

TECHNICAL SPECIFICATION



**Multicore and symmetrical pair/quad cables for digital communications –
Part 1-2: Electrical transmission characteristics and test methods of symmetrical
pair/quad cables** (<https://standards.iteh.ai>)

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INTERNATIONAL ELECTROTECHNICAL COMMISSION

**MULTICORE AND SYMMETRICAL PAIR/QUAD
CABLES FOR DIGITAL COMMUNICATIONS –****Part 1-2: Electrical transmission characteristics and
test methods of symmetrical pair/quad cables**

FOREWORD

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IEC TS 61156-1-2 has been prepared by subcommittee 46C: Wires and symmetric cables, of IEC technical committee 46: Cables, wires, waveguides, RF connectors, RF and microwave passive components and accessories. It is a Technical Specification.

This first edition cancels and replaces the first edition of IEC TR 61156-1-2 published in 2009 and Amendment 1:2014. This edition constitutes a technical revision.

This edition includes the following significant technical changes with respect to the previous edition (TR):

- a) typos and editorial corrections;

- b) the scope was updated;
- c) Figure 14: ports swapped between port 2 and port 3;
- d) new figures for balunless testing.

The text of this Technical Report is based on the following documents:

Draft	Report on voting
46C/1247DTS	46C/1259e/RVDTS

Full information on the voting for its approval can be found in the report on voting indicated in the above table.

The language used for the development of this Technical Specification is English.

This document was drafted in accordance with ISO/IEC Directives, Part 2, and developed in accordance with ISO/IEC Directives, Part 1 and ISO/IEC Directives, IEC Supplement, available at www.iec.ch/members_experts/refdocs. The main document types developed by IEC are described in greater detail at www.iec.ch/publications.

A list of all parts of the IEC 61156 series, under the general title: *Multicore and symmetrical pair/quad cables for digital communications*, can be found on the IEC website.

The committee has decided that the contents of this document will remain unchanged until the stability date indicated on the IEC website under webstore.iec.ch in the data related to the specific document. At this date, the document will be

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MULTICORE AND SYMMETRICAL PAIR/QUAD CABLES FOR DIGITAL COMMUNICATIONS –

Part 1-2: Electrical transmission characteristics and test methods of symmetrical pair/quad cables

1 Scope

This part of IEC 61156 specifies symmetrical pair/quad electrical transmission characteristics and test methods present in IEC 61156-1:2002 (Edition 2) and not carried into IEC 61156-1:2007 (Edition 3). It details characteristic impedance test methods and function fitting procedures, the open/short-circuit method and the background of unbalance attenuation measurement.

It is extended by a description of the balunless measurements technique, which is an amendment to the former technical report and is improved and incorporated into this new edition. The complete document is transferred into a technical specification.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-726, *International Electrotechnical Vocabulary (IEV) – Part 726: Transmission lines and waveguides*

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IEC 61156-1:2007, *Multicore and symmetrical pair/quad cables for digital communications – Part 1: Generic specification*

IEC TR 62152, *Transmission properties of cascaded two-ports or quadripols – Background of terms and definitions*

3 Terms, definitions, symbols, units and abbreviated terms

3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC 60050-726, IEC 61156-1, IEC TR 62152 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

3.1.1

single-ended

measurement with respect to a fixed potential, usually ground

3.2 Symbols, units and abbreviated terms

For the purposes of this document, the following symbols, units and abbreviated terms apply.

Transmission line equation electrical symbols and related terms and symbols:

R	pair resistance (Ω/m)
L	pair inductance (H/m)
G	pair conductance (S/m)
C	pair capacitance (F/m)
α	attenuation coefficient (Np/m)
β	phase coefficient (rad/m)
γ	propagation coefficient (Np/m , rad/m)
v_P	phase velocity of cable (m/s)
v_G	group velocity of cable (m/s)
τ_P	phase delay time (s/m)
τ_G	group delay time (s/m)
Z_C	complex characteristic impedance (Ω)
$\angle Z_C$	angle of the complex characteristic impedance in radians
Z_∞	high frequency asymptotic value of the complex characteristic impedance (Ω)
l	length (m)
j	imaginary denominator
Re	real part operator for a complex variable
Im	imaginary part operator for a complex variable
ω	radian frequency (rad/s)
f	frequency (Hz)
R'	first derivative of R with respect to ω
C'	first derivative of C with respect to ω
L'	first derivative of L with respect to ω
R_0	DC resistance of a round solid wire with radius r (Ω/m)
R_C	constant with frequency component of resistance which is about 1/4 of the DC resistance (Ω/m)
R_S	square-root of frequency component of resistance (Ω/m)
L_E	external (free space) inductance (H/m)
L_I	internal inductance whose reactance equals the surface resistance at high frequencies (H/m)
σ	specific conductivity of the wire material (S/m)
ρ	resistivity of the wire material (Ω/m^2)
μ	permeability of the wire material (H/m)
r	radius of the wire (m)
δ	skin depth (not to be confused with the dissipation factor $\tan \delta$) (m) $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$

$\tan \delta$	dissipation factor $\tan \delta = \frac{G}{\omega C}$
q	forward echo coefficient at the far end of the cable at a resonant frequency
p	reflection coefficient measured from the near end of the cable at a resonant frequency, $p = 10^{\frac{-PSRL}{20}} = \left \frac{Z_{CM} - Z_C}{Z_{CM} + Z_C} \right $
A_Q	forward echo attenuation at a resonant frequency (dB) $A_Q = -20 \log q $
$PSRL$	structural return loss at a resonant frequency (dB) $PSRL = -20 \log p $
K	$K = 2al - 1$ when $2al \gg 1$ (Np)
A_Q	$A_Q = 2 \times PSRL - 20 \log(2al - 1)$ (dB) where $2al$ is in Np
Z_{OC}	complex measured open circuit impedance (Ω)
Z_{SC}	complex measured short circuit impedance (Ω)
Z_{CM}	complex characteristic impedance as measured (with structure) (Ω) $Z_{CM} = \sqrt{Z_{OC} \times Z_{SC}}$
Z_{MEAS}	complex measured impedance (open or short) (Ω)
Z_{IN}	input impedance of the cable when it is terminated by Z_L (Ω)
Z_{OUT}	output impedance of the cable when the input of the cable is terminated by Z_G (Ω)
Z_N	nominal (reference) impedance of the link and/or terminals (the system) between which the cable is operating (Ω)
Z_R	(nominal) reference impedance that is used in measurement. Normally (for actual return loss results), $Z_R = Z_N$. When using a return loss measurement to approximate SRL , it is practical to choose Z_R to give the best balance in the given frequency range (Ω)
Z_T	terminated impedance measurement made with the opposite end of the cable pair terminated in the reference impedance Z_R (Ω)
ς	reflection coefficient measured in the terminated measurement method $\varsigma = \frac{Z_R - Z_C}{Z_R + Z_C}$
Z_G	termination at the cable input when defining the output impedance of the cable Z_{OUT} (Ω)
Z_L	termination at the cable output when defining the input impedance of the cable Z_{IN} (Ω)
L_0, L_1, L_2, L_3	least squares fit coefficients for angle of the complex characteristic impedance
K_0, K_1, K_2, K_3	least squares fit coefficients of the complex characteristic impedance
$ Z_C $	fitted magnitude of the complex characteristic impedance (Ω)
$ Z_{CM} $	measured magnitude of the complex characteristic impedance (Ω)
$\angle (V_{1N})$	input angle relative to a reference angle in radians

$\angle (V_{1F})$	output angle relative to the same reference angle in radians
k	multiple of 2π radians
m	indicator of matrix parameter
S_{11}	reflection coefficient measured with an S parameter test set
RL	return loss (dB)
SRL	structural return loss (dB)
CUT	cable under test

Unbalance attenuation electrical symbols:

TA	transverse asymmetry
LA	longitudinal asymmetry
R_1, R_2	resistance of one conductor per unit length (Ω)
L_1, L_2	inductance of one conductor per unit length (H)
C_1, C_2	capacitance of one conductor to earth (F)
G_1, G_2	conductance of one conductor to earth (S)
α_u	unbalance attenuation (dB)
T_u	unbalance coupling transfer function
Z_{com}	characteristic impedance of the common-mode circuit (Ω)
Z_{diff}	characteristic impedance of the differential-mode circuit (Ω)
Z_{unbal}	unbalance impedance (Ω)
x	length coordinate (m)
γ_{com}	propagation factor of the common-mode circuit (Np/m, rad/m)
γ_{diff}	propagation factor of the differential-mode circuit (Np/m, rad/m)
α_{diff}	operational differential-mode attenuation of the cable (dB)
α_{com}	operational common-mode attenuation of the cable (dB)
ΔR	resistance unbalance of the sample length (Ω)
ΔL	inductance unbalance of the sample length (H)
ΔC	capacitance unbalance to earth (F)
ΔG	conductance unbalance to earth (S)
S	summing function
U_{diff}	voltage in the differential-mode circuit (V)
U_{com}	voltage in the common-mode circuit (V)
n, f	index to designate the near end and far end, respectively

4 Basic transmission line formulae

4.1 Overview

A review of the relationships between the propagation coefficient, the complex characteristic impedance and the primary parameters R , L , G and C is useful here. Characteristic impedance is commonly thought of as being a magnitude quantity. While this concept can suffice for high frequency applications, this quantity is actually a complex one consisting of real and imaginary components or magnitude and angle. The associated propagation coefficient is

readily viewed as being complex, consisting of the real attenuation and imaginary phase coefficient components. The four secondary components are readily related to the primary components. Frequency dependence of these parameters is also developed.

The cable pair parameters are represented as frequency domain dependent quantities. The measurement methods are based on frequency domain techniques. Measurement methods based on time domain techniques and combinations of time and frequency while useful in many cases are not covered here. The present-day availability of excellent frequency domain equipment such as the network analysers and impedance meters support the frequency domain approach.

4.2 Complex characteristic impedance and propagation coefficient formulae

4.2.1 General

The frequency domain of the complex characteristic impedance Z_C relates to the primary parameters as:

$$Z_C = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (1)$$

The propagation coefficient, γ , relates to the primary parameters as:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (2)$$

4.2.2 Propagation coefficient IEC TS 61156-1-2:2023

4.2.2.1 Attenuation and phase coefficients

Formula (2) is separated into its real and imaginary parts, the attenuation coefficient α and the phase coefficient β :

$$\alpha = \sqrt{-\frac{1}{2}(\omega^2 LC - RG) + \frac{1}{2}\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}} \quad (3)$$

$$\beta = \sqrt{\frac{1}{2}(\omega^2 LC - RG) + \frac{1}{2}\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}} \quad (4)$$

Further, by factoring out $\omega\sqrt{LC}$ we obtain:

$$\beta = \omega\sqrt{LC} \sqrt{\frac{1}{2}\left(1 - \frac{R}{\omega L} - \frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}} \quad (5)$$

It can be shown that:

$$\alpha\beta = \omega\sqrt{LC} \left(\frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} \right) \quad (6)$$

4.2.2.2 Formulae useful at high frequencies

From Formulae (5) and (6) we can solve for α and thus obtain for α and β the following expressions, valid within the entire frequency range:

$$\alpha = \frac{\frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}}{\sqrt{\frac{1}{2}\left(1 - \frac{R}{\omega L} - \frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}}} \quad (7)$$

$$\beta = \omega\sqrt{LC} \sqrt{\frac{1}{2}\left(1 - \frac{R}{\omega L} - \frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}} \quad (8)$$

Formulae (7) and (8) are well suited for evaluation of high frequencies.

4.2.2.3 Formulae useful at low frequencies

For low frequency evaluations, the expressions given by Formulae (9) and (10) are suitable.

$$\alpha = \sqrt{\frac{\omega RC}{2}} \sqrt{\left(\frac{G}{\omega C} - \frac{\omega L}{R}\right) + \sqrt{\left(1 + \frac{\omega^2 L^2}{R^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}} \quad (9)$$

$$\beta = \sqrt{\frac{\omega RC}{2}} \sqrt{\left(\frac{\omega L}{R} - \frac{G}{\omega C}\right) + \sqrt{\left(1 + \frac{\omega^2 L^2}{R^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}} \quad (10)$$

4.2.3 Complex characteristic impedance

4.2.3.1 Real and imaginary parts

The complex characteristic impedance Z_C can also be separated into its real and imaginary parts as developed in Formulae (11) and (12).