TECHNICAL REPORT

ISO/TR 5168

First edition
1998-04-01

Measurement of fluid flow — Evaluation of uncertainties

Mesure de débit des fluides — Calcul de l'incertitude

Contents

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare International Standards, but in exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

— type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;

— type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;

— type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard (“state of the art”, for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 5168, which is a Technical Report of type 1, was prepared by Technical Committee ISO/TC 30, *Measurement of fluid flow in closed conduits,* Subcommittee SC 9, *Uncertainties in flow measurement*.

This document is being issued as a type 1 Technical Report because no consensus could be reached between ISO TC 30/SC 9 and ISO/TAG 4, *Metrology,* concerning the harmonization of this document with the *Guide to the expression of uncertainty in measurement,* which is a basic document in the ISO/IEC Directives. A future revision of this Technical Report will align it with the *Guide*.

This first edition as a Technical Report cancels and replaces the first edition as an International Standard (ISO 5168:1978), which has been technically revised.

Annexes A and B form an integral part of this Technical Report. Annexes C, D, E and F are for information only.

Introduction

One of the first International Standards to specifically address the subject of uncertainty in measurement was ISO 5168, *Measurement of fluid flow — Estimation of uncertainty of a flow-rate measurement*, published in 1978. The extensive use of ISO 5168 in practical applications identified many improvements to its methods; these were incorporated into a draft revision of this International Standard, which in 1990 received an overwhelming vote in favour of its publication. However, this draft revision of ISO 5168 was withheld from publication for a number of years since, despite lengthy discussions, no consensus could be reached with the draft version of a document under development by a Working Group of ISO Technical Advisory Group 4, *Metrology* (ISO TAG 4/WG 3). The TAG 4 document, *Guide to the expression of uncertainty in measurement* (GUM), was published in late 1993 as a basic document in the ISO/IEC Directives. At a meeting of the ISO Management Board in May 1995 it was decided to publish the revision of ISO 5168, *Measurement of fluid flow — Evaluation of uncertainties*, as a Technical Report.

One of the major differences between ISO/TR 5168 and the GUM is in the definitions and terminology. In addition, a substantial difference exists with respect to the concepts to be used to define practical measurement processes. In this Technical Report a normal distribution of the measurement data is assumed and Student's *t*-factor is used to determine the uncertainty. The method used to propagate elemental uncertainties to the overall uncertainty is essentially identical to that used in the GUM.

This document is published as a type 1 Technical Report instead of an International Standard because it is not consistent with the GUM. A future revision of this Technical Report will align the two documents.

Measurement of fluid flow — Evaluation of uncertainties

1 Scope

1.1 This Technical Report details step-by-step procedures for the evaluation of uncertainties in individual flow measurements arising from both random and systematic error sources and for the propagation of component uncertainties into the uncertainty of the test results. These procedures enable the following processes to be carried out:

a) estimation of the accuracy of results derived from flowrate measurement;

b) selection of a proper measuring method and devices to achieve a required level of accuracy of flowrate measurement;

c) comparison of the results of measurement;

d) identification of the sources of errors contributing to a total uncertainty;

e) refinement of the results of measurement as data accumulate.

NOTE — It is assumed that the measurement process is carefully controlled and that all calibration corrections have been applied.

1.2 This Technical Report describes the calculations required in order to arrive at an estimate of the interval within which the true value of the flowrate may be expected to lie. The principle of these calculations is applicable to any flow measurement method, whether the flow is in an open channel or in a closed conduit.

NOTE — Although this Technical Report has been drafted taking mainly into account the sources of error due to the instrumentation, it should be emphasized that the errors due to the flow itself (velocity distribution, turbulence, etc.) and to its effect on the method and on the response of the instrument can be of great importance with certain methods of flow measurement (see 5.7). Where a particular device or technique is used, some simplifications may be possible or special reference may have to be made to specific sources of error not identified in this Technical Report. Therefore reference should be made to the "Uncertainty of measurement" clause of the appropriate International Standard dealing with that device or technique.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this Technical Report. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this Technical Report are encouraged to investigate the possibility of applying the most recent editions of the standards listed below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 5725-1:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 1: General principles and definitions.*

ISO 5725-2:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 2: Basic method for the determination of repeatability and reproducibility of a standard measurement method.*

ISO 5725-3:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 3: Intermediate measures of the precision of a standard measurement method.*

ISO 5725-4:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 4: Basic methods for the determination of the trueness of a standard measurement method.*

ISO 5725-6:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 6: Use in practice of accuracy values.*

3 Definitions and symbols

For the purposes of this Technical Report, the following definitions and symbols apply.

3.1 Definitions

3.1.1 correction: Value which must be added algebraically to the indicated value to obtain the corrected result. It is numerically the same as a known error, but of opposite sign.

3.1.2 coverage: Percentage frequency at which an interval estimate of a parameter contains the true value. That is, in repeated sampling when the uncertainty interval provides 95 % coverage for each sample, over the long run the intervals will contain the true value 95 % of the time.

3.1.3 error: Result of a measurement minus the (conventional) true value of the measurement. See figure 1.

NOTE — The known parts of an error of measurement may be compensated by applying appropriate corrections. The error of the corrected result can be characterized by an uncertainty.

3.1.4 estimate: Value calculated from a sample of data as a substitute for an unknown population parameter.

For example, the experimental standard deviation (s) is the estimate which describes the population standard deviation (σ).

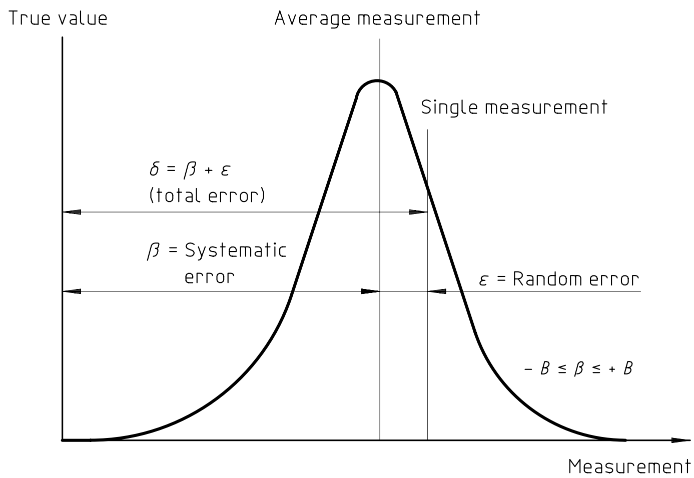

Figure 1 — Measurement error

3.1.5 fossilization: Creation of a fixed systematic error from a live random error when only a single calibration is relevant in the calibration process.

3.1.6 influence [sensitivity] coefficient: Uncertainty propagated to the result due to unit uncertainty of the measurement (see subclause 7.4).

3.1.7 observed value: Value of a characteristic determined as the result of an observation or test.

3.1.8 random error: See figure 2 and subclause 4.2.

3.1.9 random uncertainty: Component of the uncertainty associated with the random error. See figure 2.

3.1.10 statistical quality control chart: Chart on which limits are drawn and on which are plotted values of any statistic computed from successive samples of a population.

The statistics which are used (mean, range, percent defective, etc.) define the different kinds of control charts.

3.1.11 systematic error: See figures 2 and 3 and subclause 4.3.

3.1.12 systematic uncertainty: Component of the uncertainty associated with the systematic error. See figure 2.

3.1.13 Taylor's series: Power series to calculate the value of a function at a point in the neighbourhood of some reference point.

The series expresses the difference or differential between the new point and the reference point in terms of the successive derivatives of the function. Its form is:

$$f(x) - f(a) = \sum_{r=1}^{r=n-1} \frac{(x-a)^r}{r!} f^r(a) + R_n$$

where $f^r(a)$ denotes the value of the rth derivative of $f(x)$ at the reference point $x = a$. Commonly, if the series converges, the remainder R_n is made infinitesimal by selecting an arbitrary number of terms and usually only the first term is used.

3.1.14 uncertainty:

(1) Half the uncertainty interval, for a symmetrical uncertainty interval.

(2) The positive and negative components of a nonsymmetrical uncertainty interval, denoted by U^+ and U^- respectively.

3.1.15 uncertainty interval: Estimate characterizing the range of values within which the true value of a measurand is expected to lie.

3.1.16 Welch-Satterthwaite method: Method for estimating degrees of freedom of the result when combining experimental standard deviations with unequal degrees of freedom.

3.1.17 working standard: Standard, usually calibrated against a reference standard, which is used routinely to calibrate or check material measures or measuring instruments.

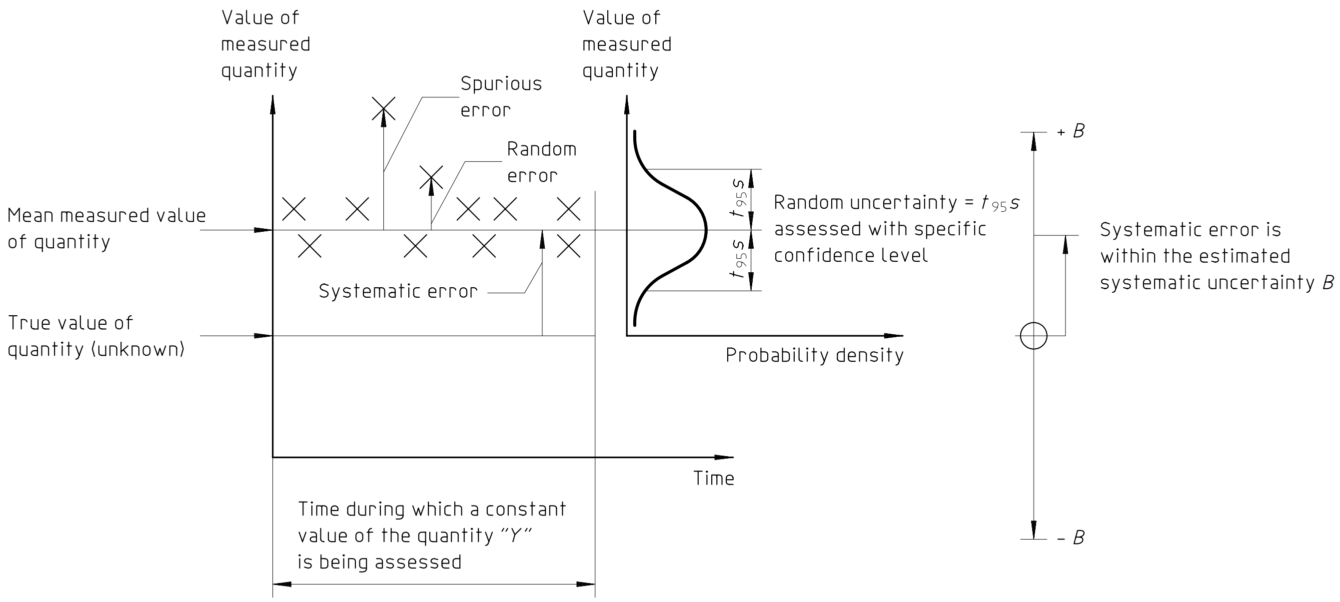

Figure 2 — Illustration of terms relating to errors and uncertainties

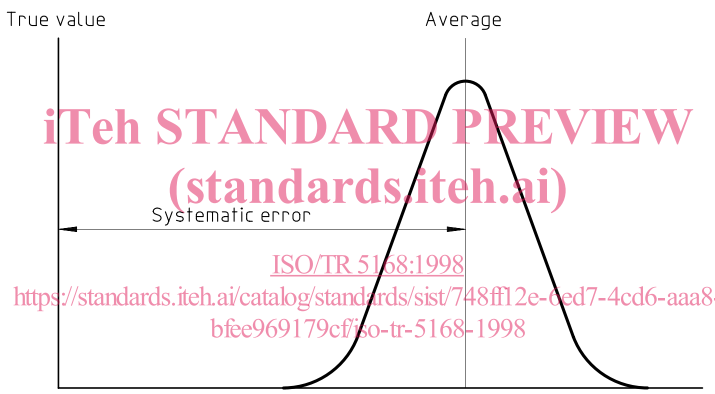

Figure 3 — Systematic error

3.2 Symbols

Symbol	Meaning
B	Systematic uncertainty of a symmetrical uncertainty interval.

$$B = \sqrt{\sum_{\text{all } j} \sum_{\text{all } i} B_{ij}^2}$$

B_{ij}	Elemental systematic uncertainty. The j subscript indicates the category, i.e.:

j = 1 calibration
= 2 data acquisition
= 3 data reduction
= 4 method
= 5 subjective or personal

The i subscript is the number assigned to a given elemental source of error. If i is more than a single digit, a comma is used between i and j.

B^+, B^-	Positive and negative systematic uncertainties of a nonsymmetrical uncertainty interval.

overbar (¯) Mean value (of a variable).

M Number of redundant instruments or tests.

N Sample size.

s^2 Unbiased estimate of the variance, σ^2.

s_{ij} Estimate of the experimental standard deviation from one elemental source. The subscripts are the same as the elemental systematic uncertainties in B_{ij}.

$$s = \sqrt{\sum_j \sum_i s_{ij}^2}$$

$s_{\bar{x}}$ Experimental standard deviation of the mean; equal to $\frac{s}{\sqrt{N}}$

s_{pooled} $= \left[\frac{\sum_{i=1}^{M} \sum_{j=1}^{N} \left(x_{ij} - \bar{x}_i \right)^2}{M\left(N-1\right)} \right]^{1/2}$

where

$\bar{x}_i$ is the arithmetic mean of all x_i at the jth datum point.

t_{95} Student's statistical parameter at the 95 % confidence level. The degrees of freedom, ν, of the sample estimate of the experimental standard deviation are needed to obtain the t values.

U^+, U^- Positive and negative uncertainties of a nonsymmetrical uncertainty interval.

U_{ADD} $= B + t_{95} s_{\bar{x}}$

U_{RSS} $= \sqrt{B^2 + \left(t_{95} s_{\bar{x}}\right)^2}$

$\bar{x}$ Arithmetic mean of the data values; x_i.

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$

x_i Value of x at the ith datum point.

x_{ij} Value of x_i at the jth datum point.

$\bar{Y}$ Arithmetic mean of the n measurements of the variable Y.

Y_i A basic measurement.

β Systematic error, the fixed, or constant component of the total error, δ.

Δ Difference between measurements.

δ Total error.

ε Random error.

Θ_i Influence coefficient $\partial R / \partial Y_i$.

μ Population mean.

σ^2 Variance, the square of the standard deviation.

Subscripts

ADD Additive model.

RSS Root-sum-square model.

NOTE — In ISO 5168:1978 and in many standards for flowrate measurement, e is used to indicate absolute uncertainty and E is used for relative uncertainty. In this Technical Report U' is used for relative uncertainty.

4 General principles of measurement uncertainty analysis

4.1 Nature of errors

All measurements have errors even after all known corrections and calibrations have been applied. The errors may be positive or negative and may be of a variable magnitude. Many errors vary with time. Some have very short periods while others vary daily, weekly, seasonally or yearly. Those which remain constant or apparently constant during the test are called systematic errors. The actual errors are rarely known; however, upper bounds on the errors can be estimated. The objective is to construct an uncertainty interval (or sometimes referred to as range) within which the true value will lie with a stated probability.

Errors are the differences between the measurements and the true value which is always unknown. The total measurement error, δ, is divided into two components: β, a fixed systematic error and a random error, ε, as shown in figure 2. In some cases, the true value may be arbitrarily defined as the value that would be obtained by a specific metrology laboratory. Uncertainty is an estimate of the error which in most cases would not be exceeded. There are three types of error to be considered:

a) random errors — see 4.2;

b) systematic errors — see 4.3;

c) spurious errors or mistakes (assumed to be identified and rejected prior to statistical analysis) — see 4.4.

It is rarely possible to give an absolute upper limit to the value of the error. It is, therefore, more practicable to give an interval within which the true value of the measured quantity can be expected to lie with a suitably high probability. This “uncertainty interval” is shown as $\left[\bar{x} - U, \bar{x} + U\right]$ in figure 4 (the interval is twice the calculated uncertainty).

Since measurement systems are subject to two types of errors, systematic and random, it follows that an accurate measurement is one that has both small random and small systematic errors (see figure 5).

4.2 Random error

Random errors are caused by numerous, small, independent influences which prevent a measurement system from delivering the same reading when supplied with the same input value of the quantity being measured. The data points deviate from the mean in accordance with the laws of chance, such that the distribution usually approaches a normal distribution as the number of data points is increased. Random errors are sometimes referred to as precision errors. The standard deviation (σ) (see figure 6) is used as a measure of the random error, ε. A large standard deviation means large scatter in the measurements. The statistic (s) is calculated from a sample to estimate the standard deviation and is called the experimental standard deviation.

$$s = \sqrt{\frac{\sum_{i=1}^{N} \left(x_i - \bar{x}\right)^2}{N-1}} \quad \ldots (1)$$

where

N is the number of measurements;

$\bar{x}$ is the average value of individual measurements x.

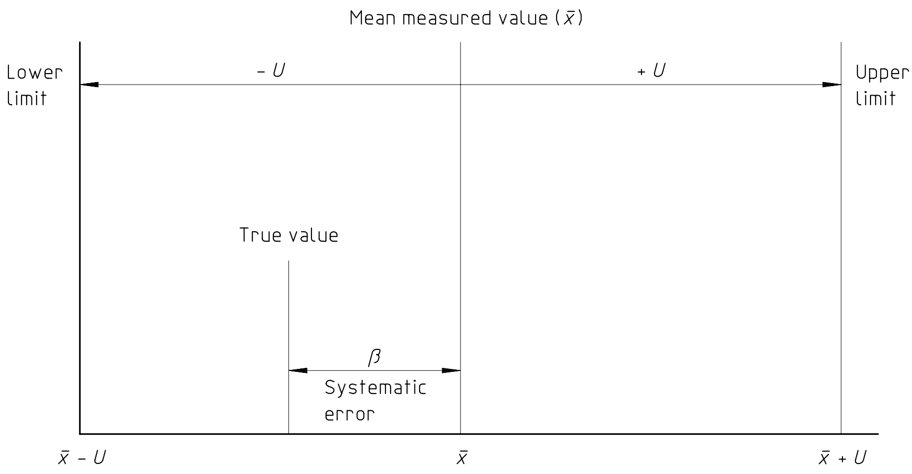

Figure 4 — Uncertainty interval $\bar{x} \pm U$ (see also figure 2)

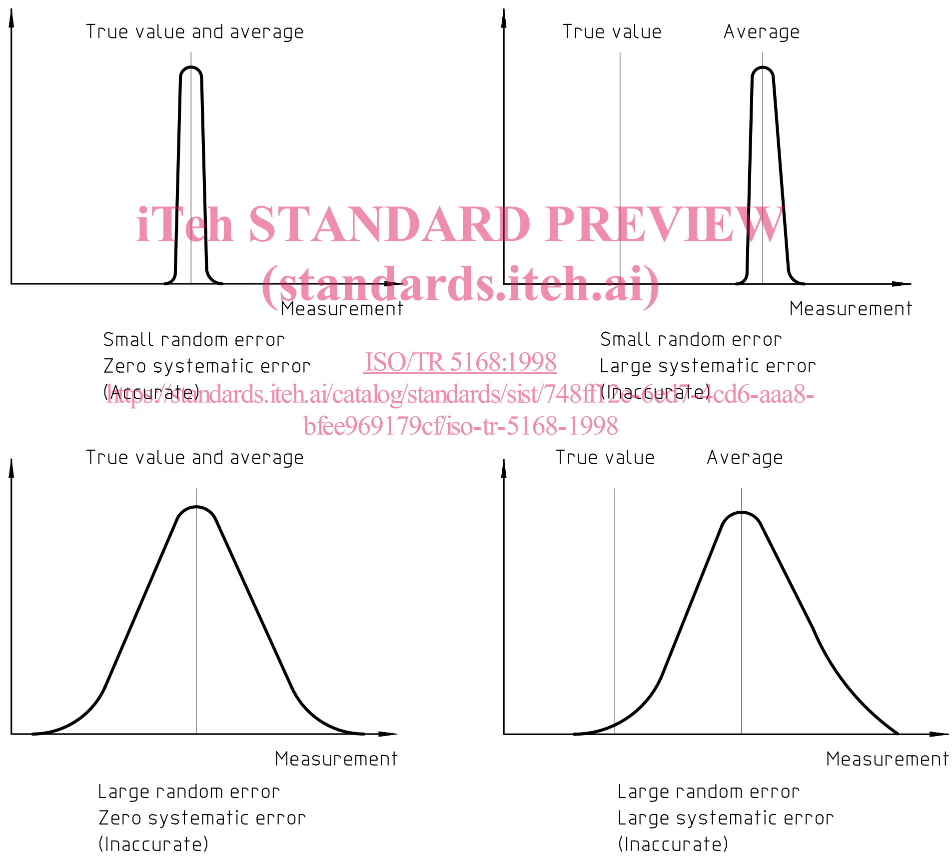

Figure 5 — Measurement error (systematic, random) and accuracy

For the normal distribution, the interval $\bar{x} \pm t_{95}\ s/\sqrt{N}$ will include the true mean, μ, approximately 95 % of the time. The random uncertainty of the mean is $t_{95}\ s/\sqrt{N}$. When the sample size is small, it is necessary to use the Student's t value at the 95 % level. For sample sizes equal to or greater than 30, two experimental standard deviations (2s) are used as an estimate of the random uncertainty in an individual measurement. This is explained in annex A.

The random uncertainty can be reduced by making as many measurements as possible and using the arithmetic mean value, since the standard deviation of the mean of N independent measurements is $\sqrt{N}$ times smaller than the standard deviation of the measurements themselves.

$$\sigma_{\text{average}} = \frac{\sigma_{\text{individual}}}{\sqrt{N}} \qquad \ldots (2)$$

and, analogously

$$s_{\overline{x}} = \frac{s}{\sqrt{N}} \qquad \ldots (3)$$

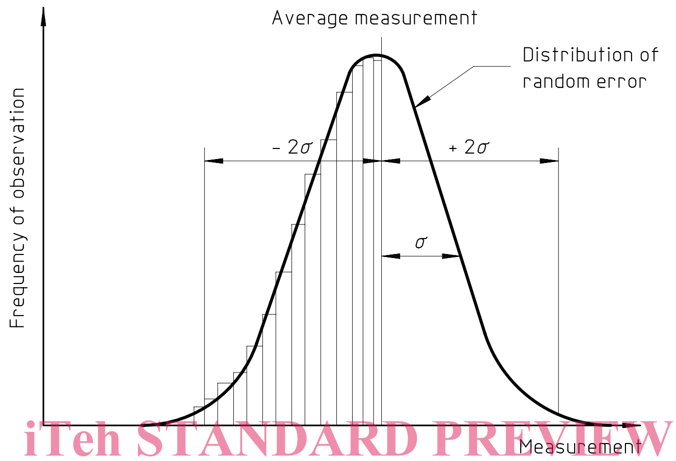

Figure 6 — Random error

4.3 Systematic error

The second component of the total error is the systematic error, β. At each flow level this error is constant for the duration of the test (figure 1). In repeated measurements of a given sample, each measurement has the same systematic error. The systematic error can be determined only when the measurements are compared with the true value of the quantity measured and this is rarely possible. Systematic errors are sometimes referred to as biases.

Every effort shall be made to identify and account for all significant systematic errors. These may arise from (1) imperfect calibration corrections, (2) imperfect instrumentation installation, (3) imperfect data reduction, and may include (4) method errors, and (5) human errors. As the true systematic error is never known, an upper limit, B, is used in the uncertainty analysis.

In most cases, the systematic error, β, is equally likely to be plus or minus about the measurement. That is, it is not known if the systematic error is positive or negative, and the systematic uncertainty reflects this as $\pm B$. The systematic uncertainty, B, is estimated as an upper limit of the systematic error, β.

4.4 Spurious errors

Spurious errors are errors, such as human mistakes or instrument malfunction, which invalidate a measurement; for example, the transposing of numbers in recording data or the presence of pockets of air in leads from a water line to a manometer. Such errors cannot be treated with statistical analysis and the measurement should be discarded. Every effort should be made to eliminate spurious errors to properly control the measurement process.

To ensure control, all measurements should be monitored with statistical quality control charts. Drifts, trends and movements leading to out-of-control situations should be identified and investigated. Histories of data from calibrations are required for effective control. It is assumed herein that these precautions are observed and that the measurement process is under control; if not, the methods described are invalid.

After all obvious mistakes have been corrected or removed, there may remain a few observations which are suspicious solely because of their magnitude.

For errors of this nature, the statistical outlier tests given in annex B should be used. These tests assume the observations are normally distributed. It is necessary to recalculate the experimental standard deviation of the distribution of observations whenever a datum is discarded as a result of the outlier test. It should also be emphasized that outliers should not be discarded unless there is an independent technical reason for believing that spurious errors may exist: data should not lightly be thrown away.

4.5 Combining elemental uncertainties

The test objective, test duration and the number of calibrations related to the test affect the classification of uncertainties into systematic and random components. Guidelines are presented in clause 6.

After all elemental error sources have been identified and classified as calibration, data acquisition, data reduction, methodic and subjective error sources and elemental standard deviations and systematic uncertainties estimated for each error source, a method for combining these elemental components into the experimental standard deviation and systematic uncertainty of the measurement is needed. The root-sum-square or quadrature combination is recommended.

$$s = \sqrt{\sum_{\text{all } j} \sum_{\text{all } i} s_{ij}^2} \qquad \text{. . (4)}$$

$$B = \sqrt{\sum_{\text{all } j} \sum_{\text{all } i} B_{ij}^2} \qquad \text{. . . (5)}$$

4.6 Uncertainty of measurements

The measurement uncertainty analysis will be completed when:

a) the systematic uncertainties and standard deviations of the measure have been propagated to uncertainty in the test result, keeping systematic and random components separate;

b) if small samples are involved, an estimate of the degrees of freedom of the experimental standard deviation of the test result has been calculated from the Welch-Satterthwaite formula (see annex A);

c) the random and systematic uncertainties are combined into a single number to express a reasonable value for the overall uncertainty.

For simplicity of presentation, a single number, U, is needed to express a reasonable limit of error. The single number, some combination of the systematic and random uncertainties, must have a simple interpretation (e.g. the largest error reasonably expected), and be useful without complex explanation. For example, the true value of the measurement is expected to lie within the interval

$$\left[\bar{x} - U, \bar{x} + U\right] \qquad \text{. . . (6)}$$

Since systematic uncertainties include those based on judgement and not on data, there is no way of combining systematic and random uncertainties to produce a single uncertainty figure with a statistically rigorous confidence level. However, since it is accepted that a single figure for the uncertainty of a measurement is often required, two alternative methods of combination are permitted:

1) linear addition:

$$U_{\text{ADD}} = B + t_{95} s_{\bar{x}} \qquad \text{. . . (7)}$$

2) root-sum-square combination:

$$U_{\text{RSS}} = \sqrt{B^2 + \left(t_{95} s_{\bar{x}}\right)^2} \qquad \text{. . .(8)}$$