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Standard Practice for Estimation of Heat Gain or Loss Through Ceilings Under Attics Containing Radiant Barriers by Use of a Computer Program¹

This standard is issued under the fixed designation C1340/C1340M; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last approval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice covers the estimation of heat gain or loss through ceilings under attics containing radiant barriers by use of a computer program. The computer program included as an adjunct to this practice provides a calculational procedure for estimating the heat loss or gain through the ceiling under an attic containing a truss or rafter mounted radiant barrier. The program also is applicable to the estimation of heat loss or gain through ceilings under an attic without a radiant barrier. This procedure utilizes hour-by-hour weather data to estimate the hour-by-hour ceiling heat flows. The interior of the house below the ceiling is assumed to be maintained at a constant temperature. At present, the procedure is applicable to sloped-roof attics with rectangular floor plans having an unshaded gabled roof and a horizontal ceiling. It is not applicable to structures with flat roofs, vaulted ceilings, or cathedral ceilings. The calculational accuracy also is limited by the quality of physical property data for the construction materials, principally the insulation and the radiant barrier, and by the quality of the weather data.

1.2 Under some circumstances, interactions between radiant barriers and HVAC ducts in attics can have a significant effect on the thermal performance of a building. Ducts are included in an extension of the computer model given in the appendix.

1.3 The values stated in either SI units or inch-pound units are to be regarded separately as standard. The values stated in each system may not be exact equivalents; therefore, each system shall be used independently of the other. Combining values from the two systems may result in non-conformance with the standard.

1.4 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the*

¹ This practice is under the jurisdiction of ASTM Committee C16 on Thermal Insulation and is the direct responsibility of Subcommittee C16.21 on Reflective Insulation.

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Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.

2. Referenced Documents

2.1 *ASTM Standards*:²

C168 Terminology Relating to Thermal Insulation

2.2 *ANSI Standards*:

X3.5 Flow Chart Symbols and Their Usage in Information Processing³

X3.9 Standard for Fortran Programming Language³

2.3 *ASTM Adjuncts*:

Computer Program for Estimation of Heat Gain or Loss through Ceilings Under Attics Containing Radiant Barriers⁴

3. Terminology

3.1 *Definitions*—For definitions of terms used in this practice, refer to Terminology C168.

3.2 *Symbols*—Symbols will be introduced and defined in the detailed description of the development.

4. Summary of Practice

4.1 The procedures used in this practice are based on the thermal response factor method for calculating dynamic heat conduction through multilayer slabs (1, 2),⁵ along with a model for convective and radiative heat exchanges inside and outside the attic.

4.2 The operation of the computer program involves the following steps:

² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

³ Available from American National Standards Institute (ANSI), 25 W. 43rd St., 4th Floor, New York, NY 10036, http://www.ansi.org.

⁴ Available from ASTM International Headquarters. Order Adjunct No. ADJC1340.

⁵ The boldface numbers in parentheses refer to the list of references at the end of this standard.

4.2.1 *Response Factors*—A separate computer program must be used to calculate the thermal response factors of the solid materials surrounding the attic. Input to this program would consist of the thermal conductivity, specific heat, density, and thickness of each layer, or the thermal resistance of the layer if it has negligible density, and the fraction of the cross-sectional area occupied by the framing. Output of such a program would be a set of response factors for use as input to the main program. The adjunct to this practice contains data files with response factors for several typical attic constructions.

4.2.2 *Data Input to the Main Program*—This input includes the response factors, total hemispherical emittances of the inside and outside surfaces of the attic envelope, solar absorptances of the outside surfaces of the attic envelope, length and width of the attic, slopes of the two roof sections, distance between attic floor and roof at edge of attic, orientation of house, vent areas and type of vents, water vapor permeances of attic surfaces, area of exposed wood inside attic, mass of wood in attic, initial moisture content of wood in attic, rate of exfiltration of air from house into attic space, latitude and longitude, time zone indicator, solar reflectance of the ground, indoor temperature, and indoor humidity.

4.2.3 *Analysis*—Using hourly weather data consisting of outdoor temperature and humidity ratio, atmospheric pressure, total horizontal and direct solar radiation, wind speed and direction, cloud amount, cloud type, and atmospheric clearness number, the computer program calculates the inside and outside temperatures of the attic envelope and the temperature of the air inside the attic. Using these temperatures, the program calculates the heat flux through the ceiling.

4.2.4 *Output*—The hourly heat flux through the ceiling is written to a file which can be used for further processing, such as seasonal or annual heat gains or losses.

5. Significance and Use

5.1 Manufacturers of radiant barriers express the performance of their products in terms of the total hemispherical emittance. The purpose of a radiant barrier is to decrease the radiation heat transfer across the attic air space, and hence, to decrease the heat loss or gain through the ceiling below the attic. The amount of decrease in heat flow will depend upon a number of factors, such as weather conditions, amount of mass or reflective insulation in the attic, solar absorptance of the roof, geometry of the attic and roof, and amount and type of attic ventilation. Because of the infinite combinations of these factors, it is not practical to publish data for each possible case.

5.2 The calculation of heat loss or gain of a system containing radiant barriers is mathematically complex, and because of the iterative nature of the method, it is best handled by computers.

5.3 Computers are now widely available to most producers and consumers of radiant barriers to permit the use of this practice.

5.4 The user of this practice may wish to modify the data input to represent accurately the structure. The computer program also may be modified to meet individual needs. Also,

additional calculations may be desired, for example, to sum the hourly heat flows in some fashion to obtain estimates of seasonal or annual energy usages. This might be done using the hourly data as inputs to a whole-house model, and by choosing house balance points to use as cutoff points in the summations.

6. Method of Calculation

6.1 Approach:

6.1.1 This calculation of heat loss or gain requires that the following be known:

6.1.1.1 The thermal conductivity, specific heat, and density of the construction materials (that is, insulation, plywood, roofing materials, sheathing, gypsum board);

6.1.1.2 The total hemispherical emittance of all materials facing the attic air space;

6.1.1.3 The solar absorptance of the exterior surfaces of the attic (that is, the roof and gables);

6.1.1.4 The geometry of the attic;

6.1.1.5 The moisture permeance and storage properties of the materials facing the attic space; and

6.1.1.6 The weather conditions.

6.1.2 The solution is a computer procedure that estimates temperatures of both sides of the components of the attic envelope and the temperature of the air in the attic space, uses these estimates of temperatures to refine estimates of convection and radiation heat transfer coefficients, reestimates the temperatures using the new heat transfer coefficients, continues iterating on the temperatures and heat transfer coefficients until convergence is reached, and uses the last estimates of temperatures to calculate the heat gain or loss through the ceiling. This procedure is repeated for each hour of the simulation period (typically a full year).

6.2 *Development of Equations*—The model that is the basis for this practice is based on the model developed by B. Peavy (3), which was later extended by Wilkes (4-6). The sketch of an attic given in Fig. 1 shows the various heat transfer mechanisms that occur within an attic. Although the sketch shows

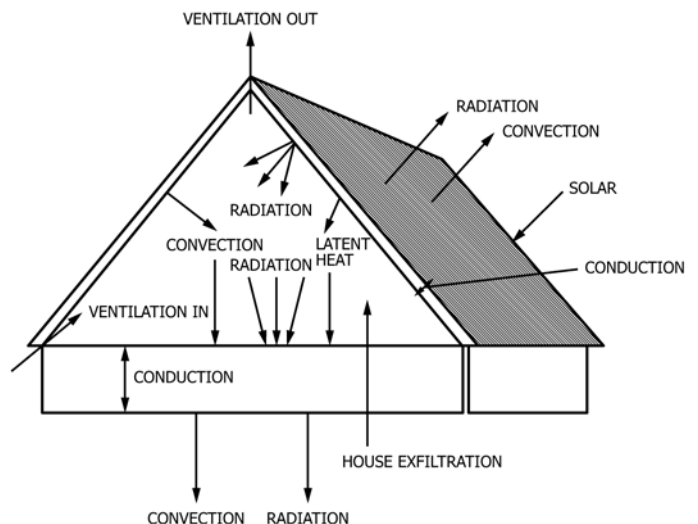


FIG. 1 Schematic of Residential Attic Showing Heat Transfer Phenomena

ventilation occurring at soffit and ridge vents, the location of the vents may be at other locations, such as at the gables. The model treats all of these phenomena through a system of heat balance equations at the interior and exterior surfaces of the ceiling, roof sections, and gables, as well as a heat balance on the air mass within the attic. To handle the case of raised trusses, short vertical walls at the eaves also are included. Each of the surfaces is assumed to be isothermal; thus, for an attic consisting of a ceiling, two roof sections, two gables, two vertical eave sections, and one air space, a total of 15 heat balance equations is used.

6.3 Equations—Conduction:

6.3.1 The model developed here utilizes the thermal response factor method to analyze conduction through building envelope sections. The thermal response factor method was developed by Mitalas and Arseneault (1) and was extended by Kusuda (2). The method is based on an exact analytical solution of the heat conduction equation for one-dimensional heat flow through a multilayer slab having temperature-independent thermal properties. The only approximation is that the surface temperatures are taken to vary linearly with time between time steps. For analysis of buildings, the time step is normally taken to be 1 h. The response factor equations relate the heat fluxes at the surfaces of the slab to the present and previous temperatures at the two surfaces. The equations are:

$$QI = \sum_{j=0}^{\infty} Z'(j) (TIS(j) - TR) - \sum_{j=0}^{\infty} Y'(j) (TOS(j) - TR) \quad (1)$$

$$QO = \sum_{j=0}^{\infty} Y'(j) (TIS(j) - TR) - \sum_{j=0}^{\infty} X'(j) (TOS(j) - TR) \quad (2)$$

where:

- QI = heat flux at inside surface at present time (note that the positive heat flow direction is from the inside to the outside), W/m^2 [Btu/h·ft²],
- QO = heat flux at outside surface at present time, W/m^2 [Btu/h·ft²],
- $TIS(j)$ = temperature at inside surface j hours previous to present time, °C [°F],
- $TOS(j)$ = temperature at outside surface j hours previous to present time, °C [°F]
- $X'(j), Y'(j), Z'(j)$ = response factors, $W/m^2 \cdot K$ [Btu/h·ft²·°F], and
- TR = reference temperature, °C [°F].

6.3.2 The response factors are determined from a sequence of calculations that involve the thermal diffusivity, thermal conductivity, specific heat, density, and thickness of each of the layers in the multilayer slab. An efficient computer program for calculating the response factors has been developed by George Walton of the National Institute of Standards and Technology (NIST) (7).

6.3.2.1 The efficiency of the response factor calculations can be increased by making use of the fact that after a sufficient number of terms, the ratio of two consecutive response factors becomes constant. This is expressed by:

$$\frac{X'(j+1)}{X'(j)} = \frac{Y'(j+1)}{Y'(j)} = \frac{Z'(j+1)}{Z'(j)} = CR \text{ for } j \geq N \quad (3)$$

- CR = the common ratio, and
- N = a sufficiently large number.

6.3.2.2 The common ratio is used to define a new set of functions, called the first order conduction transfer functions or simply the conduction transfer functions, $X(j)$, $Y(j)$, and $Z(j)$, which are given by:

$$X(0) = X'(0) \quad (4)$$

$$Y(0) = Y'(0) \quad (5)$$

$$Z(0) = Z'(0) \quad (6)$$

$$X(j) = X'(j) - CR X'(j-1) \text{ for } j \leq N \quad (7)$$

$$Y(j) = Y'(j) - CR Y'(j-1) \text{ for } j \leq N \quad (8)$$

$$Z(j) = Z'(j) - CR Z'(j-1) \text{ for } j \leq N \quad (9)$$

$$X(j) = 0 \text{ for } j > N \quad (10)$$

$$Y(j) = 0 \text{ for } j > N \quad (11)$$

$$Z(j) = 0 \text{ for } j > N \quad (12)$$

6.3.2.3 With the conduction transfer functions, the heat fluxes and surface temperatures are related by:

$$QI = \sum_{j=0}^N Z(j) (TIS(j) - TR) \quad (13)$$

$$- \sum_{j=0}^N Y(j) (TOS(j) - TR) + CR QI'$$

$$QO = \sum_{j=0}^N Y(j) (TIS(j) - TR) \quad (14)$$

$$- \sum_{j=0}^N X(j) (TOS(j) - TR) + CR QO'$$

where:

- QI' = heat flux at inside surface at previous time step,
- QO' = heat flux at outside surface at previous time step, and
- N = number of significant conduction transfer functions.

6.3.2.4 When parallel heat flow paths occur in an envelope component, separate response factors for each path may be needed. If the boundary temperatures of the two paths may be assumed to be equal, however, then the response factors may be added together as:

$$X' = A_1 X'_1 + A_2 X'_2 \quad (15)$$

where:

- A_1, A_2 = area fractions for paths 1 and 2, and
- $(X'_1, X'_2), (Y'_1, Y'_2)$ and (Z'_1, Z'_2) = the response factors for paths 1 and 2.

Parallel conduction transfer functions may be calculated from these parallel response factors, provided that the common ratio for the path with the largest number of significant terms is used.

6.3.2.5 The original derivation of the response factor technique relied upon the assumption of temperature-independent

thermal properties. An approximate method has been developed to account for the temperature dependence of the thermal properties (5). The thermal transmission coefficient of the component is taken to vary linearly with temperature as:

$$U = U_{TR} [1 + b(T - TR)] \quad (16)$$

The conduction transfer function equations then become:

$$QI = \sum_{j=0}^N Z(j) (TIS(j) - TR) \quad (17)$$

$$- \sum_{j=0}^N Y(j) (TOS(j) - TR) + CR QI$$

$$+ b/2 \sum_{j=0}^N Z(j) (TIS(j) - TR)^2 - b/2 \sum_{j=0}^N Y(j) (TOS(j) - TR)^2$$

$$QO = \sum_{j=0}^N Y(j) (TIS(j) - TR) \quad (18)$$

$$- \sum_{j=0}^N X(j) (TOS(j) - TR) + CR QO'$$

$$+ b/2 \sum_{j=0}^N Y(j) (TIS(j) - TR)^2 - b/2 \sum_{j=0}^N X(j) (TOS(j) - TR)^2$$

These equations are used in the system of heat balance equations.

6.4 Equations—Convection:

6.4.1 Convection heat transfer from the interior and exterior surfaces of the envelope components is calculated using correlations from the literature (8). The coefficients are based on correlations that have been developed for isolated isothermal flat plates. The correlations are in the form of a Nusselt number, Nu , as a function of a Rayleigh number, Ra , Grashof number, Gr , or a Reynolds number, Re , where:

$$Nu = hL/k \quad (19)$$

$$Ra = \frac{g\beta C_p \Delta T L^3}{\nu k} \quad (20)$$

$$Gr = Ra/Pr \quad (21)$$

$$Pr = \nu/\alpha \quad (22)$$

$$Re = VL/\nu \quad (23)$$

and:

h = convection heat transfer coefficient, $W/m^2 \cdot K$ [$Btu/h \cdot ft^2 \cdot ^\circ F$],

L = characteristic length of plate, m [ft],

k = thermal conductivity of air, $W/m \cdot K$ [$Btu/h \cdot ft \cdot ^\circ F$],

g = acceleration of gravity, m/s^2 [ft/h^2],

β = volume coefficient of expansion of air, K^{-1} [$^{\circ}R^{-1}$],

ρ = density of air, kg/m^3 [lb/ft^3],

C_p = specific heat of air, $J/kg \cdot K$ [$Btu/lb \cdot ^\circ F$],

ΔT = temperature difference between surface and air, K [$^{\circ}F$],

ν = kinematic viscosity of air, m^2/s [ft^2/h],

Pr = Prandtl number for air, dimensionless,

α = thermal diffusivity of air, m^2/s [ft^2/h], and

V = velocity of air stream, m/s (ft/h).

6.4.2 The model accounts for the temperature dependence of the properties of air by evaluating them at the film

temperature, which is defined as the average of the temperatures of the surface and the air. Relationships for the temperature dependent properties were obtained from NBS Circular 564 (9).

6.4.2.1 The model utilizes correlations that have been developed for various orientations of the plate with respect to gravity and for the direction of heat flow (up versus down). Correlations for both laminar and turbulent flow are used, with the choice depending upon the magnitude of the Rayleigh number for natural convection and of the Reynolds number for forced convection. Separate coefficients are calculated for natural and forced flow, and a mixed coefficient is calculated by taking the third root of the sum of the cubes of the two separate coefficients (10).

6.4.2.2 The correlations used in the model are given in Table 1. They account for the effects of surface-to-air temperature difference, heat flow direction, film temperature, surface size, and surface orientation. The correlations are contained in a subroutine called HCON. Values of surface temperature, air temperature, plate tilt angle, plate characteristic length, a flag to denote whether the plate faces up or down, and air speed past the plate are passed to the subroutine. The subroutine returns the mixed convection coefficient. For exterior surfaces, the air speed is taken to be the wind speed obtained from meteorological data. For interior surfaces, a crude estimate of air speed is obtained by dividing the attic ventilation volume flow rate by an average cross-sectional area for the attic. For the interior surfaces, a crude estimate for air speed should suffice, since natural convection should dominate over forced convection.

6.5 Equations—Radiation:

6.5.1 Radiation interchanges within the attic space are handled using the enclosure method described by Sparrow and Cess (11). With this method, each of the surfaces is assumed to be plane, opaque, gray, and isothermal, to be diffusely emitting and reflecting, and to have a uniform radiant flux over the surface. The assumption of diffuse emission and reflection for radiant barrier surfaces may be questioned. Very flat radiant

TABLE 1 Correlations for Convection Coefficients

I. Natural Convection:	
A. Horizontal surface, heat flow up	
	$Nu = 0.54 Ra^{1/4}$ for $Ra < 8 \times 10^6$
	$Nu = 0.15 Ra^{1/3}$ for $Ra > 8 \times 10^6$
B. Horizontal surface, heat flow down	
	$Nu = 0.58 Ra^{0.2}$
C. Vertical surface	
	$Nu = 0.59 Ra^{1/4}$ for $Ra < 1 \times 10^9$
	$Nu = 0.10 Ra^{1/3}$ for $Ra > 1 \times 10^9$
D. Nearly horizontal surface (tilt angle less than 2°), heat flow down	
	$Nu = 0.58 Ra^{0.2}$
E. Tilted surfaces (greater than 2° tilt), heat flow down	
	$Nu = 0.56 (Ra \sin(\Phi))^{1/4}$ Φ = tilt angle
F. Tilted surface, heat flow up	
	$Nu = 0.56 (Ra \sin(\Phi))^{1/4}$ for $Ra/Pr < Gr_c$
	$Nu = 0.14 (Ra^{1/3} - (Gr_c Pr)^{1/3})$
	+ $0.56 (Gr_c Pr \sin(\Phi))^{1/4}$ for $Ra/Pr > Gr_c$
	$Gr_c = 1 \times 10^6$ for $\Phi < 15^\circ$
	$Gr_c = 10^{(0.11870 + 0.0870 \times \Phi)}$ for $15^\circ < \Phi < 75^\circ$
	$Gr_c = 5 \times 10^9$ for $\Phi > 75^\circ$
II. Forced Convection:	
	$Nu = 0.664 Pr^{1/3} Re^{1/2}$ for $Re < 5 \times 10^5$
	$Nu = Pr^{1/3} (0.037 Re^{0.8} - 850)$ for $Re > 5 \times 10^5$

barrier surfaces would be expected to exhibit specular properties. Real radiant barrier surfaces usually are not flat, and hence the diffuse assumption is probably appropriate.

6.5.1.1 The net radiant heat flux away from surface i is given by:

$$Q_{ri} = \sum_{j=1}^N G_{ij} \sigma (T_i^4 - T_j^4) \quad (24)$$

with the following relations:

$$G_{ij} = \frac{\varepsilon_i}{1 - \varepsilon_i} \Psi_{ij} \quad (25)$$

Ψ_{ij} = inverse of the matrix χ_{ij}

$$\chi_{ij} = \frac{\delta_{ij} - (1 - \varepsilon_i) F_{ij}}{\varepsilon_i}$$

$$\delta_{ij} = 0 \text{ for } i \neq j; \delta_{ij} = 1 \text{ for } i = j$$

where:

- F_{ij} = radiation view factor from surface i to surface j ,
- ε_i = emittance of surface i ,
- σ = Stefan-Boltzmann constant, $5.6696 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ [$1.714 \times 10^{-9} \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{R}^4$], and
- N = number of surfaces in enclosure.

6.5.1.2 The heat flux equation may be cast into a form that is linear in the temperatures by factoring as:

$$Q_{ri} = \sum_{j=1}^N G_{ij} \sigma (T_i^2 + T_j^2)(T_i + T_j)(T_i - T_j) \quad (26)$$

$$= \sum_{j=1}^N HR_{ij} (T_i - T_j)$$

6.5.1.3 The quantities appearing in these equations are calculated in several subprograms: subroutine VIEW2, and functions FMN, FP, and HRAD. Function FMN calculates the view factor between two rectangular surfaces that meet at an angle and share a common edge, while function FP calculates the view factor between two parallel surfaces that have the same dimensions. Expressions for the two view factors are given by Sparrow and Cess (11). The functions themselves are based on those given by Peavy, with some necessary modifications (3). The functions have been verified by comparison with the calculations of Feingold (12). Subroutine VIEW2 calculates the view factors among the surfaces that face the attic space, using view factor algebra and calls to functions FMN and FP. The G_{ij} matrix is also calculated within VIEW2. The HR_{ij} matrix is calculated by function HRAD using G_{ij} , T_i , and T_j as inputs.

6.5.1.4 The algorithm for radiation interchanges allows each of the surfaces to have different emittances. The algorithm accounts for all interreflections within the enclosure and properly accounts for the T^4 (Stefan-Boltzmann) law for radiation exchange.

6.6 Equations—Ventilation:

6.6.1 Heat transfer to the ventilation air is treated by an extension of the method used by Peavy (3). With this method, the temperature of the air is assumed to increase (or decrease) as it moves along a flow path and picks up heat by convection

from the surrounding surfaces. This is expressed by the following first order ordinary differential equation:

$$\dot{m} C_p \frac{dT}{dx} = \sum_{j=1}^M A(j) HC(j) (TIS(j) - T) \quad (27)$$

- \dot{m} = mass flow rate of ventilation air, kg/s [lb/h],
- C_p = specific heat of air, J/kg · K [Btu/lb · °F],
- T = air temperature at position, °C [°F],
- x = normalized position (0–1) along flow path,
- $A(j)$ = area of surface j in contact with ventilation air, m^2 [ft^2],
- $HC(j)$ = convection heat transfer coefficient at surface j , $\text{W/m}^2 \cdot \text{K}$ [$\text{Btu/h} \cdot \text{ft}^2 \cdot \text{°F}$],
- $TIS(j)$ = temperature of surface j , °C [°F], and,
- M = number of surfaces in contact with ventilation air.

6.6.2 In the absence of a detailed picture of the ventilation air flow pattern within the attic, the simplifying assumption has been made that equal proportions of area of each surface are contacted as the air flows along a differential length of its flow path. Integrating this equation yields the following expression for the temperature as a function of flow path position:

$$T = T_o \exp(-ax) + b[1 - \exp(-ax)] \quad (28)$$

where:

- T_o = temperature at $x = 0$,
- $a = \sum_{j=1}^M A(j) HC(j) / \dot{m} C_p$, and
- $b = \frac{\sum_{j=1}^M A(j) HC(j) TIS(j)}{\sum_{j=1}^M A(j) HC(j)}$.

6.6.3 An average air temperature is defined by:

$$TA = \int_0^1 T dx = b[1 + (\exp(-a) - 1)/a] - T_o/a[\exp(-a) - 1] \quad (29)$$

and the temperature of the air exiting from the attic at $x = 1$ is:

$$TE = b[1 - \exp(-a)] + T_o \exp(-a) \quad (30)$$

6.6.4 The flow rate of ventilation air is determined from a combination of stack and wind pressure effects. For the stack effect calculation, the air inside the attic space is assumed to be well-mixed, and relations from Chapter 22 of the 1985 ASHRAE Handbook of Fundamentals are used (13). The volume flow rate due to the stack effect is given by:

$$\dot{V}_S = C_D A [2 gh (TA - T_o) / TA]^{1/2} \text{ for } TA > T_o \quad (31)$$

$$\dot{V}_S = C_D A [2 gh (T_o - TA) / T_o]^{1/2} \text{ for } T_o > TA \quad (32)$$

where:

- \dot{V}_S = volume flow rate due to stack effect, m^3/s [ft^3/h],
- C_D = discharge coefficient taken to be 0.65,
- A = the lesser of the net free areas of the vent inlet and outlet, m^2 [ft^2],
- h = height of the neutral pressure level from the lower opening, m [ft],

T_o = absolute temperature of inlet air, K [°R], and
 TA = average absolute temperature of air in attic, K [°R].

6.6.5 The height of the neutral pressure level is given by:

$$h = \frac{H}{1 + \left(\frac{AI}{AO}\right)^2 \frac{TA}{T_o}} \text{ for } TA > T_o \quad (33)$$

$$h = \frac{H}{1 + \left(\frac{AI}{AO}\right)^2 \frac{T_o}{TA}} \text{ for } T_o > TA \quad (34)$$

where:

H = difference in elevation between inlet and outlet vents, m [ft],

AI = net free area of inlet vents, m^2 [ft^2], and

AO = net free area of outlet vents, m^2 [ft^2].

6.6.6 The volume flow rate due to wind pressure is given by:

$$\dot{V}_w = C_F A WS \quad (35)$$

where:

C_F = discharge coefficient,

A = the lesser of the net free areas of the vent inlet and outlet, m^2 [ft^2], and

WS = wind speed, m/s [ft/h].

6.6.7 From the data given by Burch and Treado (14), the discharge coefficients for the wind pressure effect are estimated to be 0.38 for soffit and ridge vents and 0.54 for soffit and gable vents. For soffit vents only, C_F is estimated from

$$C_F = 0.089 + 0.132 \sin^2 D \quad (36)$$

where:

D = wind direction, measured from a direction parallel to the ridge, radians.

6.6.8 Mass flow rates for the stack and wind pressure effects are obtained by multiplying the volume flow rates by the density of the air. A total mass flow rate is given by:

$$\dot{m} = (\dot{m}_s^2 + \dot{m}_w^2)^{1/2} \quad (37)$$

6.6.9 Finally, the adjustment recommended by ASHRAE is made to account for unequal inlet and outlet vent areas:

$$\dot{m} = [1 + 0.4077 (1 - (A_{\min}/A_{\max})^{1.5})] (\dot{m}_s^2 + \dot{m}_w^2)^{1/2} \quad (38)$$

where the quantity in parentheses is a fit to the graph in the 1985 ASHRAE Handbook of Fundamentals (13). The air flow due to exfiltration of air from the house into the attic space is added to the flow due to the stack and wind pressure effects. The algorithm for estimating ventilation rates is embodied in a subroutine called VENT.

6.6.10 Although some other results in the literature are in qualitative agreement (15,16), there appears to be a good deal of uncertainty in this ventilation algorithm. D. M. Burch, in a private conversation with K. E. Wilkes in 1987, has suggested that the ventilation rates he and Treado measured may be in error because of problems of mixing tracer gases in an attic. D. Ober, in a private conversation with K. E. Wilkes in 1988, has suggested that a better stack ventilation rate might be obtained by using the roof temperature rather than the average air temperature as the driving force during the daytime hours.

Additional experimental data appear to be needed to develop better algorithms for ventilation.

6.7 Equations—Moisture:

6.7.1 Approximations have been built into the models to account for the latent heat effects due to sorption and desorption of moisture at the wood surfaces that face the attic space. These generally follow the suggestions given by Burch et al (17) and Cleary (18). In this model, the wood surface is assumed to be in moisture equilibrium with a thin layer of air adjacent to the surface. The humidity ratio of this layer of air is given by (18).

$$\omega_s = (b + c \cdot u + d \cdot u^2 + e \cdot u^3) \exp(T/a) \quad (39)$$

where:

ω_s = humidity ratio of air near wood surface, kg of water/kg of dry air [lb of water/lb of dry air],

T = temperature of wood surface, °C [°F],

u = moisture content of wood (dimensionless fraction),

a, b, c, d = constants from fit to equilibrium moisture content data.

6.7.2 The rate of transfer of moisture from the air in the attic space to the surface is given by:

$$\dot{m}_w = h_w (\omega_a - \omega_s) \quad (40)$$

where:

\dot{m} = mass flow rate of moisture per unit area of exposed wood surface, $kg/s \cdot m^2$ [lb/h-ft²],

ω_a = humidity ratio of air in attic space, kg of water/kg of dry air [lb of water/lb of dry air], and

h_w = mass transfer coefficient, $kg/s \cdot m^2$ [lb/h-ft²].

6.7.3 The mass transfer coefficient is obtained from the analogy between heat and mass transfer as:

$$\frac{h_c}{h_w C_p} = \left(\frac{\alpha}{D}\right)^{2/3} \approx 1 \quad (41)$$

where:

h_c = convection heat transfer coefficient, $W/m^2 \cdot K$ [Btu/h-ft²·°F],

C_p = specific heat of air, $J/kg \cdot K$ [Btu/lb·°F],

α = thermal diffusivity of air, m^2/s [ft²/h], and

D = coefficient for diffusion of water vapor through air, m^2/s [ft²/h].

6.7.4 The humidity ratio of the attic air is obtained by performing a steady-state moisture balance on the attic space, including diffusion of moisture through the boundary surfaces, convection of moisture into the attic space from the outside air and from exfiltration from the house, convection of moisture out of the attic space by the ventilation air, and moisture transfer to or from the wood surfaces. The attic moisture balance is given by:

$$\sum_{j=1}^M A(j) Perm(j) (P(j) - P_a) + \dot{m}_v (\omega_o - \omega_a) + \dot{m}_E (\omega_i - \omega_a) + \sum_{j=1}^M h_w(j) A'(j) (\omega_s(j) - \omega_a) = 0 \quad (42)$$

where:

- $Perm(j)$ = water vapor permeance of surface j ,
 \dot{m}_v = mass flow rate of ventilation air,
 \dot{m}_E = mass flow rate of exfiltration air,
 $\omega_s(j)$ = humidity ratio at surface j ,
 ω_a = humidity ratio of air in attic space,
 ω_o = humidity ratio of outside air,
 ω_i = humidity ratio of indoor air,
 $P(j)$ = partial pressure of water vapor in air on outside of surface j ,
 P_a = partial pressure of water vapor in attic air,
 $h_w(j)$ = water vapor mass transfer coefficient at surface j ,
 $A(j)$ = area of surface j , and
 $A'(j)$ = exposed wood area at surface j .

6.7.4.1 When the humidity ratio of the attic air and the mass transfer rates have been calculated, the heat transferred to the surface by latent heat effects, Q_{lat} , is given by

$$Q_{lat} = \dot{m}_w h_v \quad (43)$$

where:

h_v = latent heat of vaporization of water. For this model, h_v has been taken to have a constant value of 2 466 kJ/kg [1060 Btu/lb].

6.7.4.2 The mass transfer to a surface is used to estimate a new surface moisture content, assuming that only a thin layer of wood participates in the moisture exchanges. It should be noted that the intent of this treatment of moisture is only to estimate the effect of latent heats on the heat flow rates and not to determine the accumulation of moisture itself. An estimation of moisture accumulation rates over long periods of time would require a more detailed treatment than is used here.

6.7.5 The algorithms for calculating moisture effects are embodied in several subprograms. The humidity ratio at the wood surface is calculated by function WDHUM. Subroutine PSY is used to calculate humidity ratios and partial pressures of water vapor from known values of air temperature and relative humidity. The mass transfer coefficients are calculated in subroutine HMASS, and the moisture balance is performed in subroutine MOIST.

6.8 Equations—Heat Balances:

6.8.1 The heat balance equations combine the heat flows by various mechanisms at the inside and outside surfaces of the components of the attic envelope and on the attic air mass. The heat balance at an interior surface, that is, one facing the attic space, is obtained by summing the contributions due to conduction through the component, radiation interchanges with each of the other surfaces that it sees, convection exchanges with the attic air mass, and latent heat loads due to moisture sorption/desorption. The interior heat balances are:

$$\sum_{j=0}^N Z(i,j) (TIS(i,j) - TR) - \sum_{j=0}^N Y(i,j) (TOS(i,j) - TR) \quad (44)$$

$$+ b(i)/2 \sum_{j=0}^N Z(i,j) (TIS(i,j) - TR)^2 - b(i)/2 \sum_{j=0}^N Y(i,j) (TOS(i,j) - TR)^2 + CR(i) QI(i) + HC(i) (TIS(i,0) - TA)$$

$$+ \sum_{k=1}^M \sum_{k \neq i} HR(i,k) (TIS(i,0) - TIS(k,0)) - \dot{m}_w(i) h_v = 0$$

6.8.1.1 In Eq 44, the index i refers to the surface for which the heat balance is being written. Index k refers to the other surfaces that face the attic space, and j is the index for the conduction transfer function time sequence, with $j = 0$ representing the current time. An equation of this form is obtained for each of the surfaces facing the attic space.

6.8.2 The heat balance at the exterior surface relates the heat conducted through the envelope surface to the heat convected to the outdoor air, the heat radiated by the surface to the surroundings, and the solar radiation absorbed by the surface. Since the exterior surface of the bottom of the attic is the ceiling of the house, the balance is modified to allow convection to the indoor air, radiation to the surfaces of the room below the attic, and no direct absorption of solar radiation. The exterior heat balances have the form:

$$\sum_{j=0}^N Y(i,j) (TIS(i,j) - TR) - \sum_{j=0}^N X(i,j) (TOS(i,j) - TR) \quad (45)$$

$$+ b(i)/2 \sum_{j=0}^N Y(i,j) (TIS(i,j) - TR)^2 - b(i)/2 \sum_{j=0}^N X(i,j) (TOS(i,j) - TR)^2 + CR(i) QO'(i) + HC(i) (T_o - TOS(i,0)) + HR(i) (T_s - TOS(i,0)) + \alpha(i) Q_s(i) = 0$$

where:

- T_o = temperature of outside air,
 T_s = temperature of surroundings,
 $\alpha(i)$ = solar absorptance of surface i , and
 $Q_s(i)$ = solar radiation incident on surface i .

6.8.2.1 The heat balance on the attic air mass accounts for the heat convected to the air from each of the surfaces facing the attic space, convection of outdoor air into the space, and convection of attic air out of the space. With the model given above for the temperature rise of the ventilation air, the attic air heat balance is given by:

$$\sum_{i=1}^M A(i) HC(i) TIS(i,0) - \frac{C_1}{1+C_2C_3} TA \quad (46)$$

$$= \dot{m}_v C_p T_o \frac{C_3}{1+C_2C_3}$$

where:

- $C_1 = \sum_{i=1}^M A(i) HC(i)$,
 $C_2 = \frac{\dot{m}_v C_p}{C}$, and
 $C_3 = \exp(-1/C_2) - 1$.

6.8.2.2 The system of heat balance equations is arranged in a matrix form with the interior and exterior surface temperatures and the air temperatures at the current time step being the unknown quantities. Values for these temperatures at previous time steps are known. The matrix equation may be written as:

$$(AA)(T) = (BB) \quad (47)$$

where:

AA = a square matrix of coefficients,
 T = a vector of unknown temperatures, and
 BB = a known factor.

Detailed expressions for the elements of the matrices are lengthy. They may be found by inspecting the Fortran code for the computer program and will not be repeated here.

6.8.3 This system of equations is solved by Gauss-Jordan elimination, using a subroutine named SOLVP, which was developed by Peavy (given in Kusuda (19)). Since many of the coefficients in the square matrix and the constant vector depend upon the unknown temperatures, the system of equations is solved iteratively. When the temperatures have been determined, the heat flows are calculated using the conduction transfer function equations.

6.8.4 To avoid a numerical instability associated with the latent heat term in the heat balance on interior surfaces, the surface humidity ratio is expanded in a Taylor series such that:

$$\begin{aligned} \dot{m}_w(i) &= h_w(i) A'(i) (\omega_a - \omega_s(i)) \quad (48) \\ &= h_w(i) A'(i) \left[\left(\omega_a - \omega'_s(i) - \frac{d\omega_s(i)}{dT} (TIS(i) - TIS'(i)) \right) \right] \end{aligned}$$

where:

$\omega'_s(i)$ = is the surface humidity ratio evaluated at a previously estimated surface temperature $TIS'(i)$.

The coefficients of $TIS(i)$ are included in the (AA) matrix while the other terms are included in the (BB) vector. With this scheme, the solutions have been found to converge.

6.8.5 The system of equations is set up to allow the model to be driven by weather conditions with all the surface and air temperatures being unknown. For comparison of the model with experiments, often it is helpful to use measured values of exterior roof and ceiling temperatures as boundary conditions. The model is set up to allow any of the surface temperatures to be forced to its measured value. This is accomplished using a method suggested by D. Ober, in a private conversation with K. E. Wilkes in 1988, wherein the diagonal element of (AA) that corresponds to the forced temperature is multiplied by a very large number and the corresponding element of (BB) is set to the product of the new element of (AA) and the known temperature.

7. Computer Program

7.1 General:

7.1.1 A computer program that embodies the model described in this practice is available from ASTM as an adjunct. This program is only an example. Users of the practice may develop their own program if they wish. The computer program in the adjunct is written in Fortran 77.

NOTE 1—Identical versions of this computer program have been compiled successfully and run on mainframe and personal computers. Only minor modifications necessary for conformance to the resident operating system were required for operation.

7.1.2 The program consists of a main program and several subroutines. Other subroutines may be added to make the program more applicable to the specific problems of individual users.

7.1.3 A flow chart for the program is given in Fig. 2. The program starts by initializing all temperatures to 75°F and all heat fluxes to zero through the use of DATA statements. DATA statements also are used to set various other quantities to zero such as the elements of the (AA) matrix. Many of these elements will remain at zero while others are recalculated later in the program.

7.1.3.1 Next, material property and geometrical input data are read in. This consists of conduction transfer functions, solar absorptances, total hemispherical emittances, the length and width of the attic, roof pitches, height of eave walls, orientation of the house, vent inlet and outlet areas and type of vents, water vapor permeances, wood surface areas and participating masses, and wood moisture contents. The house exfiltration rate and the latent heat of vaporization also are read as inputs. By setting the latent heat to zero, moisture effects may be ignored. The input read in also includes a series of flags that specify whether surface temperatures are to be forced to known values or are to be calculated from weather conditions.

7.1.3.2 The program then calls subroutine VIEW2. The attic length, width, roof pitches, height of eave walls and emittances of the surfaces facing the attic space are passed into VIEW2. The subroutine utilizes function FMN to calculate view factors between two rectangular surfaces that share an edge and function FP to calculate view factors between two equal parallel rectangles. The subroutine then uses view factor algebra to calculate the view factors among the surfaces that face the attic space and the overall view factor matrix G_{ij} , which is passed back out of the subroutine.

7.1.3.3 Next, the program calculates characteristic lengths and areas of surfaces. For the ceiling, the characteristic length is taken to be the average of the length and width of the attic. For a roof surface, it is taken to be the distance from the eave to the ridge. For gables, it is taken to be the average height, and for the eave walls, it is taken to be the height. These characteristic lengths are the ones normally chosen for use in correlations for convection heat transfer coefficients.

7.1.3.4 In the next step, the program reads a line of hourly weather data. This reading includes the outdoor temperature, incident solar radiation, wind speed, wind direction, and outdoor humidity ratio. At this step, the program also is set up to read in measured values for surface temperatures and ventilation rate. If the appropriate flags are set, the program will use the indicated measured surface temperatures or ventilation rate in the calculations. If this option is elected, a separate input file is required.

7.1.3.5 If the flag is set so that the measured ventilation rate is not to be used, then the program makes a call to subroutine VENT. Quantities passed into the subroutine are outdoor air temperature, average air temperature within the air space, wind speed and direction, difference in elevation between inlet and outlet vents, inlet and outlet vent areas, and a flag to indicate the type of vent. The subroutine uses the algorithms described above for the stack and wind pressure effects to calculate

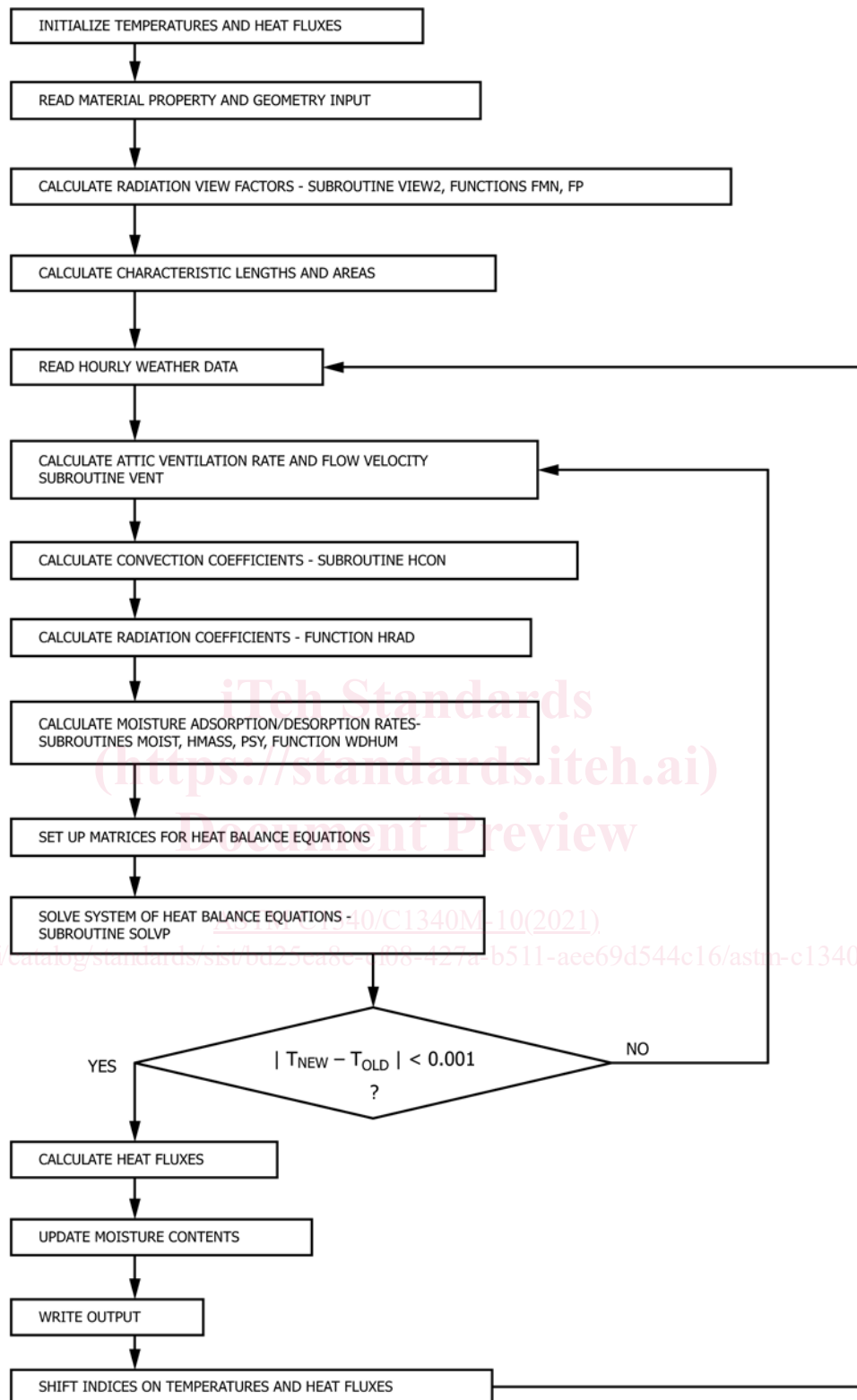


FIG. 2 Flow Chart for Attic Model

ventilation volume and mass flow rates, and the product of the mass flow rate and the specific heat of air. These last three quantities are passed back out of the subroutine. If the flag is

set to use the measured ventilation volume flow rate, VENT is bypassed and the measured volume flow rate is used to calculate the mass flow rate and the product of the mass flow

rate and the specific heat. Next, a crude estimate for the flow velocity is calculated by dividing the volume flow rate by an average cross sectional area for the attic.

7.1.3.6 The program then makes several calls to subroutine HCON to calculate convection heat transfer coefficients for the interior and exterior surfaces. Quantities passed into HCON are the previous estimates of the surface temperature and the adjacent average air temperature, surface tilt angle, characteristic length of surface, a flag to indicate whether the surface faces up or down, and the air speed past the surface. For interior surfaces, the crude estimate of ventilation flow velocity is used. For exterior surfaces, the wind speed is used (except for the exterior of the ceiling, where the air speed is set to zero). HCON utilizes the correlations described above to calculate natural, forced, and mixed convection heat transfer coefficients. The mixed coefficient is passed back out of HCON.

7.1.3.7 Radiation heat transfer coefficients are calculated using the HRAD function. The function uses the previous estimates of the two bounding temperatures and either the appropriate element of the G_{ij} matrix (for an interior surface) or the emittance (for an exterior surface).

7.1.3.8 If the latent heat is not set close to zero, the program calls subroutine MOIST, which calculates moisture sorption/desorption rates for the surfaces facing each of the air space. The quantities passed into MOIST are the previous estimates of the surface temperatures, the outdoor and indoor air temperatures, the previous estimate of the average air temperature within the space, the convection heat transfer coefficients, surface areas, wood surface areas and masses, wood moisture contents, indoor and outdoor relative humidities, water vapor permeances, attic ventilation mass flow rate, and house exfiltration mass flow rate. The subroutine calculates wood surface humidity ratios from the surface moisture content and temperature using function WDHUM. Subroutine PSY is used to calculate humidity ratios and water vapor partial pressures for the indoor and outdoor air using the given temperatures and relative humidities. Mass transfer coefficients are calculated using subroutine HMASS, which uses the analogy between heat and mass transfer.

7.1.3.9 The humidity ratio of the air in the air space then is calculated from a moisture balance on the space. This calculation is done iteratively since the moisture balance involves both the humidity ratio and the partial pressure. With the humidity ratio of the air established, the moisture flow rates from Eq 48 are calculated and pass back out of the subroutine. If the latent heat is set close to zero, all of these calculations are bypassed.

7.1.3.10 At this point, all of the quantities for the matrices (AA) and (BB) are available. The matrices are set up in the main program and are solved using subroutine SOLVP. The result of this solution is a new set of estimates for the surface and air space temperatures. The new estimates are compared with the old estimates. If any differences are greater than 0.0005°C [0.001°F], then the program goes back to subroutine VENT and new elements for the matrices of the heat balance equations are calculated using the newly estimated temperatures.

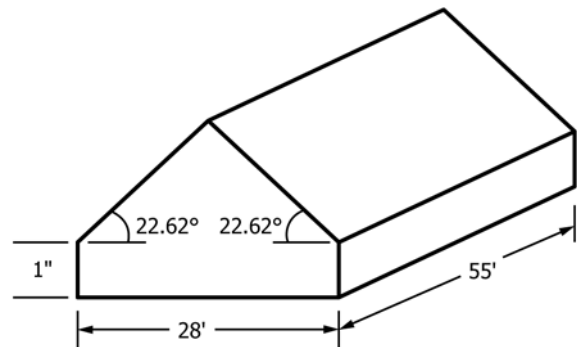


FIG. 3 Geometry of Attic for Example Problem

7.1.3.11 When all of the new estimates for surface and air temperatures agree with old estimates to within 0.0005°C [0.001°F], or when the limit of 15 iterations has been reached, heat fluxes are calculated using the conduction transfer function equations, moisture contents are updated, and results are written out for the current hour. The WRITE and FORMAT statements may need to be changed to suit the user's needs.

7.1.3.12 The time indices on the temperatures and heat fluxes are shifted, the program goes back to read another line of weather data and the calculations proceed as before for the next hour.

8. Example Calculation

8.1 The gabled attic shown in Fig. 3 was modeled using the computer program. The construction and properties of the various surfaces of the attic are given in Tables 2 and 3. The ridge of the attic is oriented in the east-west direction, the attic has soffit vents with an area of 0.64m^2 [6.84ft^2] and a ridge vent with an area of 0.32m^2 [3.42ft^2]. The interior of the house is maintained at 23°C [74°F] and 50 % relative humidity. No air from the house exfiltrates into the attic space. The house is located in Phoenix, AZ, and the reflectance of the ground is 0.2.

8.2 An input file for the computer program is given in Fig. 4, and a key to this file is given in Table 4. A file containing weather data for input to the program is given in Fig. 5, and a

TABLE 2 Construction and Properties of Attic Surfaces for Example Problem

Material	Thickness, in.	Thermal Conductivity, Btu/(h-ft ² -°F)	Specific Heat, Btu/lb-°F	Density, lb/ft ³
Ceiling:				
Gypsum board	0.5	0.0926	0.26	50
R-19 insulation	6.25	0.02741	0.19	0.6
Joists ^A	0.2917	0.06833	0.39	28
Roof (both sides):				
Shingles	0.25	0.04734	0.30	70
Felt	0.085	0.04734	0.36	70
Plywood	0.5	0.0667	0.29	34
Rafters ^A	3.5	0.06833	0.39	28
Gables (both ends):				
Hardboard siding	0.4375	0.1242	0.28	40
Studs	1.5	0.06833	0.39	28
Eave walls (both sides):				
Hardboard siding	0.4375	0.1242	0.28	40

^ATrusses are 24 in. on center.

TABLE 3 Properties of Surfaces for Example Problem

Property	Ceiling	Roof (both sides)	Gables (both ends)	Eave Walls (both sides)
Solar absorptance	...	0.9	0.7	0.7
Outside emittance	0.9	0.9	0.9	0.9
Interior emittance	0.9	0.05	0.05	0.9
Water vapor permeance	0.08	0.322	10	10
Exposed wood area/ projected area	0	1.29	1.19	1.0
Wood mass/projected area	0.0365	0.871	0.888	0.833
Initial moisture content	0.09	0.09	0.09	0.09

key to the file is given in [Table 5](#). The weather data consists of typical meteorological year weather data for one day in the summer. The data for this one day are repeated three times in the file for startup purposes.

8.3 [Fig. 6](#) contains the output file, which is an hourly listing of the ceiling heat flux, in Btu/h-ft², and [Fig. 7](#) contains a plot of the heat flux.

9. Report

9.1 The results of calculations performed in accordance with this practice may be used as design data for specific job conditions or may be used in general form to represent the performance of a particular product or system. When the results will be used for comparison of performance of similar products, it is recommended that reference be made to the specific constants used in the calculation. These references should include:

- 9.1.1 Name and other identification of products or components.
- 9.1.2 Source of weather conditions used in calculations and location being simulated.
- 9.1.3 Surface total hemispherical emittances and solar absorptances used in calculations.

9.1.4 All other input parameters, including other material properties and geometry of attic.

9.1.5 The resulting heat gain or loss.

9.1.6 Either tabular or graphical representation of the results of the calculations may be used. No attempt is made to recommend the format of this presentation of results.

10. Precision and Bias

10.1 The precision of this practice is a function of the computer equipment used to generate the calculational results. In many typical computers normally used, seven significant digits are resident in the computer for calculations. Adjustments to this level can be made through the use of *double precision*, however, for the intended purpose of this practice, standard levels of precision are adequate. The formatting of the output results however has been structured to provide a resolution of better than 0.1 % for the typical expected levels of heat flux.

10.2 Many factors influence the accuracy of a calculational procedure used for predicting heat flux results. These factors include computer resolution, accuracy of input data, and the applicability of the assumptions used in the method for the system under study. By comparison of model predictions with measured heat flux data, Wilkes (5) concluded that calculated cumulative heat flows under summer conditions were accurate to 5–10 % for attics without radiant barriers and were accurate to about 15 % for attics with radiant barriers. A significant source of uncertainty in the calculations for a radiant barrier attached near the roof was in the effective emittance of the combination of radiant barrier and exposed wood surfaces.

11. Keywords

11.1 attic radiant barrier system; computer program; heat gain; heat loss