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An American National Standard

Standard Guide for Reliability Demonstration Testing[1]

This standard is issued under the fixed designation E3291; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This standard covers fundamental concepts, applications and mathematical relationships associated with the planning of reliability demonstration tests as applied to components and materials testing.

1.2 The system of units for this guide is not specified. Quantities and examples are presented only as illustrations of a method or a calculation. Any examples used are not binding on any particular product or industry.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.4 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E456 Terminology Relating to Quality and Statistics

E2555 Practice for Factors and Procedures for Applying the MIL-STD-105 Plans in Life and Reliability Inspection

E2696 Practice for Life and Reliability Testing Based on the Exponential Distribution

E3159 Guide for General Reliability

2.2 *ISO Standards:*[3]

ISO 3534-1 Statistics – Vocabulary and symbols, Part 1: Probability and general statistical terms

ISO Guide 73 Risk management vocabulary

3. Terminology

3.1 *Definitions:*

3.1.1 Unless otherwise noted, terms relating to quality and statistics are as defined in Terminology E456. Other general statistical terms and terms related to risk are defined in ISO 3534-1 and ISO Guide 73.

3.1.2 *B_p life, n—for continuous variables*, the life at which there is a probability, *p*, (expressed as a percentage) of failure at or less than this value. **E3159**

3.1.3 *failure mode, n*—the way in which a device, process or system has failed. **E3159**

3.1.4 *hazard rate, n*—differential fraction of items failing at time *t* among those surviving up to time *t*, symbolized by $h(t)$. **E2555**

3.1.5 *mean time between failures (MTBF), n*—the average time to failure for a repairable item. **E3159**

3.1.6 *mean time to failure (MTTF), θ, n—in life testing*, the average length of life of items in a lot. **E2696**

3.1.7 *reliability, n*—the probability that a component, device, product, process or system will function or fulfill a function after a specified duration of time or usage under specified conditions. **E3159**

3.2 *Symbols:*

3.2.1 The following symbols are used extensively in the discussion.

3.2.2 *C*—confidence coefficient (decimal value between 0 and 1).

3.2.3 *n*—sample size, (positive integer at least 1).

3.2.4 *p*—failure probability (decimal between 0 and 1, or percentage between 0 and 100).

3.2.5 *R*—reliability. $R = 1 - p$ or $(100 - p)\%$ for *p* expressed as a percentage.

3.2.6 *r*—number of failures allowed ($0 \le r < n$).

3.2.7 *β*—Weibull shape parameter, also referred to as the "Weibull slope."

3.2.8 *θ—for the exponential model*, the mean (MTTF) of the distribution.

3.2.9 *η—for the Weibull model*, the characteristic life or scale parameter.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] Available from American National Standards Institute (ANSI), 25 W. 43rd St., 4th Floor, New York, NY 10036, http://www.ansi.org.

3.2.10 *λ—for the exponential model*, the failure rate; also equal to 1 / θ.

3.2.11 *f*—scatter factor equal to the ratio $B_{50}/B_{0.1}$

4. Significance and Use

4.1 Reliability demonstration testing is a methodology for qualifying or validating a product's performance capability. Demonstration methods are useful for components, devices, assemblies, materials, processes, and systems. Many industries require demonstration testing either for new product development and product introduction, in validating a change to an existing product or as part of an audit. Test plans generally try to answer the questions, "How long will a product last?" or "What is its reliability?", under stated conditions at some specific time. When time is being used as a life variable, it must be cast in some kind of "time" units. Typical time units are hours (or minutes), cycles of usage, calendar time or some variation of these. In certain cases, "time" can be accelerated in order to reduce a plan's completion time. In the automotive industry mileage may be used as the time variable. Certain means of accelerating tests involve the use of increased power, voltage, mechanical load, humidity, vibration, or temperature (often in the form of thermal cycling).

4.2 Two fundamental objectives in reliability test planning are: *(a)* demonstrating that a product meets a specific life requirement, and *(b)* demonstrating what a product can do – its life capability. In the first case, a requirement is specified; in the second case a series of test results are used to state a result at the present time – its current capability. Both cases share similar inputs and outputs.

4.3 Often a life distribution model is specified such as the Weibull, the exponential, the lognormal or the normal distribution. In addition, for the specific distribution assumed, a parameter is typically assumed (or a range of values for a parameter). For example, in the Weibull case, the shape parameter, β, is assumed; in the lognormal case the scale parameter, σ, is assumed and in the normal case the standard deviation, σ, is assumed. In other cases, a non-parametric analysis can be used. Non-parametric cases typically require a larger sample size than parametric cases. This standard will discuss conditions under which distributions and associated parameters can be assumed.

4.4 Generally, a life requirement is cast as a mission time and associated reliability, for example, to demonstrate a reliability of 99% at time *t*=1000 hours of usage. In another case the requirement might be cast as a B_p life requirement, such as the B_5 life. For example, if B_5 = 10 000 cycles are specified, this means to demonstrate a reliability of 95 % at *t* = 10 000 cycles. Other life requirements might be a mean life, a median life (B_{50}) or a failure rate not to be exceeded at a specified time *t*. In other cases, the requirement might mean withstanding a load for some duration. Demonstration necessarily means to demonstrate with some statistical confidence. Thus, a confidence value is a standard input in any plan. Commonly used confidence values are 99 %, 95 %, 90 %, and 63.2 %.

4.4.1 When a requirement and a confidence value have been stated, a derived plan will determine a sample size, *n*, a test time, *t*, and a maximum number of failures, *r*, allowed by the plan. A test concludes and is successful if the *n* units tested result in not more than *r* failures by time *t*. In another scenario, the sample size, number of failures allowed and confidence value are first stated and the plan returns the test time requirement.

4.5 The "RC" nomenclature for specifying a test requirement is often used, where *R* stands for reliability and *C* for confidence. For example, to state a requirement of 2000 hours at R99C90 means that the requirement is to demonstrate 99 % reliability at 2000 hours with 90 % confidence. Alternatively, this also means to demonstrate a B_1 life of 2000 hours with 90 % confidence.

4.6 This guide considers, the Weibull, lognormal and normal parametric cases as well as the basic non-parametric case for attribute reliability. The common exponential case is a Weibull distribution with assumed shape parameter β = 1, but is considered as a separate case, distinct from the Weibull.

5. General Introduction

5.1 Before reliability can be assessed, and a test plan specified, it is useful to know something about the nature of the failure modes that might occur. The type of failure mode is important in selecting a life distribution and any assumed parameter where variable (continuous distribution) data is used.

5.1.1 In general, there are three classes of failure modes that operate in electro-mechanical systems: *(a)* random, *(b)* wear out, and *(c)* "infant mortality". These are related to the failure rates operative in field applications.

5.2 In a random type failure mode, the failure mechanism is not related to the age of the object tested or fielded, and the failure rate is constant throughout this portion of life (constant failure rate or CFR). A new object and any unfailed object having seen usage each have the same propensity to fail in the future. Causes for this type of failure mode are often related to some external stimulus, operability robustness or to a complex of rare factors that might come together, rarely, in just the right mix to cause a failure.

5.2.1 Random stress spikes under field conditions, such as those driven by electrical, mechanical, chemical or thermal shock may cause random failures at any point in a product life cycle. Operating a product outside of specification limits at random times may render a product prone to failure at any time in its life. Other outside stimuli such as those caused by foreign object damage or biological interference are possible and may lead to random type failures.

5.3 Wear out mechanisms are related to the age of the object and have increasing failure rates with time/usage. This is called an increasing failure rate (IFR). The more usage the object has, the greater the likelihood of failures for the surviving population. Wear-out or performance degradation is generally a gradual process as usage increases. Ultimately, this results in loss of robustness and eventual failure. In electro-mechanical applications, causes of this type of failure may be driven by gradual chemical, thermal, mechanical, electrical or radiation stresses.

5.4 An "infant mortality" type failure mode relates to certain severe conditions possessed by some objects in a population rendering these select units prone to early failure, while the remaining population may last many times longer. The older a unit is, the less likely such failures would appear because the affected unit would most likely have failed early in the life cycle if it had the severe condition. Essentially, there is a failure mode and some items in the population have the failure conditions more severely than others, resulting in early failure. The failure rate in these cases is decreasing with time. This is called a decreasing failure rate (DFR). There are numerous causes for this case ranging from assembly issues to material problems.

5.5 The three general classes of failure are depicted in the so-called *bathtub curve* shown in Fig. 1. Fig. 1 is a highly idealized portrait of a product's life cycle. It suggests that there are three phases to life: *(a)* the burn-in period is the trouble shooting or corrective action phase where the failure rate is decreasing, *(b)* the random period is the useful life phase where the failure rate is constant, and *(c)* the wear out period with an increasing failure rate leads to end of life or retirement. Not every product will experience all three phases and the degree of each phase depicted may be of very dissimilar proportion.

5.5.1 *Infant Mortality*—Failures of this type are generally the result of some components that do not meet specifications or workmanship standards. These types of failures are typically not design-related issues, but quality-related issues. As a product is put into service, early failures are observed as a result of these or similar conditions. Corrections are gradually made, and the failure rate decreases for a given time until it may reach a steady state or constant rate. The infant mortality period, then, is characterized by a decreasing failure rate. The Weibull distribution with shape parameter $0<\beta<1$ is commonly used to characterize infant mortality conditions.

5.5.2 *Constant Failure Rate*—Once the failures due to components and workmanship are for the most part eliminated, the constant failure rate period is entered, and called the random failure rate period. "Random" means that failure events are proportional to time in use, but this is not dependent on age at the start of any time interval. The constant failure rate period is the most common time frame for making reliability predictions, where the exponential distribution is used. The exponential – which is equivalent to a Weibull with $\beta = 1$ – can be used to describe this phase. The number of such random failures occurring within this period is Poisson distributed with some mean number of events. A failure rate parameter, λ, in the units of failure per unit time is used to characterize this portion of life.

5.5.3 *Wear Out Period*—As components begin to fatigue or wear out, failures occur at increasing rates for a specified interval. An increasing or sometimes sharp rise in failure rate can be the result of fatigue and other physical actions. As time goes on, failures occur more and more frequently to a point where it may no longer be practical to continue operating the product. Several distributions may be appropriate to model the wear-out period. The Weibull and lognormal distributions are often used. For the Weibull distribution, an increasing failure rate occurs where the shape parameter $\beta > 1$.

5.6 For purposes of validation of a new product introduction or for a change to an existing product it is common practice to use the exponential distribution which covers the useful (random) portion of product life. This assumption is conservative in that it takes more test time and/or a larger sample size to validate a specific requirement for the random life case than for the wear out case. In general, the infant mortality case is not typical in validating procedures (although in theory, this case would be valid). The infant mortality case is more often used in working with a fielded population that shows signs of early failures. It may also be used to develop a burn-in or screening period that a product is exposed to prior to its field application, in an attempt to find early type failures before customer use.

5.7 In some cases, the failure rate may exhibit increases and decreases as a function of age. This is typically distribution-specific and would be the case in certain parameter cases when the lognormal distribution is used.

6. The Nonparametric Case

6.1 In the nonparametric case (NP) there is no life distribution being used to model variable data. Testing is of the pass-fail type and governed by the binomial distribution. In this attribute reliability case, n is the sample size, and p is the failure probability at the specified test condition. The reliability is cast as $R = 1 - p$. The parameters for this test case are C the specified confidence, R the reliability, n, the sample size, and r the number of failures in n allowed by the plan.

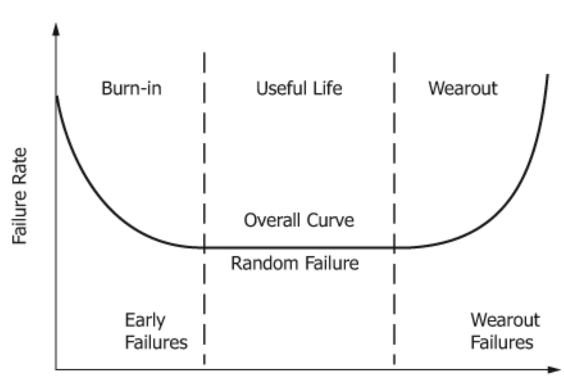

FIG. 1 The "Bathtub" Curve

6.1.1 A random variable X is said to have a binomial distribution if x is takes on integer values between 0 and n, and is a count of the number of occurrences of a defined condition, where each of the n sample units either has or does not have the condition. Each sample unit has a probability p of having the condition and the n sample values behave independently of one another. We say that X is binomially distributed with parameters n and p.

6.1.2 A "failure event" is defined as the failure of a component, a material specimen or other entity to meet a well-defined requirement. A requirement may be a variable quantity such as a number of cycles or a minimum load condition in a fatigue test, a variable dimensional requirement, or a basic attribute tested as go or no-go type of characteristic. Each test either meets or fails to meet the requirement. For pure attribute pass or fail testing, a "zero failure" test plan is a common theme. For the zero-failure test plan the following equation, based on the binomial distribution, relates sample size, n, confidence, C, and reliability, R (**1, 2**).[4]

$$R \geq \sqrt[n]{1 - C} \tag{1}$$

6.1.3 In Eq 1, C is the chosen confidence coefficient ($0 < C < 1$), n is the sample size ($n > 0$), and R is the demonstrated reliability ($0 < R < 1$) when n tests result in zero failures. Eq 1 may be solved for either C or n giving.

$$C \geq 1 - R^n \tag{2}$$

$$n = \frac{\ln(1 - C)}{\ln(R)} \tag{3}$$

6.1.4 In Eq 2, the confidence in meeting the reliability requirement R using sample size n is at least C. In Eq 3, the sample size needed to claim reliability R with confidence C is n. In each case $r = 0$ failures are observed in a sample of size n.

6.1.5 *Example 1*—Suppose $n = 22$ units have been run to a specific requirement and 0 failures were observed. Then at C=90% confidence the minimum reliability demonstrated is $\sqrt[22]{1-0.9} = 0.90$ or 90 %, using Eq 1. This is often referred to as an "R90C90" value (reliability at least 90 % with 90 % confidence). Other "RC" combinations are in use. Suppose further that $R = 90$ % is not good enough and the practitioner requires 95 % reliability, what sample size is necessary? Use Eq 3 with $R = 0.95$ and $C = 0.9$ giving $n = \ln(1 - 0.9) / \ln(0.95) = 44.89$ or $n = 45$ units required without failure. What is the confidence in meeting a reliability of at least 95 % using $n = 22$ tests where 0 failure have occurred? Use Eq 2 with $n = 22$ and $R = 0.95$ giving $C \geq 1 - 0.95^{22} = 0.676$ or about 67.6 % confidence.

6.2 Another commonly occurring case is to allow 1 failure in n (not more than 1 failure among n units tested). In that case, the relation among n, C and R, based on a binomial model and simplified from the relation between a binomial and beta distribution is (**1**):

$$nR^{n-1} - (n - 1)R^n \geq 1 - C \tag{4}$$

[4] The boldface numbers in parentheses refer to the list of references at the end of this standard.

6.2.1 Eq 4 may be solved numerically for any variable when the remaining two are known or assumed.

6.2.2 *Example 2*—Suppose 1 failure in $n = 22$ tests are observed. What is the reliability demonstrated using $C = 90$ % confidence? Use Eq 4 with $n = 22$ and $C = 0.9$, and iterating on R until the inequality in Eq 4 is just met, gives $R = 0.8344$ or about 83.4 % reliability. If the desire is to achieve 99 % reliability using 90 % confidence, what sample size is required if we allow not more than 1 failure in n? Use Eq 4 with $C = 0.9$ and $R = 0.99$. Then iterating on n until the inequality is just met find that $n = 388$.

6.3 The variables in the general case include a sample size n, confidence C, reliability R and r the number of allowable failures in n. The solution is a generalization of the $r = 1$ case discussed in section 6.2. The beta distribution equivalent to the binomial probability expression $P(X \leq r) \geq 1 - C$ is used. Let p be the failure probability. Then $R = 1 - p$. Given a confidence level C, a sample size n, and a number of allowed failures r, the relationship is (**3**):

$$C \geq \int_0^{1-R} \beta(r + 1, n - r)dy \tag{5}$$

6.3.1 Eq 5 is a cumulative beta distribution with parameters $r + 1$ and $n - r$ evaluated at the point $1 - R$. Refer to the *cdf* integral as $G(1 - R)$. Then $C = G(1 - R)$ and the parameter R is calculated using the inverse beta *cdf* evaluated at C. This is $G^{-1}(C) \geq 1 - R$. Then the demonstrated reliability R is calculated as:

$$R \geq 1 - G^{-1}(C) \tag{6}$$

6.3.2 This calculation is carried out numerically using for example a spreadsheet-type program having beta distribution capability. When R, n, and r are specified, Eq 5 is used directly to calculate the confidence demonstrated. If R, C and r are specified or if R, C and n are specified, then Eq 5 is iterated on the variable being solved for until the inequality in Eq 6 is just met. It is important, that r must be strictly less than n in all cases for these types of plans. In addition, for specified R, C and n, there may not exist a suitable r that meets the *RC* requirement. In that case, the sample size needs to increase.

6.3.3 *Example 3*—A test plan was conducted using a sample of $n = 250$ units with $r = 3$ failures observed. What reliability is demonstrated at $C = 95$ % confidence? Use Eq 6 with beta distribution parameters $r + 1 = 4$ and $n - r = 247$. This results in $R \geq 0.969$ or 96.9 %. If an R95C95 plan is desired that allows 3 failures in n, what sample size is required? Use Eq 6 with $C = 0.95$, $R = 0.95$ and $r = 3$. Iterate n in the beta parameters until the inequality in Eq 6 is just met. This results in $n = 153$ as the required sample size.

6.3.4 *Example 4*—If $n = 450$ units are available for test and R95C90 is the requirement, what number of failures is allowed? Use Eq 6 with $R = 0.95$, $C = 0.9$ and $n = 450$, iterating on r until the inequality in Eq 6 is just met. Find that $r = 16$ is the maximum number of failures allowed. Note that this combination of n and r is one of several possible combinations that would satisfy the R95C90 requirement. For example, n=282 and $r = 9$ or $n = 209$ and $r = 6$ would also work. A smaller sample size would be possible and more economical. Process capability should be taken into account, where feasible, when deciding on any plan. Table 1 is a short summary comparing sample sizes for the $r = 0$, $r = 1$ and $r = 2$ cases and several common "RC" combinations.

7. The Exponential Case

7.1 The exponential distribution case is one the most commonly required testing scenarios found in reliability. In this section, theory and methods are outlined for the exponential life case where the failure mode is assumed to be of the random type. Further related information is given in the Appendices. The exponential distribution is shown in Fig. 2. It is used for predicting the reliability of items in the constant failure period (see Fig. 1). This is typically the starting point in design reliability requirements determination.

7.2 The exponential probability density function (*pdf*) is:

$$f(t) = \frac{1}{\theta}e^{-t/\theta},\ t>0 \tag{7}$$

7.2.1 The parameter θ is often called the mean time to failure (or MTTF) and is the mean of the distribution of t for single use items or first occurrence failure times. In repairable systems, units are repaired upon failure and reinstalled until failure a second or a third time, the average time between repair cycles is called the mean time between failures (MTBF). In general, this should not be confused with the MTTF because repair does not always render a repaired item in like new condition; however, for the exponential case, it does, at least in theory, and we have that MTTF = MTBF.

7.2.2 Eq 7 may be recast using $\lambda = 1 / \theta$ where λ is called the failure rate (failures per unit time).

$$f(t) = \lambda e^{-\lambda t} \tag{8}$$

7.2.3 Integrating Eq 8 gives the cumulative distribution function (*cdf*), $F(t)$.

$$\int_0^t f(y)dy = F(t) = 1 - e^{-\lambda t} \tag{9}$$

7.2.4 Where, again, λ is substituted for $1 / \theta$. $F(t)$ is the failure probability at time *t* for objects having a constant failure rate λ. The quantity $1 - F(t) = R(t)$ is the reliability function at time *t*. Another name for reliability is survival probability, used particularly in the life sciences.

$$R(t) = 1 - F(t) = e^{-\lambda t} \tag{10}$$

7.2.5 The hazard function, $h(t)$, of any failure time distribution is the ratio $f(t)/R(t)$. It is the instantaneous failure rate at time *t* for the population of surviving units at that time. For the exponential, $h(t) = \lambda$, a constant throughout time.

7.2.6 The exponential distribution has the following key properties:

7.2.7 The mean and standard deviation are the same value.

7.2.8 Approximately 63.2 % of the area under the curve falls below the mean (θ). This further means that the failure probability at the mean life is F(θ) = 63.2 % leaving the reliability at the mean life of 100 – 63.2 = 36.8 %. This property is not greatly appreciated by practitioners and can be the source of numerous types of errors and misunderstandings concerning reliability with this model.

7.2.9 The hazard function (failure rate) is constant throughout life.

7.2.10 The quantity λt is called the cumulative hazard. It is the expected number of failures at time *t*. For given time interval *t*, the number of failures in the interval [0,*t*] is Poisson distributed with mean λt provided the random behavior remains homogeneous through time *t*. For the exponential, this is independent of the starting time of the interval.

7.2.11 *Memoryless Property*—For an exponentially distributed life, the probability that an item of age *t*, still surviving, lasts an additional time *s* without failing is identical to a brand-new item lasting through a time *s* without failure. Essentially, an item's propensity to fail is not age dependent. A surviving item at any age has the same probability of failing within or surviving a future time duration s as a new item does. Thus, there is no wear out mechanism.

7.2.12 A key property that holds only for the exponential distribution is:

$$R(t + s) = R(t)R(s) \tag{11}$$

7.2.12.1 This can be manipulated further as:

$$R(ns) = \{R(s)\}^n \tag{12}$$

7.2.12.2 For example, if we know that the reliability at 1 cycle is 0.9999 ($s = 1$), then the reliability at 1000 cycles ($n = 1000$) is 0.9999^{1000} or about 0.905.

7.2.13 *Reproductive or Closure Property*—The smallest order statistic in a sample of size *n* from an exponential distribution with mean θ is also exponentially distributed with mean θ / n. The smallest in *n* inherits the exponential property with mean proportional to $1 / n$.

7.2.14 *B-life Formula*—The B_p life of a failure distribution is that time by which *p*% of the population is expected to fail by. For the exponential distribution this is found by inverting the *cdf*, Eq 7:

$$B_p = -\theta \ln\left(1 - \frac{p}{100}\right) \tag{13}$$

7.2.15 *Simulation*—A random observation from an exponential distribution with mean θ is determined using:

$$t = -\theta \ln(u) \tag{14}$$

where:

u = a random observation from a uniform distribution on [0,1].

TABLE 1 Sample Sizes for Nonparametric RC[A] Test Plan Cases with *r* Failures Allowed

RC case	r=0	r=1	r=2
R90C90	22	38	52
R90C95	29	46	61
R90C99	44	64	81
R95C90	45	77	105
R95C95	59	93	124
R95C99	90	130	165
R99C90	230	388	531
R99C95	299	473	628
R99C99	459	662	838

[A] "R" = reliability, "C" = confidence

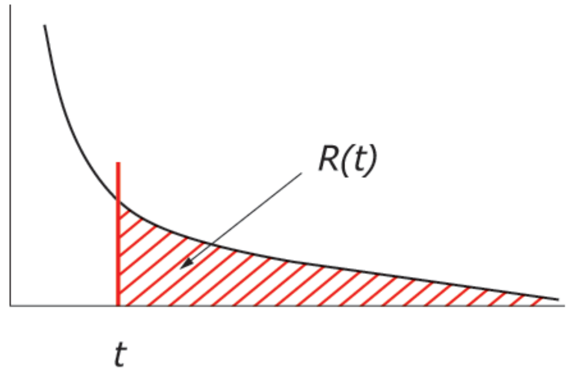

FIG. 2 The Exponential Distribution

7.2.16 Both the time units and the failure rate may be expressed in several ways. Hours or cycles are the typical time units used. Several commonly used metrics are:

(1) Failures/hr,
(2) Failures per 100k hours,
(3) Failures per million hours, and
(4) %/*K* hours.

7.2.17 The last expression, %/*K* hours, represents % failing in 1000 hours of service. For example, if the rate is 1.5 %/*K* and 10 000 units each operate for 500 hours we would expect 75 failures because 10 000 units at 500 hours each is equivalent to 5 000 units at 1000 hours each. 1.5 % of 5000 is 75 failures.

7.2.18 The standard rate in failures per hour is then 75 / (10 000 × 500) = 1.5E-5. The relationship between %/*K* and the failure rate λ in failures per hour is: %/*K* = $10^5\lambda$.

7.2.19 Another variation is "failures in time" abbreviated as FIT and equivalent to parts per million per 1000 hours of service or PPM/*K*. 1 PPM/*K* means 1 failure in 1 million units each operating for 1000 hours or 1 failure per billion operating hours. This is equivalent to FIT = 10 000(%/*K*). In the above example, the FIT number is 10 000X(1.5%/*K*) = 15 000 FIT. Table 2 shows a conversion among λ, %/*K* and FIT in the time units of hours.

7.2.20 Eq 9 can be re-parameterized in the following way substituting any B_p life in for θ. This equation is useful in some applications.

$$F(t) = 1 - (1 - p / 100)^{\frac{t}{B_p}} \quad (15)$$

7.2.21 *Example 5*—A certain type of transistor is known to have a constant failure rate with rate 0.04 %/*K*. What is the probability that one of these transistors fails before 15 000 hours of use? How long do we have to wait to expect 1% of the population to have failed? What is the FIT measure of the failure rate?

TABLE 2 Equivalent Failure Rates in Different Metrics[A]

rate, λ	%/K	FIT
1E-04	10	100000
1E-05	1	10000
1E-06	0.1	1000
1E-07	0.01	100
1E-08	0.001	10
1E-09	0.0001	1

[A] λ = 1E-5X(%/*K*) = 1E-9X(FIT); FIT = 1E4X(%/*K*)

7.2.21.1 First, convert to the failure metric to failures per hour, λ. Multiply %/*K* by 10^{-5} to get λ = 4E-7. The probability of failure by 15 000 hours is *F*(15 000) or:

$$F(15,000) = 1 - e^{-(4E-7)(15000)} = 0.006$$

7.2.21.2 The 1 % failure time is the B_1 life. Use Eq 13 with θ = 1 / λ = 1 / (4E-7) = 2 500 000 hours.

$$B_1 = -2{,}500{,}000 \ln\left(1 - \frac{1}{100}\right) = 25{,}126$$

7.2.21.3 The FIT measure of failure is $10^9\lambda = 10^9(4E\text{-}7)$ = 400 failures per billion operating hours.

7.2.22 *Example 6*—A certain type of aerospace system uses five components operating simultaneously in series. The component MTTF is known to be 250,000 cycles. The system fails if any of the 5 components fail. Its mission time is 50 cycles of usage. Calculate the system reliability at the mission time. Calculate the $B_{0.1}$ life of the system.

7.2.22.1 The system will fail when the first failure in *n* = 5 occurs. The first of *n* = 5 distribution is also exponential with MTTF = θ/*n* = 250 000 / 5 = 50 000 cycles (see 7.2.13). The probability of failure by the mission time of 50 cycles is:

$$F(50) = 1 - e^{-50/50000} = 0.001$$

7.2.22.2 Thus, the *t* = 50 cycle value is the $B_{0.1}$ system life.

7.3 *Test Planning*—A test plan for the exponential distribution requires several key design parameters.

(1) Sample size (*n*) and number of failures allowed (*r*),
(2) Objective MTTF (θ) or failure rate (λ), and
(3) Confidence level (*C*).

7.3.1 As an alternative objective, a mission time and associated reliability may be specified, and this is equivalent to specifying a B_p life. For example, if a reliability of 0.99 is required for a mission time of 1200 hours, this is equivalent to stating a B_1 life requirement of 1200 hours.

7.3.2 We may also ask what MTTF, B_p life or reliability at a stated mission time *T* has been demonstrated given a set of failure time data at the present time during a test. In addition, we can solve for the demonstrated confidence in meeting a requirement.

7.3.3 The most commonly occurring questions about test planning are: *(1)* what sample size (*n*) should I use? And *(2)* what test time (*t*) is appropriate? Associated with both of these is the number of failures allowed (*r*). This analysis starts with a relationship involving, n, r, the confidence value C and the beta distribution on the interval [0,1]. Let p be beta distributed with parameters r+1 and n-r. Its cumulative distribution is denoted $G(p)$. Set $G(p) = C$ and solve for p. This uses the inverse beta integral:

$$p_c = G^{-1}(C) \tag{16}$$

7.3.4 Alternatively, we have the beta integral itself:

$$G(p_c) = C \tag{17}$$

7.3.5 Eq 16 and Eq 17 can be solved using many software programs including spreadsheet-type programs. For clarity of notation, a subscript "c" is attached to p indicating that it satisfies Eq 16 and Eq 17. The quantity p_c is the upper 100C % non-parametric confidence bound on a failure probability p when n units are tested and r failures occur where $0 \leq r < n$.

7.3.6 Let $F(t)$ be the cumulative exponential failure probability where t is the test time and θ is an objective MTTF (see 7.2). Note that we could use λ, θ, or a B_p life, converting one from the other. Use the relation $p_c = F(t)$.

$$p_c = 1 - e^{-\frac{t}{\theta}} \tag{18}$$

7.3.7 Eq 18 then involves the key variables of the test plan and may be solved or manipulated for any variable. It is noted that Eq 18 depends on the ratio of test time to the objective MTTF (t / θ). So long as that remains the same, the test plan will be the same. This further means that for any test plan, the test time will be proportional to the objective MTTF desired. For example, if t / θ = 0.1 then we could use a test time of t = 100 to demonstrate θ = 1000 or t = 10 to demonstrate θ = 100.

7.3.8 *Example 7*—Sample size required; r, C, t, θ specified. An engineer wants to claim an MTTF of θ = 50 000 hours with 90 % confidence (C = 0.90). He is willing to allow up to r = 5 failures during the test and can run each of the samples for 500 hours. What sample size, n, is required?

7.3.8.1 First solve Eq 18 for p_c, using θ = 50 000 and t = 500. This results in p_c = 0.00995. Next, solve the beta function $G(p_c) = C$ targeting C = 0.90, using r = 5 held constant and adjusting n in the formula until $G(p_c)$ = 0.9 results. It is useful to use a spreadsheet type of calculator in executing the calculation. The exhibit below shows several steps toward the solution. We want to solve $G(p_c)$ = 0.9 for n using C = 0.9 and p_c = 0.00995.

Solution Exhibit for Example 7

n	$G(p_c) = C$
900	0.88278
910	0.88879
915	0.89169
920	0.89453
925	0.89730
926	0.89785
929	0.89948
930	**0.90002**

7.3.8.2 Here we see that n = 930 is the solution. This method may also be used if r is the unknown. In this case, n remains fixed and r is varied until Eq 16 is true.

7.3.9 *Example 8*—Number of failures (r) required; n, C, t, θ specified. Suppose in Example 7 that n = 1200 units can be tested for the increased time of t = 600. We want to determine the maximum number of failures for the same confidence level, C = 0.90, and objective MTTF of 50 000.

7.3.9.1 Again, solve Eq 18 for p_c, using θ = 50 000 and t = 600. This result is p_c = 0.011928. Next, solve the beta function $G(p_c) = C$ targeting C = 0.90, using n = 1200 held constant and adjusting r in the formula until $G(p_c)$ = 0.9 results.

Solution Exhibit for Example 8

r	$G(p_c)$=C
6	0.9886
7	0.9741
8	0.9479
9	**0.9060**
10	0.8458

7.3.9.2 Here we see that r = 9 is the solution.

7.3.10 To solve for the test time per unit required first use Eq 16 solving for p_c with an assumed confidence, C, sample size n, and number of failures allowed, r. Then using p_c and the objective MTBF, θ, solve Eq 18 for t. This results in:

$$t = -\theta \ln(1 - p_c) \tag{19}$$

7.3.11 *Example 9*—Test time required; n, r, C and θ specified. Suppose we want to demonstrate a B_1 life of 2500 cycles using 95 % confidence. We will use a sample size of n = 120 units and allow r = 2 failures. What test time should be used?

7.3.11.1 First, use Eq 13 with p = 1 and B_1 = 2500, solving for θ, and finding θ = 248 748. We simplify this by rounding up to 249 000. Next, use Eq 16 with n = 120, r = 2 and C = 0.95 finding p_c = 0.051534. Then use Eq 18, solving for t finding t = 13 174.4 cycles.

7.3.12 *Example 10*—Confidence demonstrated required; n, r, t and θ specified. If 150 units on test have resulted in 3 failures at t = 100 cycles, what confidence can we claim in stating an MTTF requirement of 2500 cycles has been met?

7.3.12.1 Use Eq 17 directly with p_c = 0.63212. This is the failure probability in theory at t=θ. Use the beta integral with parameters r + 1 and n – r or 4 and 147 respectively. The beta integral evaluates to 0.8433 making the confidence demonstrated 84.3 %. Iterating on sample size, n, shows that n = 169 would be the requirement to state at least 90% confidence with r = 3 failures.

7.3.13 *The Zero-Failure Case*—Many organizations require zero failure test plans. In such a plan, a sample of n units are tested, where the test times may be different for different units tested, and zero failures is the requirement. To determine a plan, is to determine the total test time, T, required to demonstrate an MTTF of θ using confidence C. A total time T can be calculated using $nt = T$ where t is the time per unit tested in for each of n units. Sample size and test time can be interchanged as long as $nt = T$ is preserved. Further we can have variable test times as long as $T = \Sigma t$ is preserved.

7.3.13.1 The equation governing this case is:

$$T = -\theta \ln(1 - C) \tag{20}$$

7.3.13.2 In using Eq 20, T is the total time on test for all units tested. Once we have T we can decide on the number of test units and distribute the test time in any way that is convenient.

7.3.14 *Example 11*—Determine the total test time, T, required for a plan to demonstrate that the failure rate is not more than λ = 2.5E-5 failures per cycle at confidence 95 % with zero failures required. Here, $C = 0.95$, $\theta = 1 / \lambda = 40\,000$ cycles. Using Eq 20, find that $T = 119\,829.3$ cycles. If we wanted to use equal test times for $n = 10$ units we should use a test time of $T / 10 = 11\,983$ cycles per unit.

7.3.14.1 The $C = 50$ % zero failures estimate is sometimes used as a simple point estimate for λ. It is a value of λ that makes the likelihood of obtaining zero failures in the given experiment 50 %. Using this is as if one has observed 0.693 failures in the sample. This 50 % confidence estimate of λ would then be 0.693 / T. Alternatively, when zero failures are observed in a total test time of T, the assumption of $x = 1$ failure is sometimes assumed. In that case the resulting confidence in the estimate 1 / T is about 63.2 %. When the total test time is T and zero failures have been observed, Table 3 can be used to develop the estimate of λ.

7.3.15 When using a zero-failure plan, it is important to note that these test plans have a low pass probability when the actual true parameter value is at or even substantially larger than the parameter minimum requirement. They are very conservative. For example, a zero-failure plan used to validate an MTTF of 1000 hours with $C = 95$ % confidence requires a total test time of about 3000 hours without failure. The test can be allocated in any configuration of sample size and individual test time as long as the total test time equals 3000 hours. If the actual MTTF is truly just at 1000 hours, the test plan will only achieve a 5 % pass probability. In fact, the pass probability in this scenario will only reach 50 % when the true MTTF is about 4330 – more than 4 times the parameter value minimum requirement.

7.3.16 Compare the zero-failure plan to a plan that allows 1 failure in 5. In that case the test time will be 1071 hours per unit tested making the total time between 4284 and 5355 hours. The $r = 1$ plan can result in as much as a 77.8 % increase in required test time; however, the $r = 1$ plan achieves a pass probability of 50 % at an MTTF = 2840. Fig. 3 shows comparison between the two plans with the 50 % pass probability indicated.

7.3.17 The general rule is that the true MTTF has to be much better than the minimum requirement to have a reasonable pass probability. This same trend holds for other sample sizes and requirements as well as other distributional assumptions.

8. The Weibull Case

8.1 The Weibull distribution is widely used to model field data exhibiting the full range of wear out, infant mortality and random type failure modes. Section 7 discussed failure modes of the random type – the exponential distribution. That model is a special case of the Weibull distribution as will be seen below. For the Weibull, demonstration test planning is generally used where the Weibull takes on wear out type failure modes. Infant mortality type failure modes are not generally assumed for product test. They are more likely manifested as field data cases during an initial product release. The two parameter Weibull model *cdf* is Eq 21.

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{21}$$

8.1.1 The reliability function is:

$$R(t) = 1 - F(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{22}$$

8.1.2 The two parameters shown in Eq 21 and Eq 22 are the Weibull shape parameter, β, and Weibull scale parameter, η. The parameter β is also referred to as the "Weibull slope". The parameter η is also referred to as the "characteristic life" and is formally, the 63.2th percentile of the Weibull distribution. The parameter β is related to the type of failure mode being modeled and the variation in failure times for this model. The three general failure mode classes are all possible using the Weibull distribution. The three cases are defined by values of β.

8.1.3 $\beta < 1$ is the Infant mortality condition where there is a failure mode that is manifested more severely in some units that in others and can cause early failure in those units having the severe condition. The failure rate is decreasing under this condition.

8.1.4 $\beta = 1$ is the random failure mode condition where the failure rate is constant throughout life. This is the exponential model of Section 7.

8.1.5 $\beta > 1$ is the wear out condition where units are wearing out in time with an increasing failure rate.

8.2 The Weibull has a closed form formula for its mean and variance but these quantities are not typically used for test planning purposes. The Weibull hazard function, $h(t)$, and cumulative hazard function, $H(t)$, are also available in closed form (see 7.2) as:

$$h(t) = \left(\frac{\beta}{\eta}\right)\left(\frac{t}{\eta}\right)^{\beta - 1} \tag{23}$$

$$H(t) = \int_0^t h(y)dy = \left(\frac{t}{\eta}\right)^{\beta} \tag{24}$$

8.2.1 The Weibull B_p life is:

$$B_p = \eta\left\{-\ln\left(1 - \frac{p}{100}\right)\right\}^{1/\beta} \tag{25}$$

8.2.2 The interpretation of the B_p life is that the reliability at $t = B_p$ is $(100 - p)$ %. For the cumulative distribution function, the interpretation is $F(B_p) = p$ %. The Weibull distribution also shares the reproductive property of the exponential distribution (see 7.2.13). In a sample of size n, from a Weibull distribution with parameters β and η, the first order statistic (smallest value) has a Weibull distribution with parameters β and $\eta / n^{1/\beta}$.

8.2.3 An alternative parameterization for $F(t)$ that substitutes a B_p life for η is useful in some applications:

$$F(t) = 1 - (1 - p/100)^{\left(\frac{t}{B_p}\right)^{\beta}} \tag{26}$$

8.3 In a field data analysis application or a test plan, the Weibull shape parameter may sometimes be assumed. This is done because there is historical evidence for or industrywide

TABLE 3 Rate Estimate When Observing Zero Failures in Time *T*

Assumed Failures	Rate Estimate λ	Confidence
0.693	0.693/T	50%
1	1/T	63.2%
2.3	2.3/T	90%
3	3/T	95%
4.6	4.6/T	99%

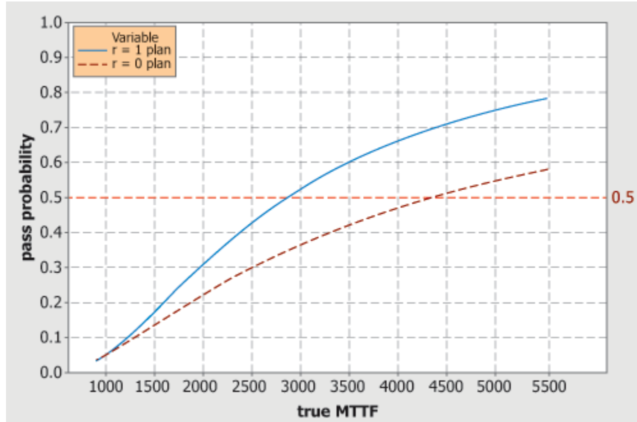

FIG. 3 Comparison of Pass Probability, β = 1, *r* = 0 and *r* = 1 in *n* = 5 tests; *C* = 95 % for MTTF ≥ 1000 hours

agreement for the assumed value. For example, bearing spall in both bench testing and in the field has been observed over and over again in many companies to exhibit a Weibull shape parameter between 1.5 and 2. In aerospace components, high cycle fatigue commonly exhibits a Weibull shape between 4.5 and 6. When β is assumed, the Weibull distribution becomes a one parameter model. That single parameter is traditionally taken as the characteristic life, η, but any other B_p life may be substituted using Eq 25 and Eq 26.

8.3.1 In the Weibull test planning application, β must be assumed. Either a specific value or a range of plausible values for β is assumed. There are two alternative quantities that can be used in place of the assumed β. Each of these is a function of β, and by their selection we are essentially assuming β. The two alternative quantities are the Weibull scatter factor, *f*, and coefficient of variation *cv*.

8.3.2 The Weibull scatter factor, *f*, used in some industries, is the ratio $B_{50} / B_{0.1}$. This is sometimes known in certain types of materials testing, for specific materials. The functional relationship between *f* and β is:

$$f = 692.8^{1/\beta} \tag{27}$$

8.3.3 Table 4 gives several values of *f* as related to β using Eq 27.

TABLE 4 Scatter Factor, f=$B_{50}/B_{0.1}$, as Related to the Shape Parameter, β, for the Weibull Distribution

β	*f*	β	*f*	β	*f*
1.00	692.80	4.50	4.28	11.00	1.81
1.25	187.28	5.00	3.70	11.50	1.77
1.50	78.30	5.50	3.28	12.00	1.72
1.75	42.00	6.00	2.97	12.50	1.69
2.00	26.32	6.50	2.74	13.00	1.65
2.25	18.30	7.00	2.55	13.50	1.62
2.50	13.68	7.50	2.39	14.00	1.60
2.75	10.79	8.00	2.27	14.50	1.57
3.00	8.85	8.50	2.16	15.00	1.55
3.25	7.48	9.00	2.07	15.50	1.52
3.50	6.48	9.50	1.99	16.00	1.51
3.75	5.72	10.00	1.92	16.50	1.49
4.00	5.13	10.50	1.86	17.00	1.47

8.3.4 Thus, if the engineer can specify a scatter factor of say *f* = 3, then β for that case will equal about 8.8. Alternatively, Eq 27 can be solved directly for β giving its exact value as a function of *f*.

8.3.5 The Weibull coefficient of variation, *cv*, is the ratio of the standard deviation to the mean. For the Weibull, this ratio is related to β and is:

$$cv = \frac{\sqrt{\Gamma(1 + 2/\beta) - (\Gamma(1 + 1/\beta))^2}}{\Gamma(1 + 1/\beta)} \tag{28}$$

8.3.6 Where Γ(*z*) denotes the gamma function (see Appendices). The functional relationship between *cv* and β can be solved using a variety of software programs, for example, a spreadsheet-type program. Table 5 gives several values of β as related to *cv* using Eq 28.

8.3.7 It is noted that for the exponential distribution (Weibull β = 1), *cv* = 1 meaning that the mean and standard deviation are equal in that case. Using Eq 27 and Eq 28 a relationship can also be established between the scatter factor and the coefficient of variation.

8.4 *Test Planning*—To specify a test plan for the Weibull model one needs to first select β. In some cases, as shown below, the test plan will be independent of β. In those invariant cases the same sample size would be adequate for any value of β. These plans are essentially identical to the non-parametric case. Most test plans are designed to demonstrate a mission time at an associated reliability using some confidence value *C*. In this standard the mission time is synonymous with the B_p

TABLE 5 Weibull Coefficient of Variation, *cv*, as related to the shape parameter, β

β	*cv*%	β	*cv*%	β	*cv*%
1.00	100.0 %	4.50	25.2 %	9.00	13.3 %
1.25	80.5 %	5.00	22.9 %	9.50	12.6 %
1.50	67.9 %	5.50	21.0 %	10.00	12.0 %
1.75	59.0 %	6.00	19.4 %	10.50	11.5 %
2.00	52.3 %	6.50	18.0 %	11.00	11.0 %
2.50	42.8 %	7.00	16.8 %	11.50	10.5 %
3.00	36.3 %	7.50	15.8 %	12.00	10.1 %
3.50	31.6 %	8.00	14.8 %	12.50	9.7 %
4.00	28.1 %	8.50	14.0 %	13.00	9.4 %

life. This further means that the reliability at mission time B_p is (100 – p) % and the failure probability is p %. For example, if the mission time is 1000 hours and the reliability desired at that time is 99 %, then p = 1 and the mission time is the B_1 life. When β, p, B_p and C have been specified, a basic plan design seeks the sample size n, test time t and the allowable number of failure r. It is possible that t and r can be specified as well, and in that case the sample size is the remaining unknown.

8.4.1 A test plan can also use the Weibull failure rate, $h(s)$, specified at time s as the requirement (instead of the B_p life). In such a case the combination of the assumed β and the specified failure rate at time s, can be converted to any convenient B_p life (such as the B_{10} life) then the test plan determination would proceed along the lines of the more traditional B_p demonstration methodology. For details on the conversion formula, see the Appendices at the end of this standard.

8.4.2 *Case 1*—β, p, B_p, C, n, and r specified; test time t, unknown. The method employs the beta distribution (not to be confused with the Weibull shape parameter β) and the Weibull *cdf*, Eq 21. With confidence C, sample size n, and maximum number of failures r specified, solve the following cumulative beta distribution for v.

$$C = \int_0^v B(r+1, n-r)dy \tag{29}$$

8.4.2.1 In Eq 29, C is the fixed confidence chosen, and $B(a,b)$ is the beta density function with parameters $a = r + 1$ and $b = n – r$. Then C is the beta *cdf* evaluated at v. Refer to this *cdf* as $G(v)$. v is found using the inverse beta *cdf* with argument C. This is:

$$v = G^{-1}(C) \tag{30}$$

8.4.2.2 Once v is determined, it is used in the Weibull *cdf* function, Eq 26, as $v=F(t)$ where t is the resulting test time. The test time t is found by inverting $v=F(t)$. This result is a closed form solution.

$$t = B_p\left(\frac{\ln(1-v)}{\ln(1-p/100)}\right)^{1/\beta} \tag{31}$$

8.4.2.3 In Eq 31, t is the required test time such that in a sample of n units not more than r failures is allowed. This uses a confidence C and an assumed shape parameter β. Note that C, n and r are being incorporated in the calculation of v and do not appear directly in Eq 31.

8.4.2.4 *Example 12*—A demonstration test for a certain type of safety mechanical device has a requirement of $B_{10} \geq 1000$ minutes using 90 % confidence. The test engineer will use β = 1.5 as this is a standard value used for this device industry wide. A sample of size n = 21 is available and the engineer is willing to allow r = 1 failure during the test. It is easy to use a spreadsheet-type program with built-in beta functions to determine the plan for this simple case. The following exhibit is an output from a spreadsheet-type program that shows the inputs, the steps and the final result.

Solution Exhibit for Example 12

C	0.9
n	21
r	1
beta, β	1.5
p	10

Solution Exhibit for Example 12

B_p	1 000
Results, steps	
v	0.172935
t (minutes)	**1 480.9**
Total time max	31 098.7

8.4.2.5 The value v = 0.172935 was determined using Eq 30 with associated n = 21, r = 1 and C = 0.9. Test time t was determined using Eq 31. In this case, t = 1481 test minutes are required for each of the 21 samples. One failure is allowed. If the plan is executed and not more than one failure occurs, then the B_{10} life of 1000 minutes has been demonstrated at 90 % confidence. The total test time in this case is a maximum of 31 099 minutes or 518.4 hours approximately. To see the effect of an increasing sample size, while maintaining the remaining test parameters the following exhibit illustrates.

Solution Exhibit for Example 12, Effect of Varying Sample Size

n	*t*	Total Time
38	989.8	37 612.4
30	1 161.6	34 848.0
25	1 314.8	32 870.0
23	1 391.7	32 009.1
21	**1 480.9**	**31 098.9**
18	1 645.9	29 626.2
16	1 784.9	28 558.4
14	1 957.6	27 406.4
12	2 179.3	26 151.6
10	2 477.0	24 770.0
8	2 903.6	23 228.8
6	3 581.1	21 486.6
3	6 210.5	18 631.5

8.4.2.6 In the above exhibit, the current test plan calls for n = 21 and t = 1481 minutes test time for each unit. If we hold everything constant and vary n we can see the effect on the test time t and the total test time. It is clear that a decreasing sample size will result in a smaller total test time, and therefore it is more efficient, at least in theory, to do a smaller number of test specimens at higher test times.

8.4.2.7 Although the confidence is maintained at 90 % in Example 12, when n = 6, the probability of failing any unit, assuming the true life just meets the minimum requirement (β = 1.5 and B_{10} = 1000 minutes) can be shown to be about 51%. This probability increases with decreasing sample size (and increasing test time). For any confidence, C, the quantity 1 – C is the probability of passing any test plan given the true life of the component being tested is just at the minimum requirement.

8.4.2.8 Consider the effect of the assumed Weibull shape parameter, β, in Example 12. In the next exhibit the columns represent the unit test time as a multiple of the B_{10} life requirement (here B_{10} = 1000 minutes). For example $n(2)$ means we are using a test time of 2 × B_{10} or 2000 minutes per unit. Rows represent the assumed Weibull β parameter. The table body contains the sample size requirement, continuing to allow one failure in n = 21. Although β < 1 is not normally used in test planning, these β values are shown for comparison purposes.

Solution Exhibit for Example 12: Sample Size vs Effect of Varying Test Time and Weibull β

β	*n*(0.5)	*n*(1)	*n*(1.5)	*n*(2)	*n*(3)
0.10	41	38	36	35	34
0.25	45	38	34	32	29

Solution Exhibit for Example 12: Sample Size vs Effect of Varying Test Time and Weibull β

β	n(0.5)	n(1)	n(1.5)	n(2)	n(3)
0.50	53	38	31	27	22
0.75	63	38	28	23	17
1.00	75	38	26	19	13
1.50	105	38	21	14	8
2.00	148	38	17	10	5
2.50	209	38	14	8	3
3.00	296	38	12	6	3
4.00	591	38	8	3	2
5.00	1 182	38	6	2	2
6.00	2 363	38	4	2	2
8.00	9 452	38	3	2	2
10.00	37 805	38	2	2	2

8.4.2.9 When the test time is less than the B_{10} requirement the sample size increases with increasing β as it is in the *n*(0.5) column. When the test time is greater than the B_{10} requirement the sample size decreases with increasing β. In the case of $t > B_{10}$ the assumption of a smaller β than is actually the case would result in a more conservative test in that if the true β were really greater than what we choose, the sample size would be less. This is equally valid for any B_p life requirement, any confidence level and any allowable number of failures. When the test time is greater than the requirement we are trying to demonstrate, the sample size decreases with increasing β.

8.4.2.10 The case where the test time is exactly equal to the B_{10} life to be demonstrated is shown in this exhibit under the column *n*(1). It is beta invariant (the same sample size for all values of an assumed β). This is equivalent to the non-parametric plan at the same confidence level, *n* and *r*.

8.4.3 *Case 2*—β, *p*, B_p, *C*, and *t* specified; sample size and/or number of allowed failures unknown. Work backwards starting with Eq 26 using test time *t* and the mission B_p life with associated *p*. The resulting calculation determines *v* which is used in the cumulative beta distribution Eq 29. In that calculation *v* is kept fixed and the parameters *n* and *r* are adjusted until the value, *C*, is just met. There may be several *n*, *r* combinations that satisfy any set of *C*, *p*, B_p and *t*. The next exhibit shows this result for varying test time *t*.

Solution Exhibit for Example 12: Determination of *n* and *r* with Varying Test Time *t* and All Other Variables Fixed from Example 12 (β = 1.5, *C* = 0.90)

t	r=0	r=1	r=2
800	31	53	72
900	26	44	61
1000	22	38	52
1100	19	33	45
1200	17	29	40
1300	15	26	36
1400	14	23	32
1481	13	**21**	30
1500	12	21	29
1600	11	19	26
1700	10	18	24
1800	10	16	22
1900	9	15	21
2000	8	14	19

8.4.3.1 The bold entry shows the original value of *n* = 21 used in the previous exhibits for Example 12, where *r* = 1 was used. The same pattern of a decreasing sample size would also hold for other values of an assumed β provided β > 1. For each row of the table the actual confidence achieved is slightly more than 90 %. That is, *n* was found for each table entry for which *C* ≥ 90 % was just achieved.

8.4.4 *Case 3*—*r* = 0. The common plan with zero failures (*r* = 0) has a closed form formula for any unknown parameter. The two key formulas are for test time and sample size. In each case there is an assumed Weibull β, a confidence *C*, a sample size *n* and a target B_p life. In addition, there is a formula for the B_p life being demonstrated at specific test time *t* per unit. These are shown in Eq 32, Eq 33 and Eq 34.

8.4.4.1 Test time unknown:

$$t \geq \left(\frac{\ln(1 - C)}{\ln(1 - p/100)} \right)^{1/\beta} \left(\frac{B_p}{n^{1/\beta}} \right) \tag{32}$$

8.4.4.2 Sample size unknown:

$$n \geq \left(\frac{\ln(1 - C)}{\ln(1 - p/100)} \right) \left(\frac{B_p}{t} \right)^{\beta} \tag{33}$$

8.4.4.3 B_p life demonstrated:

$$B_p \geq t \left(\frac{(n)\ln(1 - p/100)}{\ln(1 - C)} \right)^{1/\beta} \tag{34}$$

8.4.4.4 It may sometimes be the case that there is a sample size and there are zero failures, but the test times for each unit may be different. In such a case Eq 35 is used to figure the B_p life being demonstrated at the present time.

$$B_p \geq \left(\sum_{i=1}^{n} t_i^{\beta} \right)^{1/\beta} \left(\frac{(n)\ln(1 - p/100)}{\ln(1 - C)} \right)^{1/\beta} \tag{35}$$

8.4.4.5 *Example 13*—A specification requires a life test with *r* = 0 failures. A B_1 life of 500 cycles is to be demonstrated with 95 % confidence (R99C95). A Weibull distribution has been assumed for the life variable with a shape parameter between 2 and 4. It is feasible to carry out any test to *t* = 1000 cycles. What sample size should be used? Use Eq 33 with β's of 2, 3, and 4 (for comparison purposes). This gives sample sizes of 75, 38, and 19 respectively, rounded up. The *n* = 75 case, using β = 2, is the most conservative; while *n* = 19, using β = 4, the more relaxed. It may be a compromise to use β = 3, with *n* = 38.

8.4.4.6 *Example 14*—Suppose 8 units are available for test and we want to demonstrate a B_5 life of 300 hours using 95 % confidence (R95C95) and allow 0 failures in the sample. The Weibull distribution is assumed for the failure mode having β = 1.8, what test time should be used? Use Eq 32 finding *t* = 905.2 hours.

8.4.4.7 Table 6 contains sample sizes for *r* = 0, 1, and 2 allowable defective units, Weibull β's of 1, 2, 3, and 4, and test times equal to 0.5, 1, 2, and 3 times the B_p life requirement being demonstrated. Confidence and reliability are indicated by the "RC" column. For example, if a B_1 life of 500 hours is being demonstrated using 95 % confidence, assuming β = 3 and testing each unit for a duration of 2XB_1 allowing 1 failure in *n*, the sample size should be *n* = 60. The corresponding zero failure plan would require *n* = 38. In this sense the B_1 requirement at 95 % confidence means we want an R99C95 requirement.

8.5 *"Sudden death" Tests*—A "sudden death" test requires that the test apparatus can accommodate *n* units simultaneously. A set of *n* units on test is started simultaneously and stopped upon the first failure in *n*. In some industries this is referred to as "1st of *n*" testing, for example 1st of 4 or 1st of