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Measurement of fluid flow — Estimation of uncertainty of a flow-rate measurement

Mesure de débit des fluides — Calcul de l'erreur limite sur une mesure de débit

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FOREWORD

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Draft International Standards adopted by the technical committees are circulated to the member bodies for approval before their acceptance as International Standards by the ISO Council.

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It has been approved by the member bodies of the following countries :

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Germany

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Annexes

Measurement of fluid flow — Estimation of uncertainty of a flow-rate measurement

0 INTRODUCTION

0.1 Notation

Symbol	Description
a, b, c	Constants
$(E_R)_{95}$	Percentage random uncertainty at the 95 % confidence level
E_s	Percentage systematic uncertainty
e_i	Uncertainty in the measurement of the quantity Y_i
$e_{i,j}$	Interdependent uncertainty due to dependence between the variables Y_i and Y_j
e_R	Random uncertainty
$(e_R)_{95}$	Random uncertainty at the 95 % confidence level
e_s	Systematic uncertainty
M	Measured value
n	Number of measurements of the value of a variable
q	Flow-rate
R	The result of a measurement
s_Y	Estimate of the standard deviation of the variable Y
$s_{\overline{Y}}$	Estimate of the standard error of the mean of n independent measurements
t	Student's t
Y	Any variable
$\overline{Y}$	Arithmetic mean of the n measurements of the variable Y
δt	Systematic error
δq	Uncertainty in flow-rate measurement
θ_i	Dimensional sensitivity coefficient of the quantity Y_i
θ_i^*	Dimensionless sensitivity coefficient of the quantity Y_i
ν	Degrees of freedom
σ_Y	Standard deviation of the variable Y

0.2 Glossary

The majority of the definitions given here are taken from ISO 3534, *Statistics — Vocabulary and symbols*. Figure 1 is, however, given in order to assist in the understanding of some terms.

Where a term has been adequately defined in the main text, reference is made to the appropriate clause or sub-clause.

0.2.1 error : In a result, the difference between the measured and true values of the quantity measured.

0.2.2 random error : See 3.2.

0.2.3 systematic error : See 3.3.

0.2.4 spurious error : See 3.1.

0.2.5 constant sytematic error : See 3.3.

0.2.6 variable systematic error : See 3.3.

0.2.7 true value : The value which characterizes a quantity perfectly defined in the conditions which exist at the moment when that quantity is observed (or the subject of a determination). It is an ideal value which is assumed to exist and which could be known only if all causes of error were eliminated.

0.2.8 confidence level : See clause 2.

0.2.9 confidence limits : Each of the lower and upper limits, T_1 and T_2, of the two-sided confidence interval. For a one-sided interval, the single limit T of this interval.

0.2.10 uncertainty : The interval within which the true value of a measured quantity can be expected to lie with a stated probability : it is given as $\pm t s_Y$, with the value of t equal to that corresponding to the chosen probability.

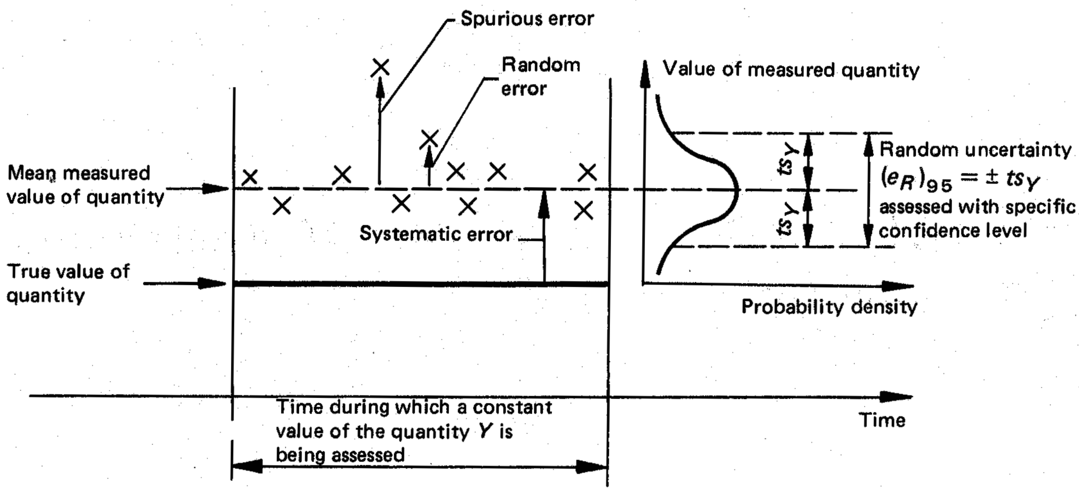


FIGURE 1

0.2.11 interdependent uncertainty : See 4.3.

0.2.12 random uncertainty : The uncertainty associated with a random error.

0.2.13 systematic uncertainty : The uncertainty associated with a systematic error.

0.2.14 standard deviation : The positive square root of the arithmetic mean of the squares of the deviations from the arithmetic mean.

0.2.15 standard deviation estimation : See 3.2.1.1.

0.2.16 sensitivity coefficient : See 4.1.

0.2.17 error limits of a measuring device; class of accuracy : The maximum possible positive or negative deviations of a measured value from the true value; the interval between them characterizes the range within which the true value will be found with a high degree of probability (greater than 95 %).

0.2.18 mean estimated error : The mean of the maximum and minimum values which it is considered a systematic error may have. (See also 3.3.1.)

0.2.19 randomize : To cause to vary according to the laws of chance.

SECTION ONE : GENERAL THEORY

1 SCOPE AND FIELD OF APPLICATION

Whenever a measurement of flow-rate (discharge) is made, the value obtained is simply the best estimate of the true flow-rate which can be obtained from the experimental data. In practice, the true flow-rate may be slightly greater or less than this value. This International Standard describes the calculations required in order to arrive at a statistical estimate of the interval within which the true flow-rate may be expected to lie.

These calculations are presented here in such a way as to be applicable to any flow measurement method, whether the flow is in open or closed ducts. In practice some simplifications may be possible when a particular type of flowmeter or flow measuring technique is used. Such simplifications are to be incorporated in the relevant clauses on "Uncertainty of measurement" in the particular standard dealing with that device or technique. For specific cases, therefore, reference should be made to the appropriate International Standard. This International Standard should be used for guidance on the general techniques to be applied.

This International Standard deals only with the statistical treatment of measurements made with one specific method in order to determine single values of either mass or volume flow-rate. No attempt is made to give guidance on how to obtain the best estimate of flow-rate from a series of measurements of different flow-rates, or on how to obtain the most accurate relation between flow-rate as a variable and any other variable (such as the power input to a pump). Consideration is, however, given to the possibility of reducing the uncertainty in the flow-rate measurement by repeating the measurement and reporting the average value as the result.

2 GENERAL PRINCIPLES

Owing to the very nature of physical measurements, it is impossible to effect the measurement of a physical quantity without error. The usefulness of the measurement is greatly enhanced if a statement of the possible error accompanies the result but is is rarely possible to give an absolute upper limit to the value of the error. It is therefore more practicable to give an interval within which the true value of the measured quantity can be expected to lie with a suitably high probability. This interval is termed the "uncertainty" of measurement and the "confidence level" associated with the uncertainty indicates the probability that the interval quoted will include the true value of the quantity being measured. It is, however, possible to calculate confidence limits only when the distribution of the measured values about the true value is known.

Although it is not possible to attach confidence limits to any assessment of a systematic error (except in special circumstances, where the error can effectively be randomized – see 3.3.1) it is nevertheless necessary to obtain some indication of the interval within which a systematic error may reasonably be expected to lie. In such cases the mean estimated error (3.3.1) is used.

It is worth noting a fundamental difference between error and the uncertainty, which is that the former is by definition unknown whereas the latter may be estimated.

2.1 Terminology

Throughout this International Standard, the terminology used is that specified in ISO 3534, *Statistics – Vocabulary and symbols*. The more important definitions are listed in the glossary (0.2).

2.2 The relation between uncertainty and confidence level

The uncertainty and the confidence with which it can be used are closely related; the wider the uncertainty, the greater is the confidence that the true measurement will be encompassed by this range. This applies even where the confidence level cannot be calculated, where the error is systematic in nature, for example. Where the shape of a probability distribution is known, it is often possible to calculate a new value for the uncertainty of measurement for a different probability from a given uncertainty and associated probability. It is, however, necessary to reach a compromise between choosing, at the one extreme, a very narrow uncertainty range with a low confidence level and, at the other, a wide uncertainty range with a high confidence level. Nevertheless, the confidence level is an essential part of the uncertainty statement, and must be included even if it has to be accompanied by an indication that it is very approximate.

Given the adequacy of the data available, the choice of the confidence level at which to work is therefore determined by the implications for those who will use the measurement result. For flow measurement, the adoption of a probability of 95 % as the confidence level to be associated with the uncertainty statement is a suitable compromise between the considerations given above, and will be the policy for this International Standard whenever confidence levels can be stated.

3 NATURE OF ERRORS

There are four types of error which must be considered :

a) spurious errors;

b) random errors;

c) constant systematic errors;

d) variable systematic errors.

3.1 Spurious errors

These are errors such as human errors, or instrument malfunction, which invalidate a measurement; for example, the transposing of numbers in recording data or the presence of pockets of air in leads from a water line to a manometer. Such errors should not be incorporated into any statistical analysis and the measurement must be discarded. Where the error is not large enough to make the result obviously invalid, some rejection criterion should be applied to decide whether the data point should be rejected or retained.

Thus, whenever it is suspected that one or more results have been affected by errors of this nature, a statistical "outlier" test should be applied. A general test is given in annex A which can be used both for a single suspect value or if more than one point is believed to be spurious. It should be noted, however, that the use of this test is rigorously permissible only when the population is normally distributed.

It is necessary to recalculate the standard deviation of the distribution of results after applying the outlier test if any data points are discarded. It should also be emphasized that outlier tests may be applied only if there is independent technical reason for believing that spurious errors may exist : data should not lightly be thrown away.

3.2 Random errors

Random errors are sometimes referred to as precision or experimental errors. They are caused by numerous, small, independent influences which prevent a measurement system from delivering the same reading when supplied with the same input value of the quantity being measured. The data points deviate from the mean in accordance with the laws of chance, such that the distribution usually approaches a normal distribution as the number of data points is increased.

When the sample size is small, it is necessary to correct the statistical results that are based on a normal distribution by means of the Student's t values, as explained in annex B. Student's t is a factor which compensates for the uncertainty in the standard deviation increasing as the number of measurements is reduced. A skewed distribution of the measurements about the mean value can be caused by variable systematic error, and must be taken into account as explained in 3.3.

3.2.1 *Calculation of uncertainty associated with random errors*

It is possible to calculate statistically the uncertainty in a measurement of a variable when the associated error is purely random in nature. To do this it is necessary to calculate the standard deviation and to decide on the confidence level which is to be attached to the uncertainty. For this International Standard the 95 % confidence level shall be used.

3.2.1.1 Standard deviation

If the error in the measurement of a quantity, Y_i, is purely random, then when n independent measurements are made of the quantity the standard deviation[1)] of the distribution of results, s_{Y_i}, is given by the equation

$$s_{Y_i} = \left[\frac{\sum\limits_{r=1}^{n} [(Y_i)_r - \overline{Y}_i]^2}{n-1} \right]^{1/2} \qquad \ldots (1)$$

where

$\overline{Y}_i$ is the arithmetic mean of the n measurements of the variable, Y_i;

$(Y_i)_r$ is the value obtained by the rth measurement of the variable, Y_i;

n is the total number of measurements of the variable, Y_i.

For brevity, s_{Y_i} is normally referred to as "the standard deviation of Y_i".

The random error in the result can be reduced by making as many measurements as possible of the variable and using the arithmetic mean value, since the standard deviation of the mean of n independent measurements is $\sqrt{n}$ times smaller than the standard deviation of the measurements themselves.

Thus, the standard deviation of the mean, $s_{\overline{Y}}$, is given by the equation

$$s_{\overline{Y}} = \frac{s_Y}{\sqrt{n}} \qquad \ldots (2)$$

3.2.1.2 Confidence levels

If the true standard deviation σ_{Y_i} is known (as n approaches infinity, s_{Y_i} approaches σ_{Y_i}), the confidence level can be related to the uncertainty of measurement as indicated in table 1.

TABLE 1 — **Confidence levels**

Uncertainty	Confidence level
$\pm\,0{,}674\,\sigma_{Y_i}$	0,50
$\pm\,0{,}954\,\sigma_{Y_i}$	0,66
$\pm\,1{,}960\,\sigma_{Y_i}$	0,95
$\pm\,2{,}576\,\sigma_{Y_i}$	0,99

1) The standard deviation as defined here is what is more accurately referred to as the "estimated standard deviation" by statisticians.

For example, the interval $\overline{Y}_i \pm 1{,}96\ \sigma_{Y_i}$ would be expected to contain 95 % of the population. That is, where a single measurement of the variable Y_i is made, and where the value of σ_{Y_i} is independently known, there would be a probability of 0,05 of the interval $(Y_i)_r \pm 1{,}96\ \sigma_{Y_i}$ not including the true value.

In practice, of course, it is possible to obtain only an estimate of the standard deviation since an infinite number of measurements would be required in order to determine it precisely, and the confidence limits must be based on this estimate. The "*t* distribution" for small samples (see annex B) should be used to relate the required confidence level to the interval.

3.3 Systematic errors

Systematic errors are those which cannot be reduced by increasing the number of measurements if the equipment and conditions of measurements remain unchanged. They may be divided into two broad groups, namely : constant systematic errors and variable systematic errors.

a) *Constant systematic errors*

These are common to all measurements made under the same conditions and are constant with time but, depending on the nature of the error, may vary with the value obtained for the measurement. Thus, for example, inaccuracy in the calibration of an instrument would lead to an error which varies over the range of the instrument, whereas a constant systematic error which is independent of the size of the reading would be caused by an incorrectly set zero in the instrument.

NOTE — If a series of flow-rate measurements were to be made (for example, in order to obtain the efficiency curve for a turbine) the former type of error would in fact be variable with flow-rate, but would still be a constant systematic error, since the error would always have the same value at the same flow-rate. It should again be noted, however, that this International Standard deals only with the measurements of a single flow-rate.

b) *Variable systematic errors*

These may arise from inadequate control during the test or experiment, being caused by, for example, changes in temperature which are not allowed for during the use of a pressure gauge which had been calibrated at a fixed temperature, or by progressive wear in the bearings of an instrument.

NOTE — Such errors will usually cause a skewed distribution of results. In practice no finite set of measurements will give a perfectly symmetrical distribution, due to sampling error, even if no variable systematic error were present. The methods for determining if the skewness of the distribution of the measurements is in excess of what would be expected from sampling error are beyond the scope of this International Standard, but it is noted that statistical tests exist which permit a decision on whether the skewness is of the order of that to be expected from the size of the sample, or whether a variable systematic error is present. If the latter, then either the source of variable systematic error must be removed (by improving the control over the experimental conditions) or the possible error must be included in the analysis of the results.

A second type of variable systematic error may occur where digital measurements are taken on a continuously varying quantity. Here, the measurement is of a series of discrete objects or events with some imprecision in the definition of the beginning and ending of the set. The uncertainty in the measurement due to its digital nature then depends on the order of the final digit. If, for example, a four-digit counter were used to count the number of cycles in a periodic wave form where each cycle is recorded separately, triggering being at the end of each cycle, the uncertainty in a measurement of 5 000 cycles would be ± 0,5 cycle, the reading being taken as 5 000,5 cycles. If, however, the counter were set to record tens of cycles, the uncertainty in the same measurement would be ± 5 cycles, the reading being taken as 5 005 cycles.

3.3.1 *Estimation of uncertainty associated with systematic errors*

The uncertainty associated with systematic errors cannot be assessed experimentally without changing the equipment or conditions of measurement. Whenever possible this should be done since the alternative is to make a subjective judgement on the basis of experience and consideration of the equipment involved. When the class of accuracy or error limits of a measuring device are specified the interval between them may be used as the systematic uncertainty of that device with a confidence level better than 95 %.[1)]

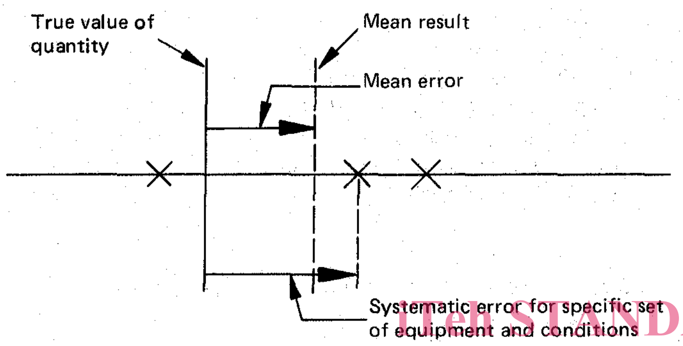
It is important to distinguish between the "estimate" of a systematic uncertainty obtained by the latter method (which is often closer to a guess than a scientific assessment) and the estimate of a random uncertainty (which can be arrived at with a stated confidence by analysing objective data). There is a general tendency to underestimate systematic uncertainties when a subjective approach is used, partly through human optimism and partly through the possibility of overlooking the existence of some sources of systematic error. Great care is therefore necessary when quoting systematic uncertainties.

It is sometimes possible partially to randomize systematic errors by repeating a measurement several times with different types of equipment or under different conditions which affect the error (see figure 2). Complete randomization is possible only by repeating the measurements using equipment based on different principles. These procedures are to be recommended

1) The error limits of a measuring device may be measured directly or determined from the guaranteed specifications of the manufacturer. If the positive and negative error limits are not equal the mean value determined by measurements using the instrument must be modified as described in b) of 3.3.1.

wherever possible since they lead not only to a higher confidence in the uncertainties but also to a lowering of the uncertainties themselves. In practice, however, it is seldom possible to carry out this type of randomization. Below, therefore, is prescribed the procedure to be followed in order to assess systematic uncertainties both by experimental and subjective methods.

If the flow-rate depends on numerous independent variables the values of which are to be measured or taken from graphs, tables or equations, systematic uncertainties associated with these variables may be treated as randomized systematic uncertainties.



The randomization of systematic errors in the measurement of a given quantity by using different sets of equipment or testing under different conditions.

FIGURE 2

The procedure to be followed for arriving at the systematic uncertainty depends on the information available on the error itself, but is the same whether a constant or a variable systematic error is being considered.

a) If the error has a unique, known value then this should be added to (or subtracted from) the result of the measurement, and the uncertainty in the measurement due to this source is then taken as zero.

b) When the sign of the error is known but its magnitude has to be estimated subjectively, the mean estimated error should be added to the result of the measurement (paying due observance to sign) and the uncertainty taken as one-half of the interval within which the error is estimated to lie. This is illustrated in figure 3, where the measured value is denoted by M and the systematic error is estimated to lie between δt_1 and δt_2 [giving a mean estimated error of $(\delta t_1 + \delta t_2)/2$].

The result, R, to be used is then given by the equation

$$R = M + \frac{\delta t_1 + \delta t_2}{2} \qquad \dots (3)$$

with an uncertainty of

$$\pm \frac{\delta t_1 - \delta t_2}{2}$$

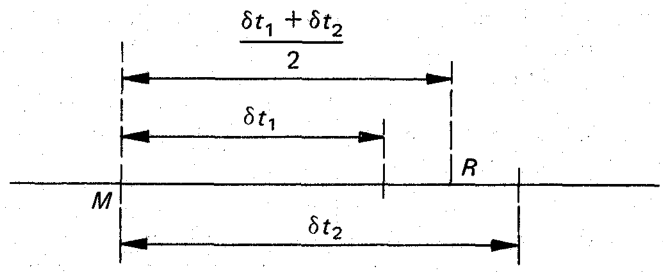


Illustration of the correction to allow for mean estimated error

Putting the mean estimated error equal to the mean of the estimated maximum and minimum values assumes implicitly that the systematic error is regarded as asymmetric.

FIGURE 3

c) When the magnitude of the systematic uncertainty can be assessed experimentally, the uncertainty should be calculated as described in 3.2 for random errors, with the measured value being adjusted as described above. Such a situation would arise where, for example, a thermometer which has not been calibrated individually is used, but where batches of identical thermometers have been previously tested to provide a mean and standard deviation of the error associated with such thermometers.

d) When the sign of the error is unknown and its magnitude is assessed subjectively, the mean estimated error is equal to zero and the uncertainty should again be taken as one-half of the estimated range of the error. This is illustrated in figure 4 below, where the notation is as above. In this case $\delta t_1 = \delta t_2$ so that the uncertainty is $\pm \delta t$.

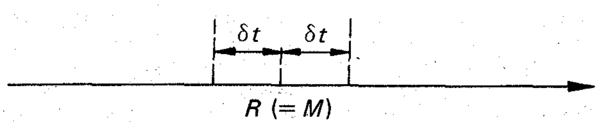


FIGURE 4

4 PROPAGATION OF ERRORS

Although it may be possible to attach values to the uncertainties in the various individual measurements used to obtain a measure of flow-rate, it is the uncertainty in the value of the flow-rate ultimately obtained which is fundamentally of interest. It is, therefore, essential to have an agreed method of combining the various uncertainties associated with each of the variables, which must be measured in order to calculate flow-rate. In open channels these would be variables such as water level and cross-section depths, and in closed ducts pipe diameter, pressure and expansibility factor for example.

Spurious errors introduce no problem since any measurement shown by the statistical tests given in annex A to be an outlier must be discarded (provided that there is independent reason for doubting the measurement). The techniques for combining random uncertainties are well developed, but if the simplest statistical formulae are to be used the different variables must be independent. Thus, every variable must be examined in order to ensure that this is so. If not, any interdependent variables must be broken down into more fundamental variables until true independence is reached.

In some cases, however, it is impractical to do this, and in others there are variables which are by their very nature interdependent but which cannot be broken down to more fundamental measurements. It is then necessary to use more complicated formulae, discussed in detail in 4.3.

It is recognized that there are conflicting opinions regarding the methods of combining uncertainties arising from systematic errors, but in order to ensure proper standardization, only the method outlined in 4.3 is to be used in flow measurement standards.

4.1 Sensitivity

Before considering methods of combining errors, it is essential to appreciate that it is insufficient to consider only the magnitudes of component uncertainties in subsidiary measurements; it is also necessary to consider the effect each measurement has on the final result. It is therefore convenient to introduce the concept of the sensitivity of a result to a subsidiary quantity as the error propagated to the result due to unit error in the measurement of the component quantity. The "sensitivity coefficient" of each subsidiary quantity is most easily obtained in one of two ways.

a) *Analytically*

When there is a known mathematical relationship between the result, R, and subsidiary quantities, Y_1, Y_2, . . ., Y_k, the dimensional sensitivity coefficient, θ_i, of the quantity Y_i, is obtained by partial differentiation.

Thus if $R = f(Y_1, Y_2, \ldots, Y_k)$, then

$$\theta_i = \frac{\partial R}{\partial Y_i} \qquad \ldots (4)$$

b) *Numerically*

Where no mathematical relationship is available or when differentiation is difficult, finite increments may be used to evaluate θ_i.

Here θ_i is given by

$$\theta_i = \frac{\Delta R}{\Delta Y_i} \qquad \ldots (5)$$

The result is calculated using Y_i to obtain R, and then recalculated using $(Y_i + \Delta Y_i)$ to obtain $(R + \Delta R)$. The value of ΔY_i used should be as small as practicable.

The sensitivity coefficient may be rendered dimensionless by writing

$$\theta_i^* = \theta_i \frac{Y_i}{R} \qquad \ldots (6)$$

In this form, the sensitivity is expressed as "percent per percent". That is, θ_i^* is the percentage change in R brought about by a 1 % change in Y_i. This is the form to be used if the uncertainties to be combined are expressed as percentages of their associated variables rather than absolute values.

4.2 Identification of sources of errors

The procedure to be followed before combining all the uncertainties is as follows :

a) identify and list all independent sources of error;

b) for each source determine the nature of the error;

c) estimate the possible range of values which each systematic error might reasonably be expected to take, using experimental data whenever possible;

d) estimate the uncertainty to be associated with each systematic error as described in 3.3.1;

e) compute, preferably from experimental data, the standard deviation of the distribution of each random error;

f) if there is reason to believe that spurious errors may exist, apply outlier tests as described in 3.1;

g) if the application of outlier tests results in data points being discarded, the standard deviations should be recalculated where appropriate;

h) compute the uncertainty associated with each random error at the 95 % confidence level;

j) calculate the sensitivity coefficient for each uncertainty;

k) list, in descending order of value, the product of sensitivity coefficient and uncertainty for each source of error.

NOTE — There are two purposes in listing these products in descending order. The first is to focus attention on the relative importance of the different sources of error so that effort may be put into reducing the uncertainty in the most important variables. Secondly, there may be several variables which contribute little or nothing to the uncertainty in the flow-rate measurement in comparison with the major sources of error, and these may be ignored in order to simplify the calculations.

4.3 Combination of uncertainties

Whenever a number of uncertainties are being combined it is possible to ignore any one which is appreciably smaller than the largest component uncertainty. As a general guide, any uncertainty which is smaller than one-fifth of the largest uncertainty in the group being combined may be ignored.

4.3.1 *Combination of random uncertainties*

In order to avoid any possible confusion, all random uncertainties used in the calculation of the uncertainty in the value of the flow-rate should be at the 95 % confidence level. The systematic uncertainties used should be estimated as described in 3.3.1.

Since the quantities in the various expressions from which the flow-rate may be calculated are not normally independent, each variable should ideally be examined individually to determine the independent variables on which it depends. It may often be impractical or indeed impossible to carry out this procedure and in such instances the formula for the calculation of the overall uncertainty should incorporate terms which allow for the dependence between the variables.

If the uncertainty in a variable Y_i is denoted by e_i the concept of interdependent uncertainties, $e_{i,j}$, may be introduced in order to produce these additional terms. The quantity $e_{i,j}$ then allows for the interdependence between variables Y_i and Y_j.

In calculating the uncertainty, e_A, in a result all uncertainties should thus be combined using the relation

$$e_A^2 = \sum_{i=1}^{k} (\theta_i e_i)^2 + 2 \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \theta_i \theta_j e_{i,j} \qquad \ldots (7)$$

where

$$e_{i,j} = \frac{4}{n-1} \sum_{r=1}^{n} [(Y_i)_r - \bar{Y}_i][(Y_j)_r - \bar{Y}_j] \qquad \ldots (8)$$

NOTE — Equation (8) holds only when the distributions of all of the sources of uncertainty, e_i, can be assumed to approach a normal distribution, and when the e_i are at the 95 % confidence level. In addition the approximation is made that the confidence limits lie at plus and minus twice the standard deviation, but this should introduce negligible error in the calculation of the overall uncertainty.

The validity of equation (8) is *seriously* affected only when an appreciable number of sources of error have marked bimodal distributions. In such a case, reference should be made to annex D. The θ_i are given by equations (4) to (6) and n is the number of independent measurements of the variable Y_i.

Three special cases are worth mentioning.

a) It is recommended that whenever possible only independent variables should be used, and in this case equation (7) reduces to

$$e_A^2 = \sum_{i=1}^{k} (\theta_i e_i)^2 \qquad \ldots (9)$$

b) When the result, R, is given by a simple sum, i.e.

$$R = Y_1 + Y_2 + \ldots + Y_k$$

then all the θ_i are unity and equation (7) becomes

$$e_A^2 = \sum_{i=1}^{k} e_i^2 + 2 \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} e_{i,j} \qquad \ldots (10)$$

c) When the result, R, is a function only of factors, then the dimensionless sensitivity coefficient for each factor is the exponent of the factor. For the relation :

$$R = K\, Y_1^a\, Y_2^b / Y_3^c$$

where Y_1, Y_2 and Y_3 are independent of each other, then

$$\theta_1^* = a; \quad \theta_2^* = b; \quad \theta_3^* = -c$$

and

$$E_A = [(a\,E_1)^2 + (b\,E_2)^2 + (c\,E_3)^2]^{1/2} \qquad \ldots (11)$$

4.3.2 *Combination of systematic uncertainties*

In order to combine the systematic uncertainties detailed in 3.3.1 the same procedure as in 4.3.1 shall be followed. In the special case where there are a large number of systematic uncertainties (see 3.3.1) they may be treated as randomized systematic uncertainties. The resulting confidence level of the overall uncertainty is at least 95 % assuming that the randomized systematic uncertainties are those associated with the independent variables to be measured or to be taken from graphs, tables or equations for the purpose of calculating the flow-rate. In general, the resulting confidence level of the overall uncertainty is better than the worst confidence level of all of the confidence levels associated with the component uncertainties.

5 PRESENTATION OF RESULTS

Despite the fact that it is preferable to list systematic and random uncertainties separately it is recongised that there are many practical reasons for presenting a single combined value in the statement of the result of a measurement, and so it is permitted to combine them using the root-sum-square method, having first calculated the overall random and systematic uncertainties separately. Combining random and randomized systematic uncertainties quadratically, the resulting overall uncertainty will have a confidence level of 95 %. Normally, however, it is not possible to attach confidence limits to the overall uncertainty presented in this way, but the confidence limits of the random component should be given.

Any rigorous presentation of results should ideally list the overall uncertainties due to random and systematic errors separately for several reasons.

Firstly, it is impossible to quote confidence levels when random and systematic uncertainties have been combined, since the concept of confidence levels cannot be applied to