Designation: ~~E308 – 18~~ E308 – 22

Standard Practice for Computing the Colors of Objects by Using the CIE System[1]

This standard is issued under the fixed designation E308; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

This standard has been approved for use by agencies of the U.S. Department of Defense.

INTRODUCTION

Standard tables (Tables 1–4) of color matching functions and illuminant spectral power distributions have since 1931 been defined by the CIE, but the CIE has eschewed the role of preparing tables of tristimulus weighting factors for the convenient calculation of tristimulus values. There have subsequently appeared numerous compilations of tristimulus weighting factors in the literature with disparity of data resulting from, for example, different selections of wavelength intervals and methods of truncating abbreviated wavelength ranges. In 1970, Foster et al. (**1**)[2] proposed conventions to standardize these two features, and Stearns (**2**) published a more complete set of tables. Stearns' work and later publications such as the 1985 revision of E308 have greatly reduced the substantial variations in methods for tristimulus computation that existed several decades ago.

The disparities among earlier tables were largely caused by the introduction of computations based on 20-nm wavelength intervals. With the increasing precision of modern instruments, there is a likelihood of a need for tables for narrower wavelength intervals. Stearns' tables, based on a 10-nm interval, did not allow the derivation of consistent tables with wavelength intervals less than 10 nm. The 1-nm table must be designated the basic table if others with greater wavelength intervals are to have the same white point, and this was the reason for the 1985 revision of E308, resulting in tables that are included in the present revision as Tables 5.

~~The 1994 revision was made in order to introduce to the user a method of reducing the dependence of the computed tristimulus values on the bandpass of the measuring instrument, using methods that are detailed in this practice.~~

1. Scope

1.1 This practice provides the values and practical computation procedures needed to obtain CIE tristimulus values from spectral reflectance, transmittance, or radiance data for object-color specimens.

1.2 Procedures and tables of standard values are given for computing from spectral measurements the CIE tristimulus values X, Y, Z, and chromaticity coordinates x, y for the CIE 1931 standard observer and X_{10}, Y_{10}, Z_{10} and x_{10}, y_{10} for the CIE 1964 supplementary standard observer.

[1] This practice is under the jurisdiction of ASTM Committee E12 on Color and Appearance and is the direct responsibility of Subcommittee E12.04 on Color and Appearance Analysis.

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[2] The boldface numbers in parentheses refer to the list of references at the end of this practice.

1.3 Standard values are included for the spectral power of six CIE standard illuminants and three CIE recommended fluorescent illuminants. Weight sets are included for tristimulus integration of nine standard or recommended CIE LED illuminants combined with the two standard CIE observers.

1.4 Procedures are included for cases in which data are available only in more limited wavelength ranges than those recommended, or for a measurement interval wider than that recommended by the CIE. This practice is applicable to spectral data obtained in accordance with Practice E1164 with 1-, 5-, 10-, or 20-nm measurement interval.

~~1.5 Procedures are included for cases in which the spectral data are, and those in which they are not, corrected for bandpass dependence. For the uncorrected cases, it is assumed that the spectral bandpass of the instrument used to obtain the data was approximately equal to the measurement interval and was triangular in shape. These choices are believed to correspond to the most widely used industrial practice.~~

1.5 This practice includes procedures for conversion of results to color spaces that are part of the CIE system, such as CIELAB and CIELUV (**3**). Equations for calculating color differences in these and other systems are given in Practice D2244.

1.6 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.7 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.8 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[3]

D2244 Practice for Calculation of Color Tolerances and Color Differences from Instrumentally Measured Color Coordinates
E284 Terminology of Appearance
E313 Practice for Calculating Yellowness and Whiteness Indices from Instrumentally Measured Color Coordinates
E1164 Practice for Obtaining Spectrometric Data for Object-Color Evaluation
E2022 Practice for Calculation of Weighting Factors for Tristimulus Integration
E2729 Practice for Rectification of Spectrophotometric Bandpass Differences

2.2 *ANSI Standard:*[4]

PH2.23 Lighting Conditions for Viewing Photographic Color Prints and Transparencies

2.3 *CIE/ISO Standards:*

ISO Standard 11664-1:2007(E)/CIE S 014-1/E:2006 Standard Colorimetric Observers[4,5]
ISO Standard 11664-2:2007(E)/CIE S 014-2/E:2006 Colorimetric Illuminants[4,5]
CIE Standard D 001 Colorimetric Illuminants and Observers (Disk)[5]

2.4 *ASTM Adjuncts:*

~~Computer disk~~Electronic file containing Tables 5[6]

3. Terminology

3.1 Definitions of terms in Terminology E284 are applicable to this practice (see also Ref (**4**)).

3.2 ~~*Definitions:*~~*Definitions—*Definitions are listed in dictionary alphabetical order which makes no distinction between capital and lower-case ordering of the letters of the alphabet, and disregards spaces between multiple-word definiens. Otherwise, order is determined by the UTF-8 value of the letter or symbol involved. Definitions that have the same meaning, but are not necessarily

[3] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[4] Available from American National Standards Institute (ANSI), 25 W. 43rd St., 4th Floor, New York, NY 10036, http://www.ansi.org.

[5] Available from CIE (International Commission on Illumination), http://www.cie.co.at or http://www.techstreet.com.

[6] ~~Computer disk of tables is~~ Electronically readable tables of tristimulus weight sets are available from ASTM Headquarters. Request Adjunct No. ~~ADJE0308A~~ADJE0308-EA. Originally approved in ~~1994. Revised in 2017.~~2022.

word-for-word identities as that contained in the committee terminology document, are notated with the designation of that document after the definition.

3.2.1 *bandpass, adj*—having to do with a passband. **E284**

3.2.2 *bandwidth, n*—the width of a passband at its half-peak transmittance. **E284**

3.2.3 *chromaticity, n*—the color quality of a color stimulus definable by its chromaticity coordinates. **E284**

3.2.4 *chromaticity coordinates, n*—the ratio of each of the tristimulus values of a psychophysical color ~~(see section 3.2.7.11)~~ to the sum of the tristimulus values. **E284**

3.2.4.1 *Discussion*—

In the CIE 1931 standard colorimetric system, the chromaticity coordinates are: $x = X/(X + Y + Z), y = Y/(X + Y + Z), z = Z/(X + Y + Z)$ ~~x; = X/(X + Y + Z), y = Y/(X + Y + Z), z = Z/(X + Y + Z);~~ in the CIE 1964 ~~supplementary~~ colorimetric system, the same equations apply with all symbols having the subscript 10 (see ~~3.2.7~~3.2.10).

3.2.5 *CIE, n*—the abbreviation for the French title of the International Commission on Illumination, Commission Internationale de l'Éclairage. **E284**

3.2.6 *CIE 1931 (x, y) chromaticity diagram, n*—chromaticity diagram for the CIE 1931 standard observer, in which the CIE 1931 chromaticity coordinates are plotted, with *x* as abscissa and *y* as ordinate. **E284**

~~3.2.7 *CIE 1964 (x_{10}, y_{10}) chromaticity diagram, n*—chromaticity diagram for the CIE 1964 supplementary standard observer, in which the CIE 1964 chromaticity coordinates are plotted, with x_{10} as abscissa and y_{10} as ordinate.~~

~~3.2.7.1 *Discussion*—~~

~~Fig. 1 shows the CIE 1931 and 1964 chromaticity diagrams, including the locations of the spectrum locus and the connecting purple boundary.~~

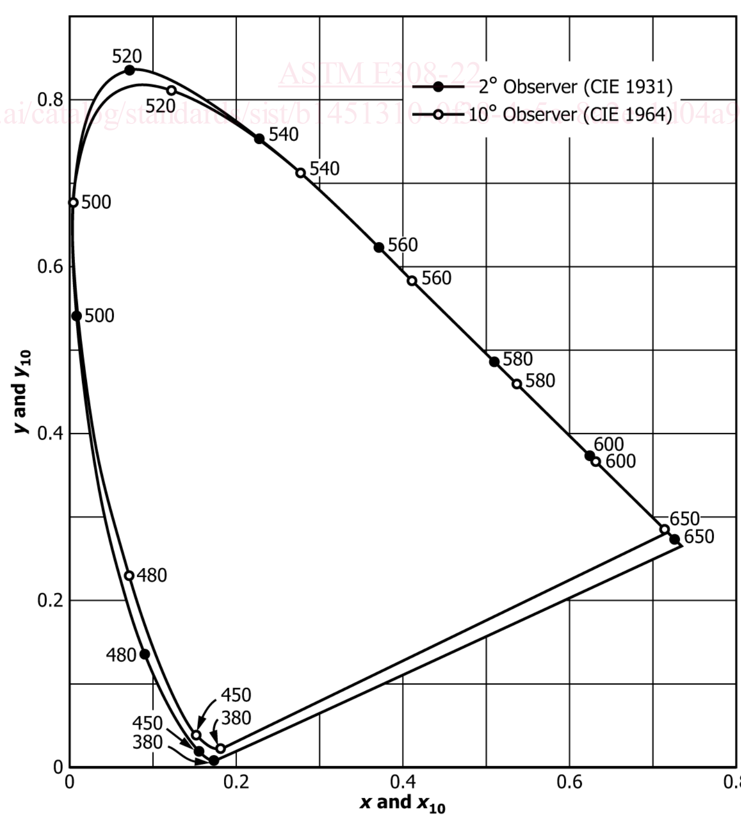


FIG. 1 The CIE 1931 *x, y* and 1964 x_{10},y_{10} Chromaticity Diagrams Ref (5) (see Note 2)

~~3.2.8 *CIE 1976 (u', v') or (u'$_{10}$, v'$_{10}$) chromaticity diagram, n*—chromaticity diagram in which the CIE 1976 *L* u* v** (CIELUV) chromaticity coordinates are plotted, with *u'* (or *u'*$_{10}$) as abscissa and *v'* (or *v'*$_{10}$) as ordinate.~~

3.2.7 *CIE 1931 standard colorimetric system, n*—a system for determining the tristimulus values of any spectral power distribution using the set of reference color stimuli, *X, Y, Z* and the three CIE ~~color–matching~~color-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ adopted by the CIE in 1931. **E284**

3.2.8 *CIE 1931 standard observer, n*—ideal colorimetric observer with color-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ corresponding to a field of view subtending a 2° angle on the retina; commonly called the "2° standard observer." **E284**

3.2.9 *CIE 1964 (x_{10}, y_{10}) chromaticity diagram, n*—chromaticity diagram for the CIE 1964 standard observer, in which the CIE 1964 chromaticity coordinates are plotted, with x_{10} as abscissa and y_{10} as ordinate. **E284**

3.2.9.1 *Discussion—*

Fig. 1 shows the CIE 1931 and 1964 chromaticity diagrams, including the locations of the spectrum locus and the connecting purple boundary.

3.2.10 *CIE 1964 ~~supplementary~~ standard colorimetric system, n*—a system for determining the tristimulus values of any spectral power distribution using the set of reference color stimuli X_{10},Y_{10},~~Z~~Z_{10} and the three CIE color-matching functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ adopted by the CIE in 1964 (see Note 1). **E284**

NOTE 1—Users should be aware that the CIE 1964 (10°) ~~supplementary~~ system and standard observer assume no contribution or constant contribution of rods to vision. Under some circumstances, such as in viewing highly metameric pairs in very low light levels (where the rods are unsaturated), the amount of rod participation can vary between the members of the pair. This is not accounted for by any trichromatic system of colorimetry. The 10° system and observer should be used with caution in such circumstances.

3.2.11 *CIE 1964 standard observer, n*—ideal colorimetric observer with color-matching functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ corresponding to a field of view subtending a 10° angle on the retina; commonly called the "10° standard observer" (see Note 1). **E284**

3.2.12 *CIE 1976 (u', v') or (u'$_{10}$, v'$_{10}$) chromaticity diagram, n*—the uniform-chromaticity-scale diagram produced by plotting in rectangular coordinates v' against u', quantities defined as follows:

$$u' = 4X/(X + 15Y + 3Z) = 4x/(-2x + 12y + 3) \tag{1}$$

$$v' = 9Y/(X + 15Y + 3Z) = 9y/(-2x + 12y + 3) \tag{2}$$

for the CIE 1931 standard colorimetric system, or v'$_{10}$ against u'$_{10}$ for the CIE 1964 standard colorimetric system, in which case in the above formulae X_{10}, Y_{10}, Z_{10} are used instead of X, Y, Z and x_{10}, y_{10} instead of x, y. **E284**

3.2.13 *CIE recommended fluorescent illuminants, n*—a set of spectral power distributions of 12 types of fluorescent lamps, the most important of which are *FL2*, representing a cool white fluorescent lamp with correlated color temperature 4200 K, *FL7*, a broad-band (continuous-spectrum) daylight lamp (6500 K), and *FL11*, a narrow-band (line-spectrum) white fluorescent lamp (4000 K).

3.2.14 *CIE standard illuminant A, n*—colorimetric illuminant, representing the full radiator at 2855.6 K, defined by the CIE in terms of a relative spectral power distribution. **E284**

3.2.15 *CIE standard illuminant C, n*—colorimetric illuminant, representing daylight with a correlated color temperature of 6774 K, defined by the CIE in terms of a relative spectral power distribution. **E284**

3.2.16 *CIE standard illuminant D_{65}, n*—colorimetric illuminant, representing daylight with a correlated color temperature of 6504 K, defined by the CIE in terms of a relative spectral power distribution. **E284**

3.2.16.1 *Discussion—*

Other illuminants of importance defined by the CIE include the daylight illuminants D_{50},D_{55}, and D_{75}. Illuminant D_{50} is used by the graphic arts industry for viewing colored transparencies and prints (see ANSI PH2.23).

3.2.17 *CIELAB color scales, n*—CIE 1976 L^*,a^*,b^* opponent-color scales, in which a^* is positive in the red direction and negative in the green direction, and b^* is positive in the yellow direction and negative in the blue direction.

3.2.18 *CIELUV color scales, n*—CIE 1976 L^*,u^*,v^* opponent-color scales, in which u^* is positive in the red direction and negative in the green direction, and v^* is positive in the yellow direction and negative in the blue direction.

3.2.19 *color, n—of an object*, aspect of object appearance distinct from form, shape, size, position or gloss that depends upon the spectral composition of the incident light, the spectral reflectance, transmittance, or radiance of the object, and the spectral response of the observer, as well as the illuminating and viewing geometry.

3.2.20 *color, n—psychophysical*, characteristics of a color stimulus (that is, light producing a visual sensation of color) denoted by a colorimetric specification with three values, such as tristimulus values.

3.2.21 *color–matching functions, n*—the amounts, in any trichromatic system, of three reference color stimuli needed to match, by additive mixing, monochromatic components of an equal–energy spectrum. **E284**

3.2.22 *fluorescent illuminant, n*—illuminant representing the spectral distribution of the radiation from a specified type of fluorescent lamp. **E284**

~~3.2.15 *CIE recommended fluorescent illuminants, n*—a set of spectral power distributions of 12 types of fluorescent lamps, the most important of which are *FL2*, representing a cool white fluorescent lamp with correlated color temperature 4200 K, *FL7*, a broad-band (continuous-spectrum) daylight lamp (6500 K), and *FL11*, a narrow-band (line-spectrum) white fluorescent lamp (4000 K).~~

3.2.23 *luminous, adj*—weighted according to the spectral luminous efficiency function $V(\lambda)$ of the CIE. **E284**

3.2.24 *opponent-color scales, n*—scales that denote one color by positive scale values, the neutral axis by zero value, and an approximately complementary color by negative scale values, common examples being scales that are positive in the red direction and negative in the green direction, and those that are positive in the yellow direction and negative in the blue direction. **E284**

~~3.2.18 *CIELAB color scales, n*—CIE 1976 L^*, a^*, b^* opponent-color scales, in which a^* is positive in the red direction and negative in the green direction, and b^* is positive in the yellow direction and negative in the blue direction.~~

~~3.2.19 *CIELUV color scales, n*—CIE 1976 L^*, u^*, v^* opponent-color scales, in which u^* is positive in the red direction and negative in the green direction, and v^* is positive in the yellow direction and negative in the blue direction.~~

3.2.25 *passband, n*—a contiguous band of wavelengths in which at least a fraction of the incident light is selectively transmitted by a light-modulating device or medium. **E284**

3.2.26 *spectral, adj—for radiometric quantities*, pertaining to monochromatic radiation at a specified wavelength or, by extension, to radiation within a narrow wavelength band about a specified wavelength. **E284**

3.2.27 *standard illuminant, n*—a luminous flux, specified by its spectral distribution, meeting specifications adopted by a standardizing organization. **E284**

~~3.2.23 *CIE standard illuminant A, n*—colorimetric illuminant, representing the full radiator at 2855.6 K, defined by the CIE in terms of a relative spectral power distribution.~~

~~3.2.24 *CIE standard illuminant C, n*—colorimetric illuminant, representing daylight with a correlated color temperature of 6774 K, defined by the CIE in terms of a relative spectral power distribution.~~

~~3.2.25 *CIE standard illuminant* D_{65}*, n*—colorimetric illuminant, representing daylight with a correlated color temperature of 6504 K, defined by the CIE in terms of a relative spectral power distribution.~~

3.2.25.1 *Discussion*—
Other illuminants of importance defined by the CIE include the daylight illuminants D_{50}, D_{55}, and D_{75}. Illuminant D_{50} is used by the graphic arts industry for viewing colored transparencies and prints (see ANSI PH2.23).

3.2.26 *standard observer, n*—an ideal observer having visual response described by the CIE color-matching functions (see CIE S 013 and Ref (**3**)).

3.2.27 *CIE 1931 standard observer, n*—ideal colorimetric observer with color-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ corresponding to a field of view subtending a 2° angle on the retina; commonly called the "2° standard observer."

3.2.28 *CIE 1964 supplementary standard observer, n*—ideal colorimetric observer with color-matching functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ corresponding to a field of view subtending a 10° angle on the retina; commonly called the "10° standard observer" (see Note 1).

3.2.28 *tristimulus values, n—of a color stimulus*, three amounts of the primary color stimuli required to make an additive match to the color stimulus under consideration. **E284**

3.2.29 *tristimulus weighting factors, $S\bar{x}$, $S\bar{y}$, $S\bar{z}$, n*—factors obtained from products of the spectral power S of an illuminant and the spectral color-matching functions $\bar{x}$, $\bar{y}$, $\bar{z}$ (or $\bar{x}_{10}$, $\bar{y}_{10}$, $\bar{z}_{10}$) of an observer, usually tabulated at wavelength intervals of 10 or 20 nm, 10 nm, used to compute tristimulus values by multiplication by the spectral reflectance, transmittance, or radiance (or the corresponding factors) and summation.

3.2.30.1 *Discussion*—
Proper account should be taken of the spectral bandpass of the measuring instrument. **E284**

3.2.29.1 *Discussion*—
Proper account should be taken of the spectral bandpass of the measuring instrument. See Practice E2729.

4. Summary of Practice

4.1 *Selection of Parameters*—The user of this practice must select values of the following parameters:

4.1.1 *Observer*—Select either the CIE 1931 standard colorimetric observer (2° observer) or the CIE 1964 supplementary standard observer (10° observer), tabulated in this practice, CIE Standard S 013 or D 001, or Ref (**3**) (see 3.2.26 and Note 1).

4.1.2 *Illuminant*—Select one of the CIE standard or recommended illuminants tabulated in this practice, CIE Standard S 014 or D 001, or Ref (**3**) (see 3.2.22).

4.1.3 *Measurement Interval*—Select the measurement interval of the available spectral data. This practice provides for 1-, 5-, 10-, or 20-nm measurement intervals. For best practice the measurement interval should be selected to be as nearly as possible equal to the instrument bandpass.

4.2 *Procedures*—For data obtained at 1- or 5-nm measurement intervals, the procedures of 7.2 should be followed.

4.3 *Procedures*—The user should ascertain whether or not the spectral data For data obtained at 10- or 20-nm measurement intervals, the tables of tristimulus weighting factors contained in Tables 5 should be used with spectral data that have been corrected for bandpass dependence. The accuracy of tristimulus values is significantly improved by incorporating a correction for bandpassFor standard methods of making such a correction see Practice E2729dependence into either the spectral data or the tables of tristimulus weighting factors (see .7.2). The procedures used depend on this and on the measurement interval.

4.2.1 For data obtained at 1- or 5-nm measurement interval, the procedures of 7.2 should be followed.

4.2.2 For data obtained at 10- or 20-nm measurement interval, the tables of tristimulus weighting factors contained in Tables 5 should be used with spectral data that have been corrected for bandpass dependence. For standard methods of making such a correction see Practice E2729.

4.2.3 A flow chart to ensure the use of proper combinations of data and tables is given in Fig. 2. The procedures of the practice are given in detail in 7.1.

4.4 A flow chart to ensure the use of proper combinations of data and tables is given in Fig. 2. The procedures of the practice are given in detail in 7.1.

4.5 *Calculations*—CIE tristimulus values *X, Y, Z* or X_{10}, Y_{10}, Z_{10} are calculated by numerical summation of the products of tristimulus weighting factors for selected illuminants and observers with the reflectance factors (or transmittance or radiance factors) making up the spectral data.

4.6 The tristimulus values so calculated may be further converted to coordinates in a more nearly uniform color space such as CIELAB or CIELUV.

5. Significance and Use

5.1 The CIE colorimetric systems provide numerical specifications that are meant to indicate whether or not pairs of color stimuli match when viewed by a CIE standard observer. The CIE color systems are not intended to provide visually uniform scales of color difference or to describe visually perceived color appearances.

5.2 This practice provides for the calculation of tristimulus values *X, Y, Z* and chromaticity coordinates *x, y* that can be used directly for psychophysical color stimulus specification or that can be transformed to nearly visually uniform color scales, such as CIELAB and CIELUV. Uniform color scales are preferred for research, production control, color-difference calculation, color specification, and setting color tolerances. The appearance of a material or an object is not completely specified by the numerical evaluation of its psychophysical color, because appearance can be influenced by other properties such as gloss or texture.

6. Procedure

6.1 *Selecting Standard Observer*—When colorimetric results are required that will be compared with previous results obtained for the CIE 1931 standard observer, use the values in Table 1 for that observer. When new results are being computed, consider using the values in Table 2 for the CIE 1964 ~~supplementary~~ standard observer, but see Note 1.

6.1.1 Whenever correlation with visual observations using fields of angular subtense between about 1° and about 4° at the eye of the observer is desired, select the CIE 1931 standard colorimetric observer.

6.1.2 Whenever correlation with visual observations using fields of angular subtense greater than 4° at the eye of the observer is desired, select the CIE 1964 supplementary standard colorimetric observer (but see Note 1).

6.2 *Selecting Standard or Recommended Illuminants*—Select illuminants according to the type of light(s) under which objects will be viewed or for which their colors will be specified or evaluated.

6.2.1 When incandescent (tungsten) lamplight is involved, use values for CIE illuminant *A*.

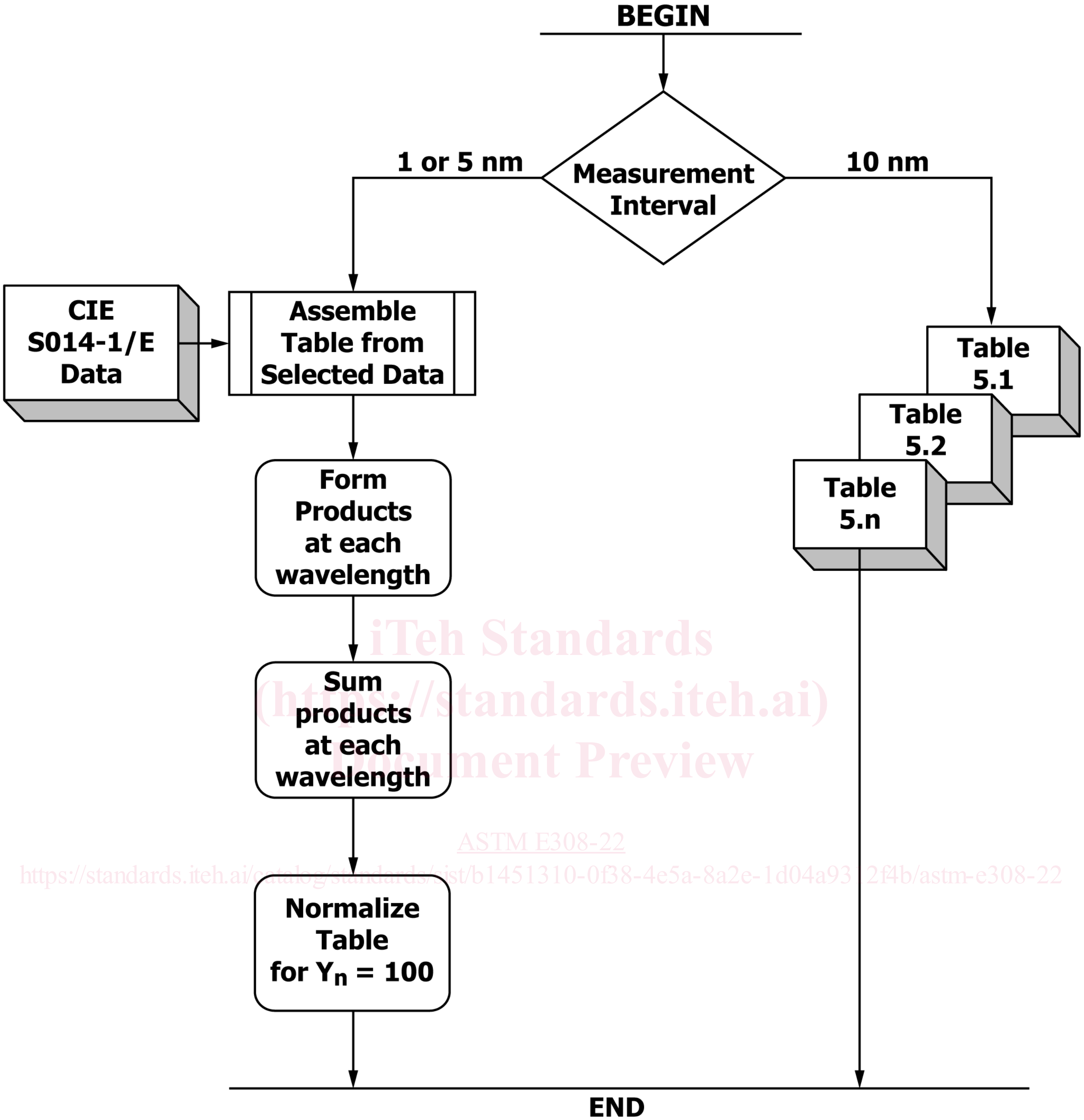
6.2.2 When daylight is involved, use values for CIE illuminant *C* or D_{65}.

6.2.3 When fluorescent-lamp illumination is involved, use 4200 K standard cool white (*FL2*) unless results are desired for 6500 K broad-band daylight (*FL7*) or 4000 K narrow-band white (*FL11*) fluorescent illumination.

6.3 *Selecting the Measurement Interval*—For greater accuracy select the 5-nm measurement interval over the 10-nm interval where spectral data are available at 5-nm intervals. Likewise, select the 10-nm measurement interval over the 20-nm interval where spectral data are available at 10-nm intervals. If the 20-nm interval is selected, users should ~~ensure~~assure themselves that the resulting accuracy is sufficient for the purpose for which the results are intended. ~~For many industrial applications use of the 20-nm interval may be satisfactory.~~

6.3.1 If the instrument used has a selectable measurement interval, select the interval that most nearly equals the bandwidth of the instrument throughout the spectrum. If the instrument has an adjustable bandwidth, adjust the bandwidth to be approximately equal to the measurement interval.

6.3.2 The measurement interval should be commensurate with the bandwidth. A much greater interval would undersample the spectrum, and a much smaller interval would not improve the accuracy of the computation.



~~NOTE 1—References to Section 7. Calculations are included.~~

Figure Note—References to Section 7. Calculations are included.

FIG. 2 Flow Chart for Selecting Methods and Tables for Tristimulus Integration

TABLE 1 Spectral Tristimulus Values (Color-Matching Functions) $\bar{x}$, $\bar{y}$, $\bar{z}$, of the CIE 1931 Standard (2°) Observer, at 5 nm Intervals from 380 to 780 nm (See Note 2 and Ref (3))

λ(nm)	$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$
380	0.0014	0.0000	0.0065
385	0.0022	0.0001	0.0105
390	0.0042	0.0001	0.0201
395	0.0076	0.0002	0.0362
400	0.0143	0.0004	0.0679
405	0.0232	0.0006	0.1102
410	0.0435	0.0012	0.2074
415	0.0776	0.0022	0.3713
420	0.1344	0.0040	0.6456
425	0.2148	0.0073	1.0391
430	0.2839	0.0116	1.3856
435	0.3285	0.0168	1.6230
440	0.3483	0.0230	1.7471
445	0.3481	0.0298	1.7826
450	0.3362	0.0380	1.7721
455	0.3187	0.0480	1.7441
460	0.2908	0.0600	1.6692
465	0.2511	0.0739	1.5281
470	0.1954	0.0910	1.2876
475	0.1421	0.1126	1.0419
480	0.0956	0.1390	0.8130
485	0.0580	0.1693	0.6162
490	0.0320	0.2080	0.4652
495	0.0147	0.2586	0.3533
500	0.0049	0.3230	0.2720
505	0.0024	0.4073	0.2123
510	0.0093	0.5030	0.1582
515	0.0291	0.6082	0.1117
520	0.0633	0.7100	0.0782
525	0.1096	0.7932	0.0573
530	0.1655	0.8620	0.0422
535	0.2257	0.9149	0.0298
540	0.2904	0.9540	0.0203
545	0.3597	0.9803	0.0134
550	0.4334	0.9950	0.0087
555	0.5121	1.0000	0.0057
560	0.5945	0.9950	0.0039
565	0.6784	0.9786	0.0027
570	0.7621	0.9520	0.0021
575	0.8425	0.9154	0.0018
580	0.9163	0.8700	0.0017
585	0.9786	0.8163	0.0014
590	1.0263	0.7570	0.0011
595	1.0567	0.6949	0.0010
600	1.0622	0.6310	0.0008
605	1.0456	0.5668	0.0006
610	1.0026	0.5030	0.0003
615	0.9384	0.4412	0.0002
620	0.8544	0.3810	0.0002
625	0.7514	0.3210	0.0001
630	0.6424	0.2650	0.0000
635	0.5419	0.2170	0.0000
640	0.4479	0.1750	0.0000
645	0.3608	0.1382	0.0000
650	0.2835	0.1070	0.0000
655	0.2187	0.0816	0.0000
660	0.1649	0.0610	0.0000
665	0.1212	0.0446	0.0000
670	0.0874	0.0320	0.0000
675	0.0636	0.0232	0.0000
680	0.0468	0.0170	0.0000
685	0.0329	0.0119	0.0000

TABLE 1 *Continued*

λ(nm)	$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$
690	0.0227	0.0082	0.0000
695	0.0158	0.0057	0.0000
700	0.0114	0.0041	0.0000
705	0.0081	0.0029	0.0000
710	0.0058	0.0021	0.0000
715	0.0041	0.0015	0.0000
720	0.0029	0.0010	0.0000
725	0.0020	0.0007	0.0000
730	0.0014	0.0005	0.0000
735	0.0010	0.0004	0.0000
740	0.0007	0.0002	0.0000
745	0.0005	0.0002	0.0000
750	0.0003	0.0001	0.0000
755	0.0002	0.0001	0.0000
760	0.0002	0.0001	0.0000
765	0.0001	0.0000	0.0000
770	0.0001	0.0000	0.0000
775	0.0001	0.0000	0.0000
780	0.0000	0.0000	0.0000

Summation at 5 nm intervals:

$\sum \bar{x}(\lambda) = 21.3714$

$\sum \bar{y}(\lambda) = 21.3711$

$\sum \bar{z}(\lambda) = 21.3715$

6.4 *Other Miscellaneous Conditions*—While the above selections cover the majority of industrial practices, the possibility exists that other conditions could be encountered. Therefore, other procedures than those included in this practice may be used provided that the results are consistent with those obtained by use of the procedures in the practice.

7. Calculations

7.1 *General Procedures*—The general procedures for computing CIE tristimulus values are summarized as follows:

7.1.1 *Procedures as Specified by the CIE*—The CIE procedures are specified in Ref (**3**) and summarized in Refs (**5-9**). The fundamental definition is in terms of integrals,

$$X = k \int_{\lambda} R(\lambda) S(\lambda) \bar{x}(\lambda) d\lambda \quad (3)$$

$$Y = k \int_{\lambda} R(\lambda) S(\lambda) \bar{y}(\lambda) d\lambda$$

$$Z = k \int_{\lambda} R(\lambda) S(\lambda) \bar{z}(\lambda) d\lambda$$

where:

$R(\lambda)$	=	the reflectance, transmittance, or radiance factor (on a scale of zero to one for the perfect reflecting diffuser),
$S(\lambda)$	=	the relative spectral power of a CIE standard illuminant, and
$\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$	=	the color-matching functions of one of the CIE standard observers.

The integration is carried out over the entire wavelength region in which the color-matching functions are defined, 360 to 830 nm. The normalizing factor k is defined as

$$k = 100 / \int_{\lambda} S(\lambda) \bar{y}(\lambda) d\lambda \quad (4)$$

The CIE notes that in all practical calculations of tristimulus values the integration is approximated by a summation, giving the equations as follows:

TABLE 2 Spectral Tristimulus Values (Color-Matching Functions) $\bar{x}_{10}$, $\bar{y}_{10}$, $\bar{z}_{10}$, of the CIE 1964 Supplementary Standard (10°) Observer, At 5 nm Intervals from 380 to 780 nm (See Note 2 and Ref (3))

λ(nm)	$\bar{x}_{10}(\lambda)$	$\bar{y}_{10}(\lambda)$	$\bar{z}_{10}(\lambda)$
380	0.0002	0.0000	0.0007
385	0.0007	0.0001	0.0029
390	0.0024	0.0003	0.0105
395	0.0072	0.0008	0.0323
400	0.0191	0.0020	0.0860
405	0.0434	0.0045	0.1971
410	0.0847	0.0088	0.3894
415	0.1406	0.0145	0.6568
420	0.2045	0.0214	0.9725
425	0.2647	0.0295	1.2825
430	0.3147	0.0387	1.5535
435	0.3577	0.0496	1.7985
440	0.3837	0.0621	1.9673
445	0.3867	0.0747	2.0273
450	0.3707	0.0895	1.9948
455	0.3430	0.1063	1.9007
460	0.3023	0.1282	1.7454
465	0.2541	0.1528	1.5549
470	0.1956	0.1852	1.3176
475	0.1323	0.2199	1.0302
480	0.0805	0.2536	0.7721
485	0.0411	0.2977	0.5701
490	0.0162	0.3391	0.4153
495	0.0051	0.3954	0.3024
500	0.0038	0.4608	0.2185
505	0.0154	0.5314	0.1592
510	0.0375	0.6067	0.1120
515	0.0714	0.6857	0.0822
520	0.1177	0.7618	0.0607
525	0.1730	0.8233	0.0431
530	0.2365	0.8752	0.0305
535	0.3042	0.9238	0.0206
540	0.3768	0.9620	0.0137
545	0.4516	0.9822	0.0079
550	0.5298	0.9918	0.0040
555	0.6161	0.9991	0.0011
560	0.7052	0.9973	0.0000
565	0.7938	0.9824	0.0000
570	0.8787	0.9556	0.0000
575	0.9512	0.9152	0.0000
580	1.0142	0.8689	0.0000
585	1.0743	0.8256	0.0000
590	1.1185	0.7774	0.0000
595	1.1343	0.7204	0.0000
600	1.1240	0.6583	0.0000
605	1.0891	0.5939	0.0000
610	1.0305	0.5280	0.0000
615	0.9507	0.4618	0.0000
620	0.8563	0.3981	0.0000
625	0.7549	0.3396	0.0000
630	0.6475	0.2835	0.0000
635	0.5351	0.2283	0.0000
640	0.4316	0.1798	0.0000
645	0.3437	0.1402	0.0000
650	0.2683	0.1076	0.0000
655	0.2043	0.0812	0.0000
660	0.1526	0.0603	0.0000
665	0.1122	0.0441	0.0000
670	0.0813	0.0318	0.0000
675	0.0579	0.0226	0.0000
680	0.0409	0.0159	0.0000
685	0.0286	0.0111	0.0000

TABLE 2 *Continued*

λ(nm)	$\bar{x}_{10}(\lambda)$	$\bar{y}_{10}(\lambda)$	$\bar{z}_{10}(\lambda)$
690	0.0199	0.0077	0.0000
695	0.0138	0.0054	0.0000
700	0.0096	0.0037	0.0000
705	0.0066	0.0026	0.0000
710	0.0046	0.0018	0.0000
715	0.0031	0.0012	0.0000
720	0.0022	0.0008	0.0000
725	0.0015	0.0006	0.0000
730	0.0010	0.0004	0.0000
735	0.0007	0.0003	0.0000
740	0.0005	0.0002	0.0000
745	0.0004	0.0001	0.0000
750	0.0003	0.0001	0.0000
755	0.0002	0.0001	0.0000
760	0.0001	0.0000	0.0000
765	0.0001	0.0000	0.0000
770	0.0001	0.0000	0.0000
775	0.0000	0.0000	0.0000
780	0.0000	0.0000	0.0000

Summation at 5 nm intervals:

$\sum \bar{x}_{10}(\lambda) = 23.3294$
$\sum \bar{y}_{10}(\lambda) = 23.3324$
$\sum \bar{z}_{10}(\lambda) = 23.3343$

$$X = k \sum_{\lambda} R(\lambda) S(\lambda) \bar{x}(\lambda) \Delta\lambda \quad (5)$$

$$Y = k \sum_{\lambda} R(\lambda) S(\lambda) \bar{y}(\lambda) \Delta\lambda$$

$$Z = k \sum_{\lambda} R(\lambda) S(\lambda) \bar{z}(\lambda) \Delta\lambda$$

with:

$$k = 100 / \sum_{\lambda} S(\lambda) \bar{y}(\lambda) \Delta\lambda \quad (6)$$

7.1.2 *Procedure Using Tristimulus Weighting Factors*—It is common industrial practice to carry out the summation to tristimulus values in two steps. In the first of these, a set of normalized tristimulus weighting factors W_x, W_y, W_z is calculated as follows:

$$W_x(\lambda) = k\, S(\lambda)\, \bar{x}(\lambda) \Delta\lambda \quad (7)$$

$$W_y(\lambda) = k S(\lambda) \bar{y}(\lambda) \Delta\lambda$$

$$W_z(\lambda) = k S(\lambda) \bar{z}(\lambda) \Delta\lambda$$

for λ = 360, ... 780 nm, (see Note 2), and where:

$$k = 100 / \sum_{360}^{780} S(\lambda) \bar{y}(\lambda) \Delta\lambda \quad (8)$$

For a given selection of illuminant, observer, measurement interval Δλ, and measurement bandpass, this calculation needs to be done only once, since the spectral reflectance (or transmittance or radiance) factor *R*(λ) is not included in the weighting factors *W*. In the second step, tristimulus values *X, Y, Z* (or X_{10}, Y_{10}, Z_{10}) are calculated using the values of *W* and *R*(λ) in the following equations:

$$X = \Sigma_{360}^{780} W_x(\lambda) R(\lambda) \quad (9)$$

$$Y = \Sigma_{360}^{780} W_y(\lambda) R(\lambda)$$

$$Z = \Sigma_{360}^{780} W_z(\lambda) R(\lambda)$$

NOTE 2—While 360 nm is recommended as the starting wavelength for summation and elsewhere in this practice, CIE data reproduced in Tables 1-4, and the spectrum locus scale of Fig. 1, begin only at 380 nm; since the missing data cannot be supplied in all cases, these references to 380 nm should remain. In the region between 360 and 379 nm, values of color matching functions are so small that their inclusion or omission in the calculations would not lead to significant differences in the resulting tristimulus values.

7.1.3 For methods of calculating weighting factors from custom sources, see Practice E2022.

7.2 *Summary of Calculations* (see Note 2)—A general outline of the procedure is given in Fig. 2 in the form of a flow chart.

NOTE 3—For reflecting materials, calculate tristimulus values from spectral data obtained relative to the perfect reflecting diffuser. For transmitting materials, calculate by use of the incident light as the reference.

7.2.1 *Procedure for 1-nm Measurement Interval*—Use the 1-nm spectral data in CIE S 014 and S 013 (or on CIE D 001 Disk) and (Eq ~~3~~6) and (Eq ~~4~~7).

7.2.2 *Procedures for Spectral ~~Data With Bandpass Correction:~~ Data:*

7.2.2.1 *Procedure for Data Obtained at 5-nm Measurement Intervals*—Prepare tables of tristimulus weighting factors for desired illuminant-observer combinations, using the spectral data in Tables 1-4 (see Note 2), and (Eq ~~5~~8) and (Eq ~~6~~9). Use the tables so prepared as described in 7.3 (see Note 4).

NOTE 4—Using the previous procedure at 10 nm or 20 nm intervals by omitting intermediate tabulated values is not allowed. Use the procedures of 7.3 instead.

7.2.2.2 *Procedures for Data Obtained at 10- or 20-nm Measurement Intervals*—Select the appropriate tables of tristimulus weighting factors from those in Tables 5 and use them as described in 7.3.

7.3 *Use of Tristimulus Weighting Factors:*

7.3.1 *Use of Data Obtained at 5-nm Measurement Intervals*—Use the color-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$, from Table 1, for the 1931 CIE standard colorimetric observer, or when desired the functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$, from Table 2, for the 1964 CIE supplementary standard colorimetric observer. Select the desired CIE standard or recommended illuminant, for example *A, C*, or one of the *D* or *F* illuminants from Table 3 or Table 4. At each wavelength multiply the tabulated value of the observer color-matching functions by the tabulated value of the relative spectral power of the illuminant $S(\lambda)$, and by the spectral reflectance (or transmittance) factor $R(\lambda)$ (or $T(\lambda)$) of the specimen. Obtain the sum of these products at 5 nm intervals over the wavelength range 360 to 780 nm and use (Eq ~~3~~6) and (Eq ~~4~~7).

7.3.2 *Use of Data Obtained at 10 nm Measurement Intervals:*

7.3.2.1 *Data Available over the Wavelength Range 360 to 780 nm*—Select the appropriate table of tristimulus weighting factors, computed for triangular bandpass and 10 nm measurement intervals, for the desired illuminant and observer, from the nine sets included in Tables 5 **(10)**.

7.3.2.2 *Data Available only for Wavelength Ranges Shorter than 360 to 780 nm*—When data for $R(\lambda)$, $T(\lambda)$, or ~~$\beta(\lambda)$~~$\beta(\lambda)$ are not available for the full wavelength range, add the weights at the wavelengths for which data are not available to the weights at the shortest and longest wavelength for which spectral data are available. That is: add the weights for wavelengths 360, ..., up to the last wavelength for which measured data are not available, to the next higher weight, for which such data are available; add the weights for wavelengths of 780, ..., down to the last wavelength for which measured data are not available, to the next lower weight, for which such data are available.

7.3.3 *Use of Data Obtained at 20 nm Measurement Intervals:*

7.3.3.1 *Data Available Over the Wavelength Range 360 to 780 nm*—Copy the 20 nm spectrum into a 10 nm framework whose indices, at 10 nm intervals, will run from 0 to 46 by copying the 22 values available to the even indices between 360 nm (index 2) and 780 nm (index 44). Extrapolate the 20 nm data to a range of 340 to 800 nm by use of the following equations:

$$R_0 = 3R_2 - 3R_4 + R_6 \tag{10}$$

$$R_n = R_{n-6} - 3R_{n-4} + 3R_{n-2} \tag{11}$$

TABLE 3 Relative Spectral Power Distributions S(λ) of CIE Standard Illuminants *A*,*C*,D_{50},D_{55},D_{65}, and D_{75} at 5-nm Intervals from 380 to 780 nm (See Note 2 and Ref (3))

λ (nm)	*A* S(λ)	*C* S(λ)	D_{50} S(λ)	D_{55} S(λ)	D_{65} S(λ)	D_{75} S(λ)
380	9.80	33.00	24.49	32.58	49.98	66.70
385	10.90	39.92	27.18	35.34	52.31	68.33
390	12.09	47.40	29.87	38.09	54.65	69.96
395	13.35	55.17	39.59	49.52	68.70	85.95
400	14.71	63.30	49.31	60.95	82.75	101.93
405	16.15	71.81	52.91	64.75	87.12	106.91
410	17.68	80.60	56.51	68.55	91.49	111.89
415	19.29	89.53	58.27	70.07	92.46	112.35
420	20.99	98.10	60.03	71.58	93.43	112.80
425	22.79	105.80	58.93	69.75	90.06	107.94
430	24.67	112.40	57.82	67.91	86.68	103.09
435	26.64	117.75	66.32	76.76	95.77	112.14
440	28.70	121.50	74.82	85.61	104.86	121.20
445	30.85	123.45	81.04	91.80	110.94	127.10
450	33.09	124.00	87.25	97.99	117.01	133.01
455	35.41	123.60	88.93	99.23	117.41	132.68
460	37.81	123.10	90.61	100.46	117.81	132.36
465	40.30	123.30	90.99	100.19	116.34	129.84
470	42.87	123.80	91.37	99.91	114.86	127.32
475	45.52	124.09	93.24	101.33	115.39	127.06
480	48.24	123.90	95.11	102.74	115.92	126.80
485	51.04	122.92	93.54	100.41	112.37	122.29
490	53.91	120.70	91.96	98.08	108.81	117.78
495	56.85	116.90	93.84	99.38	109.08	117.19
500	59.86	112.10	95.72	100.68	109.35	116.59
505	62.93	106.98	96.17	100.69	108.58	115.15
510	66.06	102.30	96.61	100.70	107.80	113.70
515	69.25	98.81	96.87	100.34	106.30	111.18
520	72.50	96.90	97.13	99.99	104.79	108.56
525	75.79	96.78	99.61	102.10	106.24	109.55
530	79.13	98.00	102.10	104.21	107.69	110.44
535	82.52	99.94	101.43	103.16	106.05	108.37
540	85.95	102.10	100.75	102.10	104.41	106.29
545	89.41	103.95	101.54	102.53	104.23	105.60
550	92.91	105.20	102.32	102.97	104.05	104.90
555	96.44	105.67	101.16	101.48	102.02	102.45
560	100.00	105.30	100.00	100.00	100.00	100.00
565	103.58	104.11	98.87	98.61	98.17	97.81
570	107.18	102.30	97.74	97.22	96.33	95.62
575	110.80	100.15	98.33	97.48	96.06	94.91
580	114.44	97.80	98.92	97.75	95.79	94.21
585	118.08	95.43	96.21	94.59	92.24	90.60
590	121.73	93.20	93.50	91.43	88.69	87.00
595	125.39	91.22	95.59	92.93	89.35	87.11
600	129.04	89.70	97.69	94.42	90.01	87.23
605	132.70	88.83	98.48	94.78	89.80	86.68
610	136.35	88.40	99.27	95.14	89.60	86.14
615	139.99	88.19	99.16	94.68	88.65	84.86
620	143.62	88.10	99.04	94.22	87.70	83.58
625	147.24	88.06	97.38	92.33	85.49	81.16
630	150.84	88.00	95.72	90.45	83.29	78.75
635	154.42	87.86	97.29	91.39	83.49	78.59
640	157.98	87.80	98.86	92.33	83.70	78.43
645	161.52	87.99	97.26	90.59	81.86	76.61
650	165.03	88.20	95.67	88.85	80.03	74.80
655	168.51	88.20	96.93	89.59	80.12	74.56
660	171.96	87.90	98.19	90.32	80.21	74.32
665	175.38	87.22	100.60	92.13	81.25	74.87
670	178.77	86.30	103.00	93.95	82.28	75.42
675	182.12	85.30	101.07	91.95	80.28	73.50
680	185.43	84.00	99.13	89.96	78.28	71.58
685	188.70	82.21	93.26	84.82	74.00	67.71
690	191.93	80.20	87.38	79.68	69.72	63.85
695	195.12	78.24	89.49	81.26	70.67	64.46
700	198.26	76.30	91.60	82.84	71.61	65.08
705	201.36	74.36	92.25	83.84	72.98	66.57
710	204.41	72.40	92.89	84.84	74.35	68.07
715	207.41	70.40	84.87	77.54	67.98	62.26
720	210.36	68.30	76.85	70.24	61.60	56.44
725	213.27	66.30	81.68	74.77	65.74	60.34
730	216.12	64.40	86.51	79.30	69.89	64.24
735	218.92	62.80	89.55	82.15	72.49	66.70
740	221.67	61.50	92.58	84.99	75.09	69.15

TABLE 3 *Continued*

λ (nm)	*A* S(λ)	*C* S(λ)	D_{50} S(λ)	D_{55} S(λ)	D_{65} S(λ)	D_{75} S(λ)
745	224.36	60.20	85.40	78.44	69.34	63.89
750	227.00	59.20	78.23	71.88	63.59	58.63
755	229.59	58.50	67.96	62.34	55.01	50.62
760	232.12	58.10	57.69	52.79	46.42	42.62
765	234.59	58.00	70.31	64.36	56.61	51.98
770	237.01	58.20	82.92	75.93	66.81	61.35
775	239.37	58.50	80.60	73.87	65.09	59.84
780	241.68	59.10	78.27	71.82	63.38	58.32

where *R* refers to the measured reflectance or transmittance and the index *zero* refers to an extrapolated value at 340 nm and the index *n* refers to the extrapolated value at 800 nm of the 10 nm interval spectrum. Use these values to calculate the missing 10 nm intervals between 360 and 780 nm, but discard these values immediately after the interpolation and use these values for no other purpose.

With the extrapolated spectrum extended to indices 0 to 46, interpolate the missing 10 nm values by use of the following equation:

$$R_j = -0.0625R_{j-3} + 0.5625R_{j-1} + 0.5625R_{j+1} - 0.0625R_{j+3} \quad (12)$$

where the range of interpolation is for every odd numbered value of *j* between 3 and 43 inclusive.

Should any interpolated value be less than zero, such value should be set to zero.

Select the appropriate table of tristimulus weighting factors, computed for triangular bandpass and 10 nm measurement intervals, for the desired illuminant and observer, from the nine sets included in Tables 5. Integrate the interpolated spectrum from index 2 to 44 (360 to 780 nm) with the chosen 10 nm table of tristimulus weighting factors, being sure to match the indices of the two multiplicative factors, spectral value and weighting factor, appropriately. The accuracy of doing so has been found to be approximately as accurate as 20 nm interpolation itself because each of the 19 missing 1-nm intervals is interpolated in each case, but in a different order.

7.3.3.2 *Data Available Only for Wavelength Ranges Shorter than 360 to 780 nm*—Interpolate the spectrum using equations Eq ~~8~~10 through Eq ~~10~~12 with the number of intervals and indices appropriate to the range of the present spectrum. Follow the teachings of 7.3.2.2 for the purpose of shortening the weighting factors to the appropriate range.

7.3.4 *Tristimulus Values*—Obtain the products of *R*(λ), *T*(λ) or *β*(λ) and the weights selected in 7.3.1 or 7.3.2, including any modifications, and sum to obtain the CIE tristimulus values *X, Y, Z* , or X_{10}, Y_{10}, Z_{10}.

7.4 *Chromaticity Coordinates*—Obtain chromaticity coordinates *x, y, z* (for the CIE 1931 standard observer) by dividing each tristimulus value *X, Y, Z* by the sum of all three: $x = X/(X + Y + Z)$; $y = Y/(X + Y + Z)$; and $z = Z/(X + Y + Z)$, or use the same procedure with all quantities having the subscript 10 for the CIE 1964 ~~supplementary~~ standard observer.

7.5 *CIE 1976 Uniform Color Spaces*—When a color space more nearly uniform than *X, Y, Z* is desired, use CIELAB or CIELUV.

7.5.1 *CIELAB or L*a*b**—This approximately uniform color space is produced by plotting in rectangular coordinates the quantities *L**, *a**, *b** defined as follows **(3)**:

$$L^* = 116 f(Q_Y) - 16 \quad (13)$$

$$a^* = 500[f(Q_X) - f(Q_Y)] \quad (14)$$

$$b^* = 200[f(Q_Y) - f(Q_Z)] \quad (15)$$

where:

$$Q_X = (X/X_n);\ Q_Y = (Y/Y_n);\ Q_Z = (Z/Z_n) \quad (16)$$

and

$$f(Q_i) = Q_i^{1/3} \ \ if\ \ Q_i > (6/29)^3 \quad (17)$$

else

$$f(Q_i) = (841/108)Q_i + 4/29 \ \ if\ \ Q_i \le (6/29)^3 \quad (18)$$

where