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INTERNATIONAL STANDARD

NORME INTERNATIONALE

Fibre optic communication subsystem test procedures − Part 2-13: Digital systems − Measurement of error vector magnitude

Procédures d'essai des sous-systèmes de télécommunication fibroniques – Partie 2-13: Systèmes numériques – Mesure de l'amplitude du vecteur d'erreur

IEC 61280-2-13:2024-07(en-fr)

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FIBRE OPTIC COMMUNICATION SUBSYSTEM TEST PROCEDURES –

Part 2-13: Digital systems – Measurement of error vector magnitude

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The text of this International Standard is based on the following documents:

Draft	Report on voting
86C/1900/CDV	86C/1924/RVC

Full information on the voting for its approval can be found in the report on voting indicated in the above table.

The language used for the development of this International Standard is English.

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This document was drafted in accordance with ISO/IEC Directives, Part 2, and developed in accordance with ISO/IEC Directives, Part 1 and ISO/IEC Directives, IEC Supplement, available at www.iec.ch/members_experts/refdocs. The main document types developed by IEC are described in greater detail at www.iec.ch/publications.

A list of all parts of the IEC 61280 series, published under the general title *Fibre optic communication subsystem test procedures*, can be found on the IEC website.

The committee has decided that the contents of this document will remain unchanged until the stability date indicated on the IEC website under webstore.iec.ch in the data related to the specific document. At this date, the document will be

- reconfirmed,
- withdrawn, or
- revised.

INTRODUCTION

The error vector magnitude (EVM) is a single, real-valued parameter that characterizes the signal quality of n-state amplitude phase shift keyed (n-APSK) signals, which are also known as vector modulated signals. Similar to the Q-factor used for intensity-modulated directly-detected optical signals, it measures the average deviations of the transmitted signal states from their ideal values. These deviations can be caused by noise and by linear and nonlinear waveform distortions. The EVM is therefore a useful quantity to characterize the quality of transmitted source signals at the input of a transmission system or the quality of received signals at the output of a transmission system [1][1].

Despite the fact that the EVM is often reported by commercial optical modulation analysers, there are only a few standards that define a procedure for calculating the EVM of optical signals.

ITU-T Recommendation G.698.2 [2], for example, specifies a maximal EVM value for polarization-multiplexed 100 Gbit/s QPSK signals generated by an optical transmitter at the input of a DWDM transmission system. These recommendations provide detailed instructions for numerical signal processing steps that are to be performed on the received signal before the EVM is calculated. The steps include removal of undesired frequency and phase offsets, spectral filtering, DC offset removal, and even the addition of artificial noise to the signal.

Similarly, OIF Implementation Agreement OIF-400ZR-01.0 [3] describes a set of signal processing steps for determining the EVM in polarization-multiplexed 400 Gbit/s 16-QAM signals, which include the addition of artificial noise, but does not specify a maximal EVM value for the transmitted signals at the input of the transmission system.

The detailed signal processing steps defined in ITU-T G.698.2 and in OIF-400ZR-01.0 are specific to the particular modulation formats and to the applications considered in these documents. They are not applicable to arbitrary n-APSK signals or to other applications.

This document specifies a general procedure for calculating the EVM of optical n-APSK signals from a set of transmitted and properly received symbols. It does not specify any signal processing steps necessary to extract the symbols from the raw received signals or optional processing steps impacting the signal quality. This document rather defines the normalization of the reference states used in the EVM calculations as well as a procedure for proper scaling of the measured signal states. It is intended to serve as a reference for instrument vendors, transmission equipment manufacturers, and users of such instruments and transmission equipment.

The procedures described in this document apply to single-polarized optical signals as well as to conventional polarization-multiplexed signals with independently modulated polarization tributaries, which are often referred to as three-dimensionally (3-D) coded signals. In general, it is not advisable to apply these procedures without modifications to four-dimensionally (4-D) coded signals, in which optical amplitude, phase and polarization state are simultaneously modulated to encode the information data [4]. At the time of writing, procedures for calculating the EVM of 4-D coded signals were still under study.

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[1] Numbers in brackets refer to the Bibliography.

FIBRE OPTIC COMMUNICATION SUBSYSTEM TEST PROCEDURES –

Part 2-13: Digital systems – Measurement of error vector magnitude

1 Scope

This part of the IEC 61280-2 series defines a procedure for calculating the root-mean-square error vector magnitude of optical n-APSK signals from a set of measured symbols. It specifically defines the normalization of the reference states and a procedure for optimal scaling of the measured symbol states.

The procedure described in this document applies to single-polarized optical signals as well as to conventional polarization-multiplexed signals with independently modulated polarization tributaries. In general, it is not advisable to apply these procedures without modification to signals, in which optical amplitude, phase, and polarization state are simultaneously modulated to encode the information data.

This document does not specify any signal processing steps for extracting the symbols from the received optical signals, because these steps depend on the optical receiver and can vary with the type of the transmitted n-APSK signal. These and optional additional signal processing steps are defined in application-specific documents.

2 Normative references

There are no normative references in this document.

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

- IEC Electropedia: available at https://www.electropedia.org/
- ISO Online browsing platform: available at https://www.iso.org/obp

3.1
digital modulation
modulation of an optical sinusoidal carrier by a digital signal

Note 1 to entry: Digital modulation is generally an amplitude shift keying, a frequency shift keying, a phase shift keying or their combination.

[SOURCE: IEC 60050-713:1998, 713-07-12, modified – addition of "optical".]

3.2
binary (digital) signal
digital signal in which each signal element has one of two permitted discrete values

[SOURCE: IEC 60050-704:1993, 704-16-03]

3.3
n-ary (digital) signal

digital signal in which each signal element has one of n permitted discrete values

[SOURCE: IEC 60050-704:1993, 704-16-05]

3.4
n-state amplitude phase shift keying

n-APSK

digital modulation in which each element of a modulating signal is represented by one of n specified combinations of phase and amplitude of a sinusoidal oscillation

[SOURCE: IEC 60050-713:1998, 713-07-13, modified – Note 1 to entry deleted.]

3.5
quadrature phase shift keying

QPSK

quadrature phase modulation phase shift keying in which the phase shift takes four values that are multiples of 90°

[SOURCE: IEC 60050-702:2018, 702-06-43]

3.6
n-state quadrature amplitude modulation

n-QAM

an n-state amplitude phase shift keying which can be obtained by amplitude shift keying of two carriers in quadrature, the modulated signals being added

Note 1 to entry: In some cases, n is equal to 2^{2p}, where p is an integer, and the signal constellation points form a square (e.g. for square n-QAM).

[SOURCE: IEC 60050-713:1998, 713-07-14, modified – Note 1 to entry added.]

3.7
signal constellation (in digital modulation)

scatter of n points representing in an amplitude-phase diagram the modulated signal in n-state amplitude phase shift keying

Note 1 to entry: The signal constellation is often plotted in a two-dimensional IQ diagram, in which the two axes represent the in-phase and quadrature components of the amplitude phase shift keyed signals.

[SOURCE: IEC 60050-713:1998, 713-07-15, modified – Note 1 to entry added.]

3.8
input signal (of a transmission system)

transmitted source signal

signal applied to the input port of the sending terminal equipment of a transmission system

[SOURCE: IEC 60050-704:1993, 704-04-11]

3.9
reference signal (of a transmission system)

ideal undistorted version of the transmitted source signal

3.10

output signal (of a transmission system)
received source signal
signal emitted from an output port of the receiving terminal equipment of a transmission system

Note 1 to entry: Ideally, the output signal of a transmission system should be an undistorted version of the corresponding input signal.

[SOURCE: IEC 60050-704:1993, 704-04-12]

3.11

polarization multiplex transmission
polarization multiplexed transmission
method of transmission employing multiplexing of two orthogonally polarized signals at the input terminal of a transmission path and complementary demultiplexing at the output terminal

[SOURCE: IEC 60050-704:1992, 704-08-09, modified – "polarization" added to the term; "of two orthogonally polarized signals" added to the definition.]

3.12

decision circuit (for a digital signal)
circuit that decides the probable value of a signal element of a received digital signal

[SOURCE: IEC 60050-704:1992, 704-16-12]

3.13

symbol (in digital modulation)
one of the n states of the modulated signal in n-state amplitude phase shift keying

3.14

error vector magnitude
EVM
difference between the measured signal and a reference

Note 1 to entry: A reference is a perfectly modulated signal.

[SOURCE: ISO/IEC 24769-2:2013, 3.1.1]

3.14.1

RMS error vector magnitude
EVM$_{\text{rms}}$
E_{rms}
root-mean-square average of the error vector magnitudes of N symbols of an n-APSK signal

Note 1 to entry: The value of the RMS EVM is greater than zero and is usually expressed in percent.

4 Background and terminology

4.1 General

Clause 4 provides background information on the EVM calculations and defines the terminology used in this document.

The error vector magnitude (EVM) is a single, real-valued parameter that measures the average deviations of the various signal states in n-state amplitude phase shift keyed (n-APSK) signals from their ideal values. Its value is zero for an ideal n-APSK signal and larger than zero for real (i.e. distorted) n-APSK signals. The EVM is frequently expressed in percent.

Frequently, n-APSK signals are also referred to as vector modulated signals (see 4.2), because they can be represented as vectors in a two-dimensional constellation diagram, as described in 4.3. The average EVM of a transmitted signal is determined from a fairly large number of transmitted symbols (e.g. larger than 1 000) by first calculating the deviation of the transmitted state (i.e. the measured state) from its corresponding ideal state individually for each transmitted symbol, as described in 4.6, and then averaging these deviations as the root-mean-square of the individual deviations, as described in 4.7.

The resulting quantity is usually referred to as the root-mean-square EVM and abbreviated as RMS EVM or EVM_{rms}. The RMS EVM can be viewed as a generalization of the Q-factor, which is often used to characterize the quality of binary and n-ary intensity-modulated signals. In fact, RMS EVM and Q-factor are closely related, as described in Annex A.

Important elements of the EVM calculation are the normalization of the reference states, which is specified in 4.4, and the scaling of the measured states, which is specified in 4.5 and 4.8.

4.2 Vector modulated signals

In general, vector modulated signals are composed of an in-phase component, characterized by a time-varying amplitude $A_\text{I}(t)$, and a quadrature component, characterized by a time-varying amplitude $A_\text{Q}(t)$. Both components are modulated on the same optical carrier frequency, with the optical phase of the quadrature component being shifted by 90° relative to the in-phase component. Hence, the time-varying optical amplitude of vector modulated signals can be represented by a complex function $A_\text{c}(t)$, as shown in Formula (1).

$$A_\text{c}(t) = \sqrt{P_\text{S}}\,\left[A_\text{I}(t) + \text{j}A_\text{Q}(t)\right]\,\text{e}^{\,\text{j}\left[\omega_\text{s}t + \varphi_\text{s}(t)\right]} \tag{1}$$

where

P_S	is the average optical power of the signal;
$A_\text{I}(t)$	is the in-phase component of the modulated signal;
$A_\text{Q}(t)$	is the quadrature component of the modulated signal;
$\omega_\text{s} = 2\pi f_\text{s}$	is the angular frequency of the unmodulated optical signal (i.e. optical carrier);
$\varphi_\text{s}(t)$	represents additional optical phase variations.

NOTE 1 In Formula (1), the amplitudes $A_\text{I}(t)$ and $A_\text{Q}(t)$ are normalized so that the time average $\langle A_\text{I}^2(t) + A_\text{Q}^2(t)\rangle$ is equal to 1. This normalization is different from the one used for calculating the EVM.

Equivalently, $A_\text{c}(t)$ can be represented by a 2-dimensional vector $\mathbf{A}_\text{v}(t)$, as shown in Formula (2), where $A_\text{I}(t)$ and $A_\text{Q}(t)$ define the components of this vector.

$$\mathbf{A}_\text{v}(t) = \sqrt{P_\text{S}}\begin{bmatrix} A_\text{I}(t) \\ A_\text{Q}(t) \end{bmatrix}\text{e}^{\,\text{j}\left[\omega_\text{s}t + \varphi_\text{s}(t)\right]} \tag{2}$$

In quadrature phase shift keying (QPSK), for example, $A_\text{I}(t)$ and $A_\text{Q}(t)$ are independent binary amplitude modulated signals (whose symbol periods are properly synchronized), whereas in 16-state quadrature amplitude modulation (16-QAM), $A_\text{I}(t)$ and $A_\text{Q}(t)$ are both quaternary amplitude modulated signals.

NOTE 2 Vector modulated signals are often generated by two independent optical amplitude modulators (e.g. Mach-Zehnder modulators) that are connected in parallel to the same light source and operated in such a way that the optical phase in one of the modulators is delayed by 90° relative to that in the other modulator. More information on the generation and detection of vector modulated signals can be found in IEC TR 61282-16 [6].

4.3 Constellation diagram

The time varying signal components $A_\mathrm{I}(t)$ and $A_\mathrm{Q}(t)$ of a vector modulated signal can be plotted in a two-dimensional graph, according to Formula (2). Typically, the abscissa represents the in-phase component $A_\mathrm{I}(t)$ and the ordinate the quadrature component $A_\mathrm{Q}(t)$. In general, these plots display only one pair of amplitude values $\left[A_\mathrm{I}(t_k), A_\mathrm{Q}(t_k)\right]$ for each transmitted symbol, which corresponds to a two-dimensional state vector $\mathbf{S}(k)$, as shown in Formula (3).

$$\mathbf{S}(k) = \begin{bmatrix} A_\mathrm{I}(t_k) \\ A_\mathrm{Q}(t_k) \end{bmatrix} \tag{3}$$

where

k is an integer, with $k = 1, 2, 3, \ldots, N$.

The time t_k at which the amplitudes $A_\mathrm{I}(t)$ and $A_\mathrm{Q}(t)$ are sampled shall be chosen to best represent the state of the transmitted n-APSK symbol. However, no decision shall be made on the probable value of the transmitted symbol (i.e. the samples shall be taken prior to a decision circuit). Moreover, the signal amplitudes of all analysed symbols shall be sampled at the same position within each symbol period T_s, so that all sampling times are spaced by an integer multiple of T_s, as described by Formula (4).

$$t_k = k\,T_\mathrm{s} + \Delta t \tag{4}$$

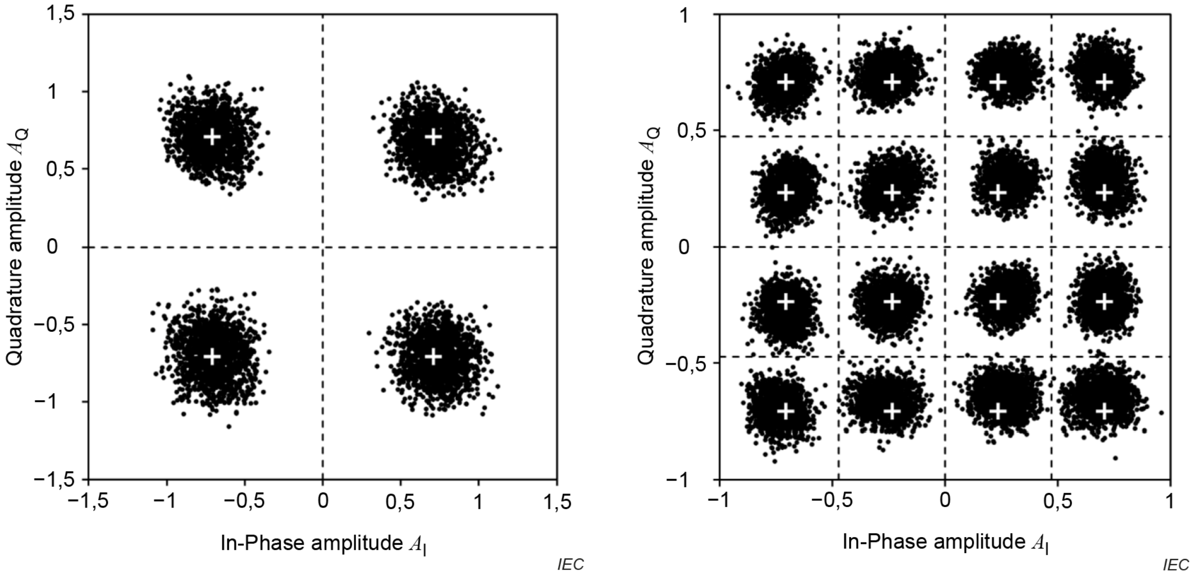
where

Δt is the time offset in each symbol period;

k is an integer, with $k = 1, 2, 3, \ldots, N$.

The scatter plot of the state vectors $\mathbf{S}(k)$ of a vector modulated signal is called a constellation diagram. Figure 1 displays the constellation diagrams of two widely used n-APSK signals: a transmitted QPSK signal and a transmitted square 16-QAM signal.

NOTE The signal amplitudes displayed in Figure 1 are scaled according to the procedures described in 4.5 and 4.8.



a) QPSK **b) Square 16-QAM**

Key

Solid black dots	Measured signal states (scaled as described in 4.5 and 4.8)
White crosses	Reference states (see 4.4)
Dashed lines	Midpoints between the in-phase and quadrature components of the reference states

Figure 1 – Constellation diagrams of measured QPSK and 16-QAM symbols

4.4 Normalization of the reference constellation

The signal constellation of an ideal n-APSK signal is represented by n different points in the constellation diagram, which correspond to n different reference vectors $\mathbf{R}(m)$, m = 1, 2, ..., n. The reference vectors shall be normalized so that the longest vector has unity length, as shown in Formula (5).

$$\max_{m=1,\dots,n}\left\{\left|\mathbf{R}(m)\right|\right\}=1 \qquad (5)$$

The reference states are unitless.

NOTE The reference states are sometimes normalized so that the average power of all possible reference states is equal to one, i.e. $\frac{1}{n}\sum_{i=1}^{n}\left|\mathbf{R}(i)\right|^{2}=1$. This normalization is often used in EVM calculations of electrical n-APSK signals [5]. For QPSK signals, it is identical to the normalization specified in 4.4, but for n-APSK signals of higher cardinality, like 16-QAM signals, the two normalizations lead to substantially different EVM values [1]. The normalization defined in this document is commonly used for optical signals and is identical to the one used in OIF-400ZR-01.0 [3].

4.5 Scaling of the measured vectors

The measured state vectors $\mathbf{S}(k)$ of the transmitted n-APSK signal in general are scaled differently than the reference vectors $\mathbf{R}(m)$. For a useful comparison between $\mathbf{S}(k)$ and $\mathbf{R}(m)$, rescale the measured states $\mathbf{S}(k)$, i.e. multiply with a common scale factor α. This scaling is impeded by the fact that the state vectors $\mathbf{S}(k)$ are typically scattered around the ideal constellation points and can even exhibit significant offsets from these points. Since the EVM characterizes the deviation of the measured state vectors $\mathbf{S}(k)$ from their ideal states $\mathbf{R}(m)$, the optimal scale factor α is the one that minimizes the average deviations of the scaled state vectors $\alpha\mathbf{S}(k)$ from the associated reference vectors $\mathbf{R}[m(k)]$, for all $k = 1, \ldots, N$. The calculation of the optimal scale factor is specified in 4.8 and 4.9.

The scaled state vectors are represented by $\mathbf{S}_\alpha(k) = \alpha\,\mathbf{S}(k)$, where α denotes the common scale factor.

4.6 Error vector magnitude of individual symbols

To calculate the EVM of the measured signal, each of the measured state vectors $\mathbf{S}_\alpha(k)$ is associated with a reference vector $\mathbf{R}(m)$ from the set of n possible states, ideally with the transmitted reference vector.

If the sequence of the transmitted reference states is known, which is the case when a well-defined test signal has been transmitted, the reference vectors can be determined from this sequence by properly correlating the received symbols with the transmitted states. If the transmitted states are unknown, which is often the case, each $\mathbf{S}_\alpha(k)$ is associated with the reference vector $\mathbf{R}(m)$ that is closest to $\mathbf{S}_\alpha(k)$ in the constellation diagram.

NOTE For signals with only a few possible states n (e.g. for QPSK signals), it is often straightforward to associate a measured state with the transmitted reference state. However, this association becomes increasingly more difficult with increasing number of states n, especially when the measured signal is noisy or otherwise distorted. Improperly associated states generally lead to an underestimation of the EVM. However, the number of improperly associated states is usually much smaller than that of properly associated states (e.g. because of the exponential distribution of noise), so that the impact on the RMS EVM can be disregarded in many cases.

The association of each $\mathbf{S}_\alpha(k)$ with a corresponding $\mathbf{R}(m)$ thus establishes a relationship between k and m, which can be described by a function $m(k)$, so that each measured state vector $\mathbf{S}(k)$ is associated with a reference vector $\mathbf{R}[m(k)]$.

The EVM of the k-th received symbol is given by the distance $D(k)$ between $\mathbf{S}_\alpha(k)$ and $\mathbf{R}[m(k)]$, as described by Formula (6) and illustrated in Figure 2.

$$D(k) = \left|\mathbf{S}_\alpha(k) - \mathbf{R}\left[m(k)\right]\right| \tag{6}$$