INTERNATIONAL

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Standard Practice for Calculating Precision Limits Where Values Are Calculated from Other Test Methods[1]

This standard is issued under the fixed designation D4460; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 Material and mixture properties such as air voids and voids in mineral aggregates (VMA) are calculated from two or three test results, combined in simple mathematical relationships. The standard deviation equations for these calculated values can be developed using a mathematical process called "propagation of errors" (also called "propagation of uncertainty"). This practice includes uncertainty equations for four forms or material and mixture equations: when two test results are (*1*) added or subtracted, (*2*) multiplied together, (*3*) one divided by the other, and (*4*) two test results divided by a third.

1.2 This approach to calculating standard deviation equations is only valid when the distributions of the test results from the two standards are independent (that is, not correlated).

1.3 The accuracy of a calculated standard deviation is dependent on the accuracy of the standard deviations used for the individual test result methods.

1.4 Values for the mean and standard deviation for each test method are needed to determine the standard deviation for a calculated value.

1.5 Examples of how to use these equations are shown in Appendix X1.

1.6 A brief explanation of how standard deviation equations are derived for more complicated material and mixture equations is also included.

1.7 The text of this standard references notes and footnotes which provide explanatory material. These notes and footnotes (excluding those in tables and figures) shall not be considered as requirements of the standard.

1.8 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.9 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

C127 Test Method for Relative Density (Specific Gravity) and Absorption of Coarse Aggregate

C128 Test Method for Relative Density (Specific Gravity) and Absorption of Fine Aggregate

D1188/D1188M Test Method for Bulk Specific Gravity and Density of Compacted Asphalt Mixtures Using Coated Samples

D2172/D2172M Test Methods for Quantitative Extraction of Asphalt Binder from Asphalt Mixtures

D2726/D2726M Test Method for Bulk Specific Gravity and Density of Non-Absorptive Compacted Asphalt Mixtures

D4125/D4125M Test Methods for Asphalt Content of Asphalt Mixtures by the Nuclear Method

D6307 Test Method for Asphalt Content of Asphalt Mixture by Ignition Method

D6752/D6752M Test Method for Bulk Specific Gravity and Density of Compacted Asphalt Mixtures Using Automatic Vacuum Sealing Method

E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods

3. Terminology

3.1 For definitions of terms used in this document, consult Practice E177, or a standard dictionary, or a statistical text.[3,4,5]

[1] This practice is under the jurisdiction of ASTM Committee D04 on Road and Paving Materials and is the direct responsibility of Subcommittee D04.94 on Statistical Procedures and Evaluation of Data.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] Geary, R. C., "The Frequency Distribution of a Quotient," *Journal of the Royal Statistical Society*, Vol 93, 1930, pp. 442–446.

[4] Fieller, E. C., "The Distribution of the Index in a Normal Bivariate Population," *Biometrika*, Vol 24, 1932, pp. 428–440.

[5] Ku, H. H., "Notes on the Use of Propagation of Error Formulas," *Journal of Research of the National Bureau of Standards*, Vol 70C, No. 4, 1966, pp. 331–341.

4. Significance and Use

4.1 Precision statements for calculated values can be developed using this approach. Users can also evaluate how an individual test method's precision influences the variability of calculated values.

4.2 The standard deviation of a calculated value that is the sum, difference, product, or quotient of two or more test method results, each with their own precision statement, can be calculated so long as the individual variables (that is, test results) are independent and the standard deviations are small relative to their mean values. These restrictions are usually met in ASTM methods. In those cases where these restrictions are not met, other methods can be used. Only cases complying with the restrictions are covered in this standard.

5. Procedure

5.1 Standard deviations that can be used for precision limits for a calculated value can be calculated from the equations in this section. The appropriate equation format is selected based on how independent individual test results are combined to calculate the property. Examples in Appendix X1 illustrate how the equations are used.

5.2 *Addition or Subtraction of Two Test Results:*

5.2.1 This subsection applies when the equation for the calculated values uses the general formula:

$$\text{value} = x + y \quad (1)$$

where:

x = result from first test method, and
y = result from second test method.

5.2.2 The standard deviation of the calculated value can be estimated with the following equation:

$$\sigma_{x \pm y} = \sqrt{\sigma_x{}^2 + \sigma_y{}^2} \quad (2)$$

where:

$\sigma_{x \pm y}$ = standard deviation for calculated values based on either an addition or subtraction of test results,
σ_x = standard deviation for first test method, and
σ_y = standard deviation for second test method.

5.3 *Product of Two Test Results:*

5.3.1 This subsection applies when the equation for the calculated values uses the general formula:

$$\text{value} = x \times y \quad (3)$$

5.3.2 The standard deviation of the calculated value can be estimated with the following equation:

$$\sigma_{xy} = \sqrt{\bar{y}^2\sigma_x{}^2 + \bar{x}^2\sigma_y{}^2} \quad (4)$$

where:

σ_{xy} = standard deviation for calculated values based on the product of two other test results,
σ_x = standard deviation for first test method,
$\bar{x}$ = mean or average value for first test method,
σ_y = standard deviation for second test method, and
$\bar{y}$ = mean or average value for second test method.

5.4 *Quotient of Two Test Results:*

5.4.1 This subsection applies when the equation for the calculated values uses the general formula:

$$\text{value} = x / y \quad (5)$$

5.4.2 It is important to note that the location (that is, numerator or denominator) of the test method variables in this equation used to calculate the value for which the standard deviation is calculated is important.

5.4.3 The standard deviation of the calculated value can be estimated with the following equation:

$$\sigma_{\frac{x}{y}} = \sqrt{\frac{\bar{y}^2\sigma_x{}^2 + \bar{x}^2\sigma_y{}^2}{\bar{y}^4}} \quad (6)$$

where:

$\sigma_{x/y}$ = standard deviation for calculated values based on the quotient of two tests,
σ_x = standard deviation for the test method in the numerator,
$\bar{x}$ = mean or average value for the test method in the numerator,
σ_y = standard deviation for the test method in the denominator, and
$\bar{y}$ = mean or average value for the test method in the denominator.

5.5 *Multiplication and Division Using Three Test Results:*

5.5.1 This subsection applies when the equation for the calculated values uses the general formula:

$$\text{value} = xy / z \quad (7)$$

5.5.2 It is important to note that the location (that is, numerator or denominator) of the test method variables in this equation used to calculate the value for which the standard deviation is calculated is important.

5.5.3 The standard deviation of the calculated value can be estimated with the following equation:

$$\sigma_{xy/z} = \sqrt{\frac{x^2(y^2\ \sigma_z^2 + z^2\ \sigma_y^2) + y^2z^2\sigma_x^2}{z^4}} \quad (8)$$

where:

$\sigma_{xy/z}$ = standard deviation for calculated values based on the product of two test results divided by a third,
σ_x = standard deviation for the first test method in the numerator,
$\bar{x}$ = mean or average value for the first test method in the numerator,
σ_y = standard deviation for the second test method in the numerator,
$\bar{y}$ = mean or average value for the second test method in the numerator,
σ_z = standard deviation for the test method in the denominator, and
$\bar{z}$ = mean or average value for the test method in the denominator.

5.6 *Propagation of Errors:*

5.6.1 The general form for the variance (that is, standard deviation squared) of a calculated value, $\sigma^2{}_{\text{value}}$, is the sum of the square of the partial derivatives of the equation used to calculate the value:

$$\sigma^2_{value} = \left(\frac{\delta_{value}}{\delta_x}\right)^2 \sigma^2_x + \left(\frac{\delta_{value}}{\delta_y}\right)^2 \sigma^2_y + \left(\frac{\delta_{value}}{\delta_z}\right)^2 \sigma^2_z \ldots \quad (9)$$

5.6.2 Rather than work through the math, a web search for "error propagation calculators" yields a variety of readily available programs (widgets, see Note 1) that will perform the mathematics. Most programs allow the user to input the equation used to calculate the value, specify a mean value and standard deviation for each test result used in the equation, and return the calculated standard deviation.

5.6.3 The same search will also return a few results for programs that will return generate the actual standard deviation equation.

NOTE 1—A widget was developed by Mshelikoff and can be found at: https://www.wolframalpha.com/widgets/.

6. Keywords

6.1 precision limits; standard deviation

APPENDIX

(Nonmandatory Information)

X1. EXAMPLE OF CALCULATED PRECISION LIMITS

X1.1 VMA Standard Deviation

X1.1.1 The equation for calculating VMA is:

$$VMA = \left(100 - \frac{G_{mb} P_s}{G_{sb}}\right) \quad (X1.1)$$

where:

VMA = voids in mineral aggregate (that is, calculated value),
G_{mb} = bulk specific gravity of compacted specimen (Test Method D2726/D2726M, D1188/D1188M, or D6752/D6752M),
G_{sb} = aggregate bulk specific gravity (Test Methods C127 and C128), and
P_s = percent stone (that is, aggregate in mixture) and is calculated as (100 – P_b), where P_b is the percent asphalt binder (Test Method D2172/D2172M, D4125/D4125M, or D6307).

X1.1.2 While G_{sb} is obtained from testing the coarse and fine aggregate portions of various stockpiles at the beginning of the mix design phase, once the gradation is selected, the G_{sb} value becomes "fixed." That is, the aggregate specific gravity is usually treated as a constant for subsequent mix design and production calculations. Only the G_{mb} and P_s vary based on daily test results during production. Therefore, the appropriate form for the VMA calculation would appear to be the standard deviation equation for the product of two test results (5.3):

$$\sigma_{xy} = \sqrt{\bar{y}^2\sigma_x^2 + \bar{x}^2\sigma_y^2} \quad (X1.2)$$

X1.1.3 But in the VMA equation, G_{sb} does appear in the denominator. The nature of the partial derivatives needed for the propagation of errors results in the constant becoming a part of the resulting standard deviation equation. Therefore, 5.3 does not provide the correct format. The propagation of error mathematical process is needed to formulate the correct VMA standard deviation equation:

$$\sigma_{xy} = \frac{\sqrt{\bar{y}^2\sigma_x^2 + \bar{x}^2\sigma_y^2}}{G_{sb}} \quad (X1.3)$$

X1.1.4 Table X1.1 shows the mean values and the individual ASTM test method within-laboratory standard deviations. The mean values were obtained using quality control data. The project provided 77 sets of test results for compacted specific gravity, ignition oven asphalt binder content, and the calculated VMA value for each data set.

X1.1.5 The individual VMA values calculated and reported daily throughout the duration of the project were used to calculate the actual VMA standard deviation for project (n = 77). The equation, using the individual test method standard deviations and the mean project values estimates the VMA standard deviation as 0.164, which compares favorably to the standard deviation of individually calculated VMA values during production, which was 0.157.

TABLE X1.1 Mean Values

Mean Value	Within-Lab Standard Deviation	Test Method
G_{sb} = 2.686	Constant	
G_{mb} = 2.364	0.0035	D2726/D2726M
P_b = 4.61	0.122	D6307 (ignition oven)
P_s = 100 − P_b = 95.39	Use P_b standard deviation	

$$\sigma_{xy} = \frac{\sqrt{((2.364^2)(0.122^2) + (95.39^2)(0.0035^2))}}{2.686} = 0.164$$