symbolized by $h(t)$. **E2555**

3.1.4.1 *Discussion*—$h(t)$ is also referred to as the instantaneous failure rate at time *t* and called a hazard function. It is related to the probability density (*pdf*) and cumulative distribution function (*cdf*) by $h(t) = f(t)/(1 - F(t))$, where $f(t)$ is the *pdf* and $F(t)$ the *cdf*.

3.1.5 *mean time between failures (MTBF), n*—the average time to failure for a repairable item.

3.1.5.1 *Discussion*—A repairable system is one that can be repaired and returned to service following a failure. When an item is repaired, it may not necessarily be returned to service in as good as new condition. There may be a reduction in life in a repaired item making the item not as robust as a new item.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] Available from American National Standards Institute (ANSI), 25 W. 43rd St., 4th Floor, New York, NY 10036, http://www.ansi.org.

Any failure-repair sequence may continue for several cycles, further reducing longevity of service following each repair time. Often the more times the item is repaired, the smaller will be the expected remaining life until the next repair. However, some repairable systems (for example, electronic) may just have some components replaced from time to time rendering the unit as good as new. In those cases, MTBF is the same thing as MTTF.

3.1.6 *mean time to failure (MTTF), θ, n—in life testing*, the average length of life of items in a lot. **E2696**

3.1.7 *reliability, n*—the probability that a component, device, product, process or system will function or fulfill a function after a specified duration of time or usage under specified conditions.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *non-repairable system, n*—a system that is intended for a single use and discarded/replaced following its first failure.

3.2.2 *repairable system, n*—a system that is intended to be used through multiple failure-repair cycles.

4. Significance and Use

4.1 The theory of reliability is used for estimating and demonstrating the probability of survival at specific times or for specific usage cycles for simple components, devices, assemblies, processes, and systems. As reliability is one key dimension of quality, it may be more generally used as a measure of quality over time or over a usage or demand sequence.

4.1.1 Many industries require performance metrics and requirements that are reliability-centered. Reliability assessments may be needed for the determination of maintenance requirements, for spare parts allocation, for life cycle cost analysis and for warranty purposes. This guide summarizes selected concepts, terminology, formulas, and methods associated with reliability and its application to products and processes. Many mathematical relationships and methods are found in the annexes. For general statistical terms not found in Section 3, Terminology E456 and ISO 3534-1 can be used for definitional purposes and ISO Guide 73 for general terminology regarding risk analysis.

4.2 The term "system" implies a configuration of interacting components, sub-assemblies, materials, and possibly processes all acting together to make the system work as a whole. Parts of the system may be linked in combinations of series and parallel configuration and redundancy used in some parts to improve reliability. Additional conditions of complex engineering may have to be considered.

4.3 Process reliability concerns the assessment of any type of well-defined process. This can include manufacturing processes, business processes, and dispatch/demand type processes. Assessment typically measures the extent to which the process can continually perform its intended function without "upset" as well as process robustness.

4.4 A number of reliability metrics are in use. For example, *mean time to failure* (MTTF) is a common measure of average life or average time to the first time a unit fails. For this reason it is said to apply to *non-repairable systems*. Other life percentiles (or quantiles) are in use such as for example a B_p life or that life at which there is *p* % expected failure. Thus, the B_{50} or median life is the life at which 50 % of items would be expected to fail as well as survive; The $B_{0.1}$ life is the life at which would be expected a 1 in 1000 failure probability 0.1 % failure) and a 99.9 % reliability.

4.4.1 Failure rate and average failure rate are also common metrics in reliability. With failure rates, it is important to understand that a rate may be changing with time and this may be increasing, decreasing or some combination of these over the life of a product or service. The failure rate may also be constant.

4.5 Bench testing of a device is used to obtain early reliability assessment or to demonstrate a specific reliability requirement or a related metric. There are a number of key methodologies that are used for this purpose. Demonstration testing may be dependent on the assumption of a distribution of failure time or may be carried out using nonparametric methods.

4.6 When a system is repaired following failure and placed back into service, we refer to the object as a *repairable system*. A key metric for this is the *mean time between failure* (MTBF); and this is not to be confused with MTTF. When a system is repaired, it may not be the case that its expected remaining life is as good as a new one. There may be a reduction in expected life following a repair and this may continue with continuing repair cycles. The MTBF metric applies to all such sequences of repair and restoration cycles over a service life period. This includes the first time to failure, the 2nd time, the 3rd time, etc.

5. Life Concepts

5.1 Before reliability can be assessed, the measure of life must be selected. Table 1 shows a sample of units that are commonly used as a measure of life.

5.1.1 Variations of these units can be found as for example the difference between an aircraft total engine operating time (EOT) and its time/hours in flight or engine flight hours (EFH). *Cycles* are dependent on ordinary time in that any cycle may last for any length of time. In another case, continued life may be driven more by calendar time.

5.1.2 A dispatch of a product or service can be used to compute the product's *dispatch reliability* (for example, relative frequency of failure free dispatches without a change to a schedule). *Demand cycle* is different than ordinary cycles in that a product may only be demanded infrequently but must be in serviceable condition when called on (for example, fire extinguishers, ambulance vehicles). *Calendar time* may be applicable in situations where a product is exposed to field or

TABLE 1 Common Measures of Product Life

Life Unit	Example
operating time	hrs., minutes, days
cycles of usage	flights, dispatches
calendar time	days since new; shelf life
demand cycles	unit is demanded occasionally

environmental conditions during most of its life and would be subject to chemical, thermal or other actions causing performance degradation over time. *Shelf life* is applied to many chemical and biological products and is a prime example of the more general "service life" concept.

5.1.3 A product's *service life* is a duration of life (in the appropriate life units) over which the manufacturer believes the product is serviceable or useful. The term *useful life* is also used synonymously. In some industries, the concept of "*mission*" is used interchangeably. A *duty cycle* is often used to describe the fraction of the time and under what conditions that a product is called on for its intended use relative to some arbitrary time period. For example jet engines operate at a lower level of stress during the cruise portion of any flight, whereas during the takeoff and landing portion of the flight, the duty requirements are greater. In a duty cycle profile, a product may be exposed to a distribution of stress during a typical usage cycle.

5.1.4 In many types of products, components or subsystems, the unit may be subject to *life limiting*. The unit must be replaced with a new one immediately upon reaching the life limit, if not failed. Such units have increasing failure rates with age and the life limit is judiciously selected at a point prior to reaching the unacceptable failure rate. Life limiting is different than service life in that the former applies to non-repairable items (for example, one use only, then dispose and replace). A concept related to life limiting is a *replacement or preventative maintenance interval*. Replacement intervals are commonly found in electro-mechanical applications such as in machine hardware, automotive or aerospace applications.

5.2 *Maintenance Schedules or Intervals*—The continued useful life of many types of products is dependent on appropriate maintenance. Such maintenance is often specified contractually or as part of a warrantee stipulation. Inappropriate product use or operation of the product outside of an intended usage range may retard or negate the desirable effect of a maintenance interval.

5.3 *Failure Modes and Failure Rate*—In using reliability calculations consideration should be given to the type of *failure mode* that is expected for the specific product and its intended application. Three broad classes of failure modes are in common use. Table 2 describes these. See (**1**)[4] and (**2**) for further detail around the failure mode concept.

5.3.1 The term "*infant mortality*," borrowed from the biological science, is now common in engineering. Each of these three classes may contain numerous more specific failure modes – depending on the type of product considered. Associated with each of the three broad classes of failure mode are the three types of *failure rate*.

5.3.2 A failure rate (also called *force of mortality*) is a measure of the rate of failure of currently surviving units at a specific time. For infant mortality cases, the failure rate decreases with time. The explanation is that the presence of special causes will cause failure, typically early in the life cycle; the longer a unit survives, the less likely it is infected with the said special cause and hence the failure rate decreases as the unit ages.

5.3.3 Infant mortality is the reason for conducting a "*burn-in*" application where products are exposed to usage prior to field introduction in order to identify potential early failures prior to field use by a customer. For example, this practice is common for personal computer (PC) manufacturers who want to ensure their machines do not have special cause type defects and will function immediately upon a customer's use.

5.3.4 A *random* failure mode is one that may occur at any time over a service life period but generally may be a rare event. The frequency of such failures is not age-dependent and is only a function of duration time or size of the observation region (that is, how long the unit is observed for). Random failures occur at a constant failure rate throughout a service life. Examples include errors of operation; installation and maintenance mistakes; foreign object damage (including hard objects, liquids, or biological interferences); other contamination and damage due to extreme environmental conditions including extreme or excessive conditions of product use. Also, several rare events that collectively can cause a failure (particularly in large systems) may manifest itself as a random type failure mode. Numerous other random causes may be found.

5.3.5 A *wear-out* failure mode is generally caused by gradual performance degradation with usage or time, ultimately resulting in failure. In electro-mechanical applications, causes of this type of failure may be driven by chemical, thermal, mechanical or electrical stress until some endurance limit is reached causing the failure. Cases of rare catastrophic shocks are more likely random events than gradual degradation. In all types of products and services understanding the type of failure mode and the potential cause is important in design considerations and in installing improvements that would make the product more robust.

5.3.6 In terms of the broader product development cycle the three classes of failure modes are depicted using the *bath-tub curve* shown conceptually in Fig. 1. Early in the development cycle, a new product may exhibit certain failure modes, for any number of reasons that are classified as infant mortality. As the causes of these early failures are removed or corrected and the product develops and is improved, it moves into a period of random failure with a constant failure rate. This random period is sometimes used as a basis for warrantee development. With increasing usage, products fall into the wearout period and performance degradation. In this period, there is an increasing failure rate.

5.3.7 The depiction in Fig. 1 is not to be construed as applying to every type of product, nor its shape the standard form. Some products may only see a random-wearout life

[4] The boldface numbers in parentheses refer to the list of references at the end of this standard.

TABLE 2 General Failure Mode Classes

Class	Description
Infant Mortality	Early failures due to special causes that typically only apply to some units in a population.
Random	Failures due to random causes that can happen at any time.
Wearout	Failures due to wear or degradation action such as chemical, thermal, mechanical, or electrical.

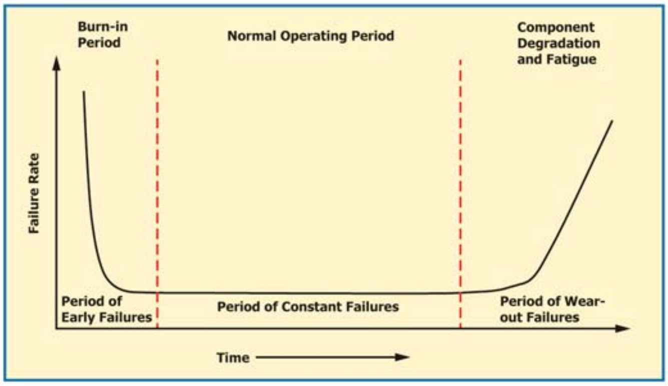

FIG. 1 The "Bathtub Curve"

cycle. In other cases, the failure rate may rise to a maximum and then gradually decline.

5.3.8 In working with large systems where there may be several failure modes related to the failure of different components of the system, each one causing the system to fail, the failure rates for the several failure modes can be added to give the composite system failure rate.

5.3.9 When a failure rate is variable (a function of time), as for example with infant mortality cases or the wearout portion of life, the average failure rate over an interval can be calculated. If the life distribution is a known form, such as a Weibull model, the instantaneous failure rate curve as well as the average failure rate over an interval can be calculated in closed form (in some cases). For infant mortality cases, the average failure rate will be greater than the instantaneous rate; for the wear-out case, the average failure rate will be less than the instantaneous rate.

5.4 *Reliability Metrics and Functions*—A number of metrics and requirements for reliability are in use. Table 3 lists commonly applied reliability metrics.

5.4.1 If t is a variable time, the metric $R(t)$ is the reliability function at time t and related to the assumed distribution such as a Weibull or a lognormal distribution. If t is a discrete variable, such as demand cycles, $R(t)$ may be based on the binomial or Poisson models where the failure probability on any cycle is constant throughout life. Where a degradation of life occurs with increasing demand cycles, a discrete version of the Weibull can be used.

TABLE 3 Reliability Metrics/Requirements

Metric	Description
R(t)	Reliability at time t.
MTTF and MTBF	MTTF or mean time to failure is the mean of the failure time distribution (1st failure). MTBF or mean time between failures is the mean failure time between failures for reparable systems (1st, 2nd, 3rd, failure etc.).
B_p Life	The pth percentile of the life distribution; for example $B_{0.1}$ or B_{10} life are the 0.1th and 10th percentiles of the life distribution.
Failure Rate	Generally applicable for random failure modes in the units of "events" or defects per unit where "unit" means some observational region such as time, space, area, volume, etc.
Dispatch Reliability	The probability that a unit will be available and in good operating condition when demanded.

5.4.2 For variable data, where a life distribution such as a Weibull is used, the mean of the failure time distribution is commonly called the *mean time to failure* or MTTF (**3**), and understood as the mean of the first failure times. Generally it is used with non-repairable systems or single use components. For repairable systems, where an object has a recurrence of life following each repair, the mean of the recurrence life cycles is called the *mean time between failure* or MTBF. When the failure mode under study is of the random type, MTTF and MTBF are theoretically the same thing.

5.4.3 For variable data, the B_p life ($0 < p < 100$) of a failure time distribution is that life at which there is a reliability (survival probability) of (100-p) % or a failure probability of p %. Care should be exercised when the MTTF is used as an indicator of reliability since, for random type failure modes, there is an approximate 63.2 % chance of a failure prior to the mean time. For example if a particular component has been shown to have an MTTF of 10 000 hours for a certain random failure mode, then under these conditions the reliability at 1000 hours is approximately 90 % (see Annex A1). Suppose further that the customer demands the reliability at t = 1000 to be 99 %, what would the MTTF have to be to achieve the 99 % reliability? For a reliability of 99 % at t = 1000 hours, the MTTF would have to be approximately 99 500 hours. This illustrates that the MTTF by itself should not be taken as a reliability benchmark – without calculating the reliability at some critical time t. In addition, the use of maintenance or replacement intervals can affect both MTTF and MTBF (see Annex A1).

5.4.4 Other reliability functions may be defined and many of the important related functions are discussed in the annexes to this standard. References (**4-9**) contain additional general information about these functions for various types of statistical distributions as well as plentiful information on reliability more generally.

5.5 *Reliability Data*—Field performance data are the principle indicator of reliability. How often failures occur, at what times, their severity and for what reasons are the key reliability intelligence. In development activity including improvement efforts, bench testing is the main indicator of reliability, but it may be difficult to emulate all possible field conditions.

5.5.1 When a unit is either tested on a "bench" or observed in the field, there are two conditions that can occur: (*1*) the unit has failed at a specific time t; and (*2*) the unit is still in good running condition at time t. If a unit has failed in some way at a specific time, that is called a complete failure case. If the unit is still working properly, it is called a right suspension or a "run-out" if the actual failure time, now unknown, is in the future (in statistics, this is referred to as censored on the right). In some cases it is only known that a unit has failed and not its specific time. The unit's condition may have been discovered at a time after it had failed – called a left suspension. In another case it may only be known that a unit has failed sometime between two times – called an interval suspension. Interval type data is common in certain types of component bench testing.

6. Reliability Estimation and Calculation Methods

6.1 *Simple Binomial and Exponential Reliability Functions:*

6.1.1 There are cases where a product or service is demanded periodically and could possibly fail (for any number of reasons) at the time it is demanded but not dependent on its previous usage. The binomial distribution may be used to express the reliability in its simplest form and this is seen to be related to the more familiar exponential reliability function. For the binomial, the failure probability p is assumed to be constant throughout life, and failure on any demand assumed independent of the past. Under these conditions, the reliability following n successful uses of a unit is (see E2334):

$$R(n) = (1 - p)^n \quad (1)$$

6.1.2 Taking the natural log of both gives:

$$\ln\{R(n)\} = n\{\ln(1 - p)\} \quad (2)$$

6.1.3 When p is small (say $p < 0.01$), $-\ln(1 - p)$ will be approximately equal to p. Upon simplification this gives:

$$R(n) = e^{-np} \quad (3)$$

6.1.4 Let a be the average time between demand cycles. Then after n demand cycles, the total time that has passed is approximated by $t = na$ making $n = t/a$. Substituting this for n gives:

$$R(t) = e^{-tp/a} \approx e^{-\lambda t} \quad (4)$$

6.1.5 In the last expression the quantity p/a is in the units of failures per unit time and this is rate constant, λ, here assumed constant throughout life. Under homogeneous conditions of continued usage the above expression can be used to find the conservative upper confidence bound for the rate λ, or the required time t that would validate an assumed or desired rate. For confidence C the relationship starts with:

$$e^{-\lambda t} \geq 1 - C \quad (5)$$

Solving for λ:

$$\lambda \leq \frac{-\ln(1 - C)}{t} \quad (6)$$

6.1.6 Eq 6 is the upper confidence bound on the rate parameter, λ when zero events have been observed in an interval t. Cases where r failures have been observed in the interval or where a test is aborted following the rth failure are discussed in the Annex A6.

6.2 *Reliability Demonstration Testing:*

6.2.1 Test planning concerns how many units to run, for how long, and at what operational parameters (temperature, moisture, etc.) in order to demonstrate that a reliability requirement has been met at some confidence level. This can be done using parametric or nonparametric methods. In addition, there can be attribute type test plans and variable type test plans. In cases of variable data where a specific life distribution is used, such as a Weibull or lognormal, the value of a parameter of that distribution is sometimes assumed when some engineering/scientific knowledge justifies this assumption.

6.2.2 There are one or more specified output requirements to be demonstrated by using a plan. Generally, but not always, a confidence requirement is used, such as 90 % or 95 % confidence, that would apply to the final stated result. When confidence is not specified, it may be assumed to be approximately 50 % as for example, when a point estimate of reliability metric is used. There are no general industry wide standards as far as confidence is concerned. A confidence level of 95 % or 90 % is very commonly used, but some industries or applications may require a different value.

6.2.3 Confidence may also depend on the specific application, as for example when safety is a concern. Users should seek out industrial benchmarks in their specific areas.

6.2.4 A test plan generally consists of a sample size, the test "duration" and a *life requirement*. There may also be a requirement for a maximum number of failures allowed by the plan. *Life requirement* implies how long a specific device should last and with what reliability at the stated life. The life requirement may be stated as a *service life* with an associated reliability at the end of life. *Service Life* means the useful functioning life at the end of which the device is repaired, overhauled, or disposed of. The units of service life can be a variable time such as hours or cycles of operation, or can be demand variable for the device as for example a safety device that has to work when it is called for.

6.2.5 The life requirement can also be a mean life, a median life (B_{50}), a failure rate or a general life point at which there is a stated reliability. For the latter case, an example requirement might be to demonstrate 99 % reliability at a service life of 2000 hours. Always, when such a requirement is specified, it must be accompanied by a set of specified test conditions. These conditions are designed to emulate field conditions or to be somewhat more severe than typical field conditions. In other cases, a duty cycle is determined that specifies a distribution of field stress that a device would be expected to see in practice. The test plan would incorporate the duty cycle in some way.

6.2.6 In many cases of testing, it is not economical to implement a given test plan under ordinary usage time or cycles. In such cases, an accelerated test is used. In an accelerated test, one or more variables are adjusted to an equivalent longer duration of actual device use. The simplest such case is where a severity multiplier factor is determined at which one test hour or cycle is equivalent to some number, k, of actual or typical hours of cycles. This is entirely device and application dependent and in many cases a more complicated relationship between the accelerated variable and the actual time is needed. There are many such accelerated models used in practice. Reference **(10)** contains many of these methods.

6.2.7 In cases where a distribution assumption is made, a further assumption may be to assume a value for a parameter of that distribution. For example, if a Weibull model is chosen and valid, users might assume the value for the shape parameter (also called the Weibull *slope* or β). For the lognormal distribution, the scale parameter, σ, is sometimes assumed. If the normal distribution is used, the standard deviation might be assumed. In each of these cases, when the associated parameter is assumed, it is possible to design a test plan for demonstrating any quantile of the distribution, with any degree of confidence, given the assumed parameter.

6.2.8 Demonstrating a reliability requirement may be more difficult (costly) if all of the parameters of the assumed distribution are unknown. A distribution parameter, such as discussed above, is sometimes assumed because there might be engineering or scientific knowledge from prior performance, from industry experience, or from material properties that supports such an assumption.

6.2.9 Another assumption that is sometimes useful is the *scatter factor* of a distribution. The *scatter factor*, *f*, is the ratio of the B_{50} life to the $B_{0.1}$ life for the assumed distribution. For the Weibull and lognormal distributions, the scatter factor is functionally related to the Weibull shape parameter, β, and the lognormal scale parameter, σ, respectively (see Annex A2 and Annex A4). In many cases of materials testing, engineers may know the approximate scatter factor under the general conditions of a specific test.

6.3 For pure attribute pass/fail testing, a "zero failure" test plan is a common theme. The following basic equation, based on the binomial distribution, relates sample size, *n*, confidence, *C*, and reliability, *R*, (**11**).

$$R \geq \sqrt[n]{1 - C} \tag{7}$$

6.3.1 From Eq 7 when any two values are known or assumed, the third may be solved for.

6.3.2 A second common case for pass/fail type data is when a single failure in the sample is allowed. In that case the relation among *n*, *C* and *R*, based on a binomial model, is:

$$nR^{n-1} - (n - 1)R^n \geq 1 - C \tag{8}$$

6.3.3 Eq 8 may be solved numerically for any variable when the remaining two are known or assumed. The general case where $r > 1$ failures are allowed in *n* is discussed in Annex A7.

6.4 Demonstration or "*substantiation*" testing is used to show that a B_p life is at least some specified value with some specified confidence. A distribution for the failure time is assumed and often that distribution is Weibull, lognormal or extreme value. The Weibull model may be imposed on Eq 7 resulting in the following equations relating reliability at mission time t_m to confidence, *C*, sample size, *n*, test time, *t*, and assumed Weibull shape parameter β, (**12**):

$$R(t_m) \geq (1 - C)^{\frac{1}{n}\left(\frac{t_m}{t}\right)^{\beta}} \tag{9}$$

$$n = \left(\frac{\ln(1 - C)}{\ln(1 - p/100)}\right)\left(\frac{B_p}{t}\right)^{\beta} \tag{10}$$

$$B_p \geq t n^{1/\beta}\left(\frac{\ln(1 - p/100)}{\ln(1 - C)}\right)^{1/\beta} \tag{11}$$

6.4.1 In Eq 9 the demonstrated reliability at mission time t_m is related to confidence, *C*, sample size, *n*, test time, *t*, and assumed Weibull shape, β. In that case *n* units are tested to time *t* and all survive. In Eq 10 the sample size is related to the B_p requirement, the test time, *t*, confidence, *C*, and assumed Weibull shape β. In Eq 11 the B_p life is related to the sample size, *n*, confidence, *C*, test time, *t*, and Weibull shape, β. In each case there is a test time, *t*, and *n* units are tested to that time without failure. More general Weibull plans, where $r > 0$ failures are allowed are briefly discussed in the Annex A7.

6.4.2 For the lognormal model with location parameter μ, and scale parameter σ, similar equations can be developed. In what follows the lognormal scale parameter is known or assumed.

Compute:

$$q = 1 - \sqrt[n]{1 - C} \tag{12}$$

For test time *t*, the lower bound at confidence *C* is:

$$\mu \geq \mu_0 = \ln(t) - \sigma F^{-1}(q) \tag{13}$$

The function $F^{-1}(q)$ is the inverse standard normal distribution function evaluated at *q* (see Eq 12). Refer to this lower confidence bound on the lognormal scale value as μ_0. Then the lower bound on mission reliability at mission time t_m is:

$$R(t_m) \geq 1 - F\left(\frac{\ln(t_m) - \mu_0}{\sigma}\right) \tag{14}$$

Let $Z_{p/100}$ be the standard normal quantile value at cumulative probability *p*/100. Then the lower bound on the B_p life is:

$$B_p \geq e^{\left(Z_{p/100} - F^{-1}(q)\right)\sigma + \ln(t)} \tag{15}$$

NOTE 1—In Eq 14, when $t_m = t$, the mission reliability at time *t* reduces to $1 - q$, which is the non-parametric result. The sample size required to state a lower confidence bound on a lognormal quantile B_p is:

$$n = \frac{\ln(1 - C)}{\ln\left\{F\left(\frac{\ln(B_p/t) - Z_p\sigma}{\sigma}\right)\right\}} \tag{16}$$

6.4.3 All of the above formulas are zero failure plans and there are many additional variations on this topic. It is also possible to derive similar plans that allow a maximum number of failures, $r > 0$. Some detail is discussed in Annex A7. Further variations of the Weibull model can be found in (**12-15**).

6.5 In certain types of testing, it may be possible to test several units at a time. For example, this method is used in the bearing industry and is called "*sudden death*" or "*first of n*" testing (**16**). The method is particularly useful when the failure mode has a Weibull distribution. In that case the first failure time or the minimum failure time in *n* units tested also has a Weibull distribution with the same shape parameter (β) as the parent Weibull of the individual failure times. If η is the Weibull scale parameter of the individual failure times, then $\eta/n^{1/\beta}$ will be the scale parameter of the *first of n* distribution. This is the so called reproductive property of the Weibull distribution. The first of *n* methodology is efficient in that failures will occur more rapidly when multiple units are tested at the same time.

6.5.1 For example if $k = 4$ sets of $n = 6$ units are tested until the first failure in 6 occur in each set of 4, one then has 4 failures from which to estimate the scale parameter, $\eta/n^{1/\beta}$, of the 1st of 6 distribution. From that estimate, the individual Weibull scale parameter may be estimated.

6.6 Reliability considerations are best addressed in the design phase of product development and where failure rate requirements are available so that engineers can factor these into a design. Reliability allocation methods attempt to distribute product strength in various ways so that the entire system just meets the requirement. There are many ways to do this depending on the system and its requirements, and in each scenario cost is typically a factor (there will generally be reliability/cost tradeoff). Some popular methods include the following.

6.6.1 Choosing materials or components, or both, that have superior reliability or material performance properties.

6.6.2 Allocation using various combinations or series and parallel networks and redundancy.

6.6.3 Derating a device or a system means specifying the operational conditions below actual capability.

6.6.4 The use of Failure Mode and Effects Analysis (FMEA) to identify failure modes, their frequency and severity (and possible latent type failure modes) during development activity.

6.7 The probability plotting technique is most appropriate for field reliability data of the variable type. In a probability plot, failure times are plotted with consideration given to the number and type of suspensions that are among the data set and the type of distribution that is assumed to apply to the data. What results is an estimate and plot of the assumed cumulative distribution function versus time (*cdf* versus *t*). The plot is typically scaled so that the assumed distribution plots as a straight line as a function of time. In certain cases the slope of the resulting line has meaning with respect to the assumed distribution. For example, in a Weibull plot, the slope measures the shape parameter (β). Most software packages will create probability plots of various kinds and also return a statistic that measures goodness of fit for the model being used.

6.7.1 In a probability plot, such as Fig. 2, the estimate of the failure probability at each failure point needs to be determined. These estimates are called plotting positions, and there are several methods in use for this. Plotting positions also depend on how suspensions are distributed among the failure data points. Two (of many) commonly used plotting positions are the mean rank, and median rank methods. The simplest way to think of a mean rank is the case where there are no suspensions. If the sample size is n, then the plotting position (estimate of the *cdf* at that data value) associated with the ith order statistic is $i/(n + 1)$. This is theoretically the expected fraction falling below the ith order statistic in any sample of size n, for any distribution. To calculate median ranks, a beta distribution is used and exact formulas are not available in closed form. A convenient approximation is often used. Again, associated with the ith order statistic, the median rank plotting position is approximately $(i – 0.3)/(n + 0.4)$. Once again, these formulas apply to cases with no suspensions. Where suspensions are distributed throughout the data, these plotting positions must be adjusted. Software packages that provide probability plots will do these calculations automatically.

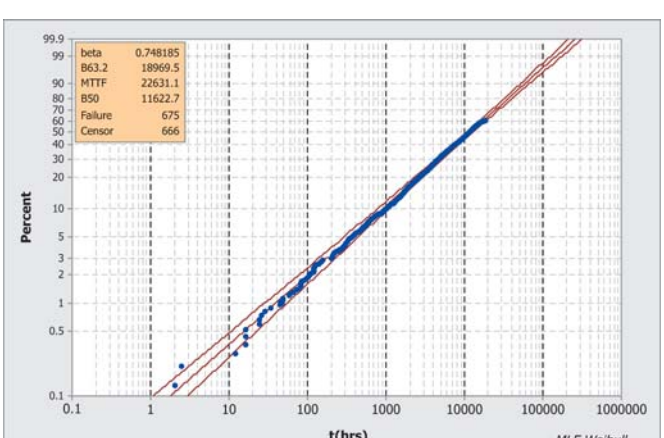

FIG. 2 Weibull Probability Plot

6.7.2 The most commonly used probability plot is the normal probability plot where the normal distribution is being used. This plot is appropriate for all types of data that can be assumed normal. For reliability type data, it is typically the Weibull, lognormal or extreme value distributions that are used. The calculation method used in a probability plot can also vary. In Fig. 2, the Weibull distribution is being used, and a maximum likelihood estimation method is used to estimate the two parameters of the Weibull model. Median rank plotting positions are also being used.

6.7.3 In constructing a probability plot it is also possible to create confidence bands around the model (straight line portion). There are several ways to do this including parametric and nonparametric methods, and Monte Carlo simulation. Many software packages offer at least several options to create confidence bands on probability plots. A common method uses the estimated standard errors of the parameter estimates and takes advantage of the asymptotic normality property of the maximum likelihood parameter estimates (MLE). The standard error estimates are typically supplied in the form of matrix – the Fisher Information matrix – that provides estimates of the variances and covariance of the MLE's. These values are then used in standard formulas to create confidence bands for any desired level of confidence (**8**). Most software packages will provide confidence bands automatically. For more detail on The Fisher Information matrix, see Annex A6.

7. Systems Reliability

7.1 A system is a set of interconnected and possibly interacting components or subsystems, or both, that functions as a whole. Systems can take many configurations, but design typically considers two fundamental types. Any two parts of a system are said to be connected either in a series or a parallel configuration. A series configuration is similar to a chain containing some number of links. The chain (system) fails if at least one of the links fail – all links are required to hold any specific load or the chain will fail. An active parallel system will continue to perform in an unfailed state if at least one of the several components works. In systems design this is referred to as an active redundant system – all parts see service but only one is required to maintain system life. This design is commonly used in safety applications.

7.1.1 A standby redundant system is the case where several units are connected in parallel but only one is actually seeing any service, the redundant units being in a dormant state until the one unit fails. This may be further complicated by imperfect switching from one unit to the next. The simplest systems are either series or parallel configurations. Systems gain in complexity as combinations of series and parallel subsystems are connected in various ways.