ASTM INTERNATIONAL

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Standard Test Method for
Measuring <u>Net</u> Heat Flux Using a Water-Cooled Calorimeter[1]

This standard is issued under the fixed designation E422; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method covers the measurement of a steady <u>net</u> heat flux to a given water-cooled surface by means of a system energy balance.

1.2 *Units*—The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate* ~~safety~~ safety, health, and ~~health~~environmental *practices and determine the applicability of regulatory limitations prior to use.*

1.4 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]
E176 Terminology of Fire Standards
E230/E230M Specification for Temperature-Electromotive Force (emf) Tables for Standardized Thermocouples
E235 Specification for Type K and Type N Mineral-Insulated, Metal-Sheathed Thermocouples for Nuclear or for Other High-Reliability Applications
E456 Terminology Relating to Quality and Statistics
E459 Test Method for Measuring Heat Transfer Rate Using a Thin-Skin Calorimeter
E3057 Test Method for Measuring Heat Flux Using Directional Flame Thermometers with Advanced Data Analysis Techniques

3. Terminology

3.1 *Definitions*—Refer to Terminologies E176 and E456 for definitions of terms used in this test method.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *absorbed heat flux, n*—incident radiative heat flux less the reflected radiative flux, W/m^2.

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[1] This test method is under the jurisdiction of ASTM Committee E21 on Space Simulation and Applications of Space Technology and is the direct responsibility of Subcommittee E21.08 on Thermal Protection.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

3.2.2 *convective heat flux, n*—the addition or loss of energy per unit area into the sensing surface due to convection, = $h*(T_{fs}-T_s)$, W/m^2.

3.2.3 *control volume, n*—user defined volume over which an energy balance is determined.

3.2.4 *emitted heat flux, n*—energy per unit area emitted from a hot surface – $\varepsilon*\sigma*T^4$, W/m^2.

3.2.5 *incident radiative heat flux (irradiance; $q_{inc,r}$), n*—radiative heat flux (energy per unit area) impinging on the surface of the calorimeter from an external environment, W/m^2.

3.2.6 *heat flux or energy flux, n*—energy per unit area, W/m^2.

3.2.7 *net heat flux or net energy flux, n*—net energy divided by the sensing surface area transferred to the calorimeter face; it is equal to the [absorbed radiative heat flux + convective heat flux] – [re-radiation from the exposed surface].

3.2.8 *reflected heat flux, n*— that part of the incident radiative flux that is not absorbed by or transmitted into the surface of the calorimeter, W/m^2.

3.2.9 *verified, n*—the process of checking that a data acquisition channel correctly measures an input value, to a pre-set, acceptable level condition defined by the user.

3.3 *Symbols:*
A—sensing surface area of calorimeter, m^2
Cp—water specific heat, $J/(kg\text{-}K)$
h—convective heat transfer coefficient, $W/m^2\text{-}K$
m—mass flow rate of coolant water, kg/sec
q—heat flux, W/m^2
T—temperature, K
T_{01}—calorimeter water inlet bulk temperature during operation, K
T_{02}—calorimeter water exhaust bulk temperature during operation, K
T_1—calorimeter water inlet bulk temperature before operation, K
T_2—calorimeter water exhaust bulk temperature before operation, K
$\Delta T_0 = T_{02} – T_{01}$—calorimeter water bulk temperature rise during operation, K
$\Delta T_1 = T_2\text{-}T_1$—calorimeter water apparent bulk temperature rise before operation, K
ε—emissivity of sensing surface
σ—Stefan-Boltzmann constant, 5.67E-08 $W/m^2\text{-}K^4$

3.4 *Abbreviations:*

3.4.1 *TC*—thermocouple

3.4.2 *PPE*—personal protective equipment

3.4.3 U_{95}—total uncertainty to 95 % confidence

3.4.4 t_{95}—"Students t"

3.4.5 B_T—total bias uncertainty

3.4.6 S_T—total systematic or precision uncertainty

3.4.7 *DOF*—degrees of freedom

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4. Summary of Test Method

4.1 A measure of the <u>net</u> heat flux to a given water-cooled surface is based upon the following measurements: (*1*) the water mass flow rate and (*2*) the temperature rise of coolant water. The <u>net</u> heat flux is determined numerically by multiplying the water coolant flow rate by the specific heat and rise in temperature of the water and dividing this value by the surface area across which heat has been transferred.

4.2 The apparatus for measuring <u>net</u> heat flux by the energy-balance technique is illustrated schematically in Fig. 1. It is a typical constant-flow water calorimeter used to measure stagnation region <u>net</u> heat flux to a flat-faced specimen. Other calorimeter shapes can also be easily used. The <u>net</u> heat flux is measured using the central circular sensing area, shown in Fig. 1. The water-cooled annular guard ring serves the purpose of preventing heat transfer to the sides of the calorimeter and establishes flat-plate flow. An energy balance on the ~~system~~ <u>control volume that surrounds the sensing surface</u> (the centrally located calorimeter in Fig. 1) requires that the energy crossing the sensing surface (*A*, in Fig. 1) of the calorimeter be equated to the energy absorbed by the calorimeter cooling water. Interpretation of the data obtained is not within the scope of this discussion; consequently, such effects as recombination efficiency of the surface and thermochemical state of the boundary layer are outside the scope of this test method. It should be noted that recombination effects at low pressures can cause serious discrepancies in <u>net</u> heat flux measurements (such as discussed in Ref (**1**))[3] depending upon the surface material on the calorimeter.

4.3 For the particular control volume cited, the energy balance can be written as follows:

$$E_{CAL} = [mC_p(\Delta T_0 - \Delta T_1)]/A \tag{1}$$

where:

~~E_{CAL}~~	~~=~~	~~energy flux transferred to calorimeter face, $W \cdot m^{-2}$~~
E_{CAL}	≡	<u>energy flux (or net heat flux) transferred to calorimeter face, $W \cdot m^{-2}$,</u>
m	=	mass flow rate of coolant water, $kg \cdot s^{-1}$,
C_p	=	water specific heat, $J \cdot kg^{-1} \cdot K^{-1}$,
~~ΔT_0~~	~~=~~	~~$T_{0_2} - T_{0_1}$, calorimeter water bulk temperature rise during operation, K,~~
ΔT_0	≡	<u>$T_{0_2} - T_{0_1}$, calorimeter water bulk temperature rise during operation, K,</u>
~~ΔT_1~~	~~=~~	~~$T_2 - T_1$ = calorimeter water apparent bulk temperature rise before operation, K,~~
ΔT_1	≡	<u>$T_2 - T_1$, calorimeter water apparent bulk temperature rise before operation, K,</u>
T_{0_2}	=	<u>calorimeter</u> water exhaust bulk temperature during operation, K,
T_{0_1}	=	<u>calorimeter</u> water inlet bulk temperature during operation, K,
T_2	=	<u>calorimeter</u> water exhaust bulk temperature before operation, K,
T_1	=	<u>calorimeter</u> water inlet bulk temperature before operation, K, and
A	=	sensing surface area of calorimeter, m^2.

4.4 An examination of Eq 1 shows that to obtain a value of the energy transferred to the calorimeter, measurements must be made of the water coolant flow rate, the temperature rise of the coolant, and the surface area across which heat is transferred. With regard
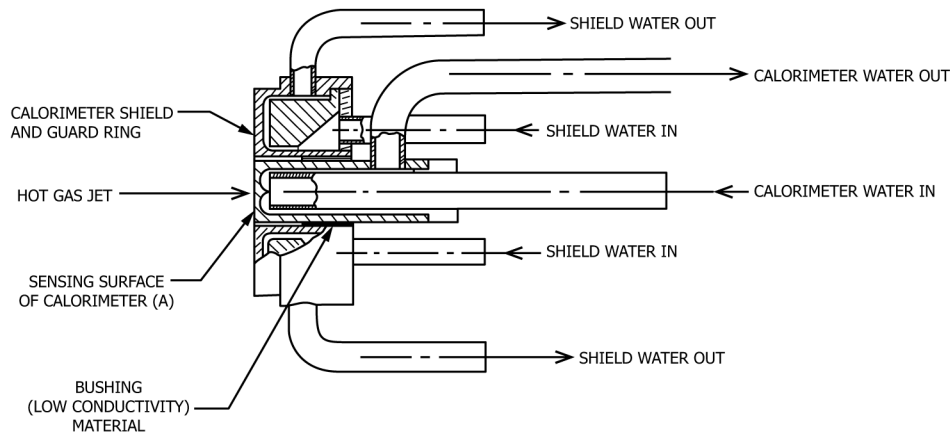

FIG. 1 Steady-State Water-Cooled Calorimeter.

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[3] The boldface numbers in parentheses refer to the list of references at the end of this test method.

to the latter quantity, it is assumed that the surface area to which heat is transferred is well defined. As is indicated in Fig. 1, the design of the calorimeter is such that the heat transfer area is confined by design to the front or directly heated surface. To minimize side heating or side heat losses, a water-cooled guard ring or shroud is utilized and is separated physically from the calorimeter by means of an air gap and low conductivity bushing such as nylon. The air gap is recommended to be no more than 0.5 mm on the radius. Thus, if severe pressure variations exist across the face of the calorimeter, side heating caused by flow into and out of the air gap will be minimized. Also, since the water-cooled calorimeter and guard ring operate at low surface temperatures (usually lower than ~~100°C)~~100 °C) heat losses across the gap by radiant interchange are negligible and consequently no special calorimeter surface gap finishes are necessary. Depending upon the size of the calorimeter surface, large variations in net heat flux may exist across the face of the calorimeter. Consequently, the measured net heat flux represents an average net heat flux over the surface area of the water-cooled calorimeter. The water-cooled calorimeter can be used to measure ~~heat flux~~ net heat flux levels over a range from 10 kW/m^2 to 60 MW/m^2.

5. Significance and Use

5.1 The purpose of this test method is to measure the net heat flux to a water-cooled surface for purposes of calibration of the thermal environment into which test specimens are placed for evaluation. The measured net heat flux is one of the important parameters for correlating the behavior of materials. If the calorimeter and holder size, shape, and surface finish are identical to that of the test specimen, the measured net heat flux to the calorimeter is presumed to be the same as that to the sample's heated surface. ~~The measured heat flux is one of the important parameters for correlating the behavior~~If the calorimeter configuration (holder size, shape, finish, etc.) is not identical to that of the test specimen, then the measurement results may need to be modified to account for those differences. See Appendix X1~~of materials.~~.

5.2 The water-cooled calorimeter is one of several calorimeter concepts used to measure net heat flux. The prime drawback is its long response time, that is, the time required to achieve steady-state operation. To calculate energy added to the coolant water, accurate measurements of the rise in coolant temperature are needed, all energy losses should be minimized, and steady-state conditions must exist both in the thermal environment and fluid flow of the calorimeter.

5.3 Regardless of the source of energy input to the water-cooled calorimeter surface (radiative, convective, or combinations thereof) the measurement is averaged over the ~~surface active~~ surface-active area of the calorimeter. If the water-cooled calorimeter is used to measure only radiative flux or combined convective-radiative ~~heat flux~~ net heat flux rates, then the surface reflectivity of the calorimeter shall be measured over the wavelength region of interest (depending on the source of radiant energy). If nonuniformities exist in the gas stream, a large surface area water-cooled calorimeter would tend to smooth or average any variations. Consequently, it is advisable that the size of the calorimeter be limited to relatively small surface areas and applied to where the ~~heat flux~~ net heat flux is uniform. Where large samples are tested, it is recommended that a number of smaller diameter water-cooled calorimeters be used (rather than one large unit). These shall be located across the heated surface such that a ~~heat flux~~ net heat flux distribution can be described. With this, a more detailed ~~heat flux~~ net heat flux measurement can be applied to the specimen test and more information can be deduced from the test.

6. Apparatus

6.1 *General*—The apparatus shall consist of a water-cooled calorimeter and the necessary instrumentation to measure the heat transferred to the calorimeter. Although the recommended instrumentation accuracies are state-of-the-art values, more rugged and higher accuracy instrumentation may be required for high pressure and high ~~heat flux~~ net heat applications. A number of materials can be used to fabricate the calorimeter, but ~~OFHC (oxygen~~ oxygen free high ~~conductivity)~~ conductivity (OFHC) copper is often preferred because of its superior thermal properties. The user should decide before fabrication and use of the water-cooled calorimeter what the total acceptable uncertainty is for the application of interest. Some applications can accommodate larger uncertainties than others, and typically the smaller the required uncertainty, the higher the cost. For the water-cooled calorimeter, there are four parameters that can be uncertain: the water flow rate, the temperature difference, the water specific heat, and the sensing surface area. The acceptable total uncertainty should be expressed as xx % of reading or maximum value good to a certain confidence level. An example (but not a recommended value) would be a ±5 % uncertainty to 95 % confidence level.

6.2 *Coolant Flow Measurement*—The water flow rate to each component of the calorimeter shall be chosen to cool the apparatus adequately and to ensure accurately measurable rise in water temperature. The error in water flow rate measurement ~~shall~~should be not more than ~~±2 %.~~±2 %, assuming this value has been apportioned with the total acceptable uncertainty preferred. Suitable equipment that can be used is listed in ~~Ref~~Refs (**2, 3**) and includes turbine flowmeters, variable area flowmeters, etc. Care must be exercised in the use of all these devices. In particular, it is recommended that appropriate filters be placed in all water inlet lines to prevent particles or unnecessary deposits from being carried to the water-cooling passages, pipe, and meter walls. Water flow rates and pressure shall be adjusted to ensure that no bubbles are formed (no boiling). If practical, the water flowmeters shall be

placed upstream of the calorimeter in straight portions of the piping. The flowmeter device shall be checked and calibrated periodically. Pressure gages, if required, shall be used in accordance with the manufacturer's instructions and calibration charts.

6.3 *Coolant Temperature Measurement*—The method of temperature measurement must be sufficiently sensitive and reliable to ensure accurate measurement of the coolant water temperature rise. There are two calorimeter water temperature sensors in the calorimeter, one to measure inlet water temperature and the other to measure outlet water temperature. Procedures similar to those given in Specification E235, Type ~~K, and Ref~~ K (chromel/alumel, range 0 °C to 1260 °C, accuracy ±2.2 °C or 0.75 % of reading in °C, whichever is greater, for standard tolerances), and Ref (~~(3~~**4**)) should be followed in the calibration and preparation of temperature sensors. Specification E230/E230M provides information on thermocouples (TCs) with a smaller temperature range and higher accuracy than Type K TCs (for example, Type T, copper/constantan, range is 0 °C to +370 °C, ±1.0 °C or ±0.75 % of reading in °C, whichever is greater, for standard tolerances) that might be more appropriate if the water coolant temperature does not rise too high. There are also special tolerances (that is, more accurate) available for both Type T and Type K TC wire. The bulk or average temperature of the coolant shall be measured at the inlet and outlet lines of each cooled ~~unit.~~ unit, as accurately as possible given the required total uncertainty, the sensors available, and the data acquisition system used. The error in measurement of temperature difference between inlet and outlet shall be determined when the total aceptable uncertainty is established, and when uncertainty (or accuracy) limits are portioned for the four uncertain parameters (temperature, water flow rate, specific heat of water, and sensing surface area). A reasonable value for total temperature uncertainty is about ±1-2 % of the reading in K (Ref (**5**~~not more than ±1 %.~~)). The user must realize that the temperature measurement uncertainty includes the sensor accuracy and the uncertainty from the data acquisition system, not just the sensor alone. At temperatures of interest in this standard, the uncertainty contribution from the data acquisition system can be ±1 °C (Ref (**5**)), which is comparable to the sensor uncertainty. See 13.5. The water temperature indicating devices shall be placed as close as practical to the calorimeter's heated surface in the inlet and outlet lines. However, care must be exercised so as not to place the temperature sensors where there is energy exchange between the incoming (cold) water and the outgoing (heated) water. This occurs most readily at flow dividers and at the calorimeter sensing surface. No additional apparatus shall be placed in the line between the temperature sensor and the heat source. The temperature measurements shall be recorded continuously to verify that steady-state operation has been achieved. ~~Reference~~ Refs ((**2**, **3** ~~lists~~) list) a variety of commercially available temperature sensors. Temperature sensors which are applicable include liquid-in-glass thermometers, thermopiles, thermocouples, and thermistors. During operation of the heat source, care should be taken to minimize deposits on the temperature sensors and to eliminate any possibility of sensor heating because of specimen radiation to the sensor. In addition, all water lines should be shielded from direct-flow impingement or radiation from the test environment.

6.3.1 If at all practical, a ~~thermocouple~~TC shall be placed on the water-cooled side of the heated calorimeter surface. Although this surface temperature (water side) measurement is not used directly in the calculation of net heat flux it is necessary for the calculation of the surface temperature (front face) used in the correction of the measured net heat flux to walls of different temperatures.

6.4 *Recording Means:*

6.4.1 Since measurement of the energy transfer requires that the calorimeter operate as a steady state device, all calculations will use only measurements taken after it has been established that the device has achieved steady operating levels. To assure steady flow or operating conditions, the ~~above mentioned~~ above-mentioned parameters shall be continuously recorded such that instantaneous measurements are available to establish a measure of steady-state operation. Wherever possible, it is highly desirable that the differential temperature (ΔT) be made of the desired parameters rather than absolute measurements.

6.4.2 In all cases, parameters of interest, such as water flow rates and cooling water temperature rises should be automatically recorded throughout the measurement period. Recording speed or sampling frequency will depend on the variations of the parameters being recorded. Digital data acquisition systems (DASs) are widely available and cost effective. It is important to sample the data fast enough to ensure one can establish steady state, but slow enough such that the sample rate is not faster than the calorimeter response time. Test Method E459 recommends that the sample rate for a digital data acquisition system be no more than 40 % of the calorimeter response time. When a strip chart recorder is used, the response time of the recorder shall be ~~1 s~~ 1 s or less for full-scale deflection. Timing marks should be an integral part of the recorder with a minimum requirement of 1/s.

7. Hazards

7.1 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

8. Calibration and Standardization

8.1 It is desirable, but most often not possible, to obtain estimates of the precision and bias of all sensors and instruments. For example, it is not possible to obtain bias errors for thermocouples mounted on surfaces or inside a water flow system because the bias errors depend on specifics of the setup, which varies for each different test. As a result, most often manufacturers provide information for the sensor only. For example, ASTM's specifications for thermocouple accuracy are provided in Specification E230/E230M as "tolerances." An example for Type K thermocouples is a standard tolerance of "the greater of ±2.2 °C or ±0.75 % of the reading in °C". There is no mention of a precision or bias in that tolerance; it would be very difficult to obtain such numbers. Also, most manufacturers do not provide an estimate of the confidence level associated with the accuracy specification. For example, one does not often see a specification like this: the accuracy is ±5 % with a confidence level of 95 %. The manufacturer only provides this: the accuracy is ±5 %. Fortunately, many specifications imply the accuracy specification assumes the largest confidence, that is, about 99 %.

8.2 *Thermocouple Calibration*—The user should purchase thermocouples or thermocouple wire from a reputable source and confirm that the sensors at least meet ASTM tolerances shown in Specification E230/E230M. Individual thermocouples can be calibrated in a certified laboratory if more accurate values are needed. But those calibrations only apply to a bare wire or shielded thermocouple not attached to a surface or in a water flow stream. If more information is desired about bias errors, a separate analysis may be needed on the setup specific to that test. See also Section 13.

8.3 *Flow Meter Calibration*—Flow meters likely also come with an accuracy specified as % of reading or % of the maximum range. Most often, no additional information is provided on the precision and bias of the sensor, just a total uncertainty. See also Section 13.

8.4 *Data Acquisition System Calibration or Verification*—The DAS also needs to be checked. The DAS should be on a regularly scheduled re-calibration schedule (for example, yearly) to check the accuracy of the voltmeter and other parts of the DAS. When used much more frequently than yearly, the DAS should be checked to verify the channels being used are within an "acceptable" range. To accomplish this, one can use a NIST calibrated voltage simulator which provides a very accurate input into each channel at a range of values. For example, if the test has 30 channels of thermocouple data, one can insert input values into each channel at fixed intervals spanning the temperature range expected during the test. Multiple scans (for example, 1000) are taken at each temperature, for each channel. Data from those 1000 scans are analyzed to obtain the average and standard deviation of each channel at each temperature. For those channels that meet an acceptable value, the channel is assumed satisfactory. No further adjustment or calibration is performed, and the channel is considered "verified" for operation. In this manner, all DAS channels can be "verified" for use. See also Section 13.

9. Procedure

9.1 It is essential that the environment be at steady-state conditions prior to testing if the water-cooled calorimeter is to give a representative measure of the net heat flux.

9.2 After a sufficient length of time has elapsed to assure constant mass flow of water as well as constant inlet and outlet water temperature, place the system into the heat-source environment. Steady-state operation has been assured if the inlet and exhaust water temperature, and water flow rates are steady and not changing with time. In particular, the water flow rates should not change during operation. After removing the calorimeter from the environment, record the inlet water temperature and flow rates so that they can be compared with pretest values. Changes between pre- and post-test water temperature rise may indicate deposit buildups on the calorimeter back-face or cooling passages which may alter the results of the measurement of energy transfer.

9.3 To ensure consistent net heat flux data, it is recommended that measurements be repeated with the same apparatus. A further check on the measurement of net heat flux using a water-cooled calorimeter would be to use a different mass flow of water through the calorimeter for different test runs. No significant difference in net heat flux measurements should be noted with the change in water flow rate for different test runs.

10. Calculation or Interpretation of Results

10.1 The quantities as defined by Eq 1 shall be calculated based on the bulk or average temperature rise of the coolant water for each water-cooled section of the calorimeter. The choice of units shall be consistent with the measured quantities.

10.2 Variance analyses of heat-source conditions shall provide a sound basis for estimation of the reproducibility of the thermal environment. Refs (46) and (57) may provide a basis for error analysis of the measurements.

11. Report

11.1 In reporting the results of the measurement tests, the following steady-state data shall be reported:

11.1.1 Dimensions of the calorimeter configuration active surface and guard ring,

11.1.2 Calorimeter coolant water flow rate,

11.1.3 Temperature rise of calorimeter coolant water,

11.1.4 Water specific heat as a function of temperature,

11.1.5 Calculated net heat flux,

11.1.6 Front surface temperature (if measured or calculated), and

11.1.7 Variance of results.

12. Precision and Bias

12.1 *Statements of Precision and Bias*—"*Precision*" is defined (in Terminology E456) as the degree of agreement of repeated measurements of the same property, expressed in terms of dispersion of test results about the arithmetical-mean result obtained by repetitive testing of a homogeneous sample under specified conditions. The precision of a test method is expressed quantitatively as the standard deviation computed from the results of a series of controlled determinations. "*Bias*" is a systematic error that contributes to the difference between a population mean of the measurements or test results and an accepted or reference value. (Note that the "reference" value (or "true" value) is never known precisely.) Precision and bias definitions noted above apply to a test method that is intended to repeatedly measure the same sample or test result. This is not what this test method (E422) is intended to do. Results from Test Method E422 will generate different results during each test because each test will be different, even if all attempts are made to generate identical results. More detail is provided in the next section on these topics, Measurement Uncertainty, Section 13, and in Section 8, Calibration and Standardization.

12.2 *Precision and Bias*—It is not possible to provide a statement on precision or bias for the following reasons.

12.2.1 *Precision*—It is not possible to specify the precision of the procedure in Test Method E422 for measuring the net heat flux to the water-cooled calorimeter face because tests cannot be precisely repeated, so the net heat flux will be slightly different for each test.

12.2.2 *Bias*—It is not possible to specify the bias of the procedure in Test Method E422 for measuring the net heat flux to the water-cooled calorimeter face because tests cannot be precisely repeated, so the net heat flux will be slightly different for each test.

13. Measurement Uncertainty

13.1 There are a number of methods that can be used for the determination of measurement uncertainty. A recent summary of the various uncertainty analysis methods is provided in Ref (6). Ref. (7) has a detailed account of uncertainty analysis for engineers. The American Society of Mechanical Engineers' (ASME's) earlier performance test code PTC 19.1-1985 (78) has been revised and was replaced by Ref (89) in 1998. In Refs (78) and (89), uncertainties were separated into two types: "bias" or "systematic" uncertainties (B) and "random" or "precision" uncertainties (S). Systematic uncertainties (Type B) are often (but not always) constant for the duration of the experiment. Random uncertainties are not constant and are characterized via the standard deviation of the random measurements, thus the abbreviation 'S.'

13.2 ASME's new standard (89) proposes use of the following model:model to combine bias and precision uncertainties:

$$U_{95} = \pm t_{95} \left[(B_T/2)^2 + (S_T)^2 \right]^{\frac{1}{2}} \tag{2}$$