



Designation: E3076 – 18^{ε1}

Standard Practice for Determination of the Slope in the Linear Region of a Test Record¹

This standard is issued under the fixed designation E3076; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reappraisal. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reappraisal.

^{ε1} NOTE—Footnote 3 was editorially corrected in May 2023.

1. Scope

1.1 This practice presents an automated, objective linear-fitting method for determining the slope of the linear portion of a test record. The method assumes that there is a linear region early in the test record where the value of the y variable increases roughly in proportion to the x variable and the slope of the record decreases after the linear region. The practice determines the best linear fit to the data based on the least normalized residual and provides metrics for evaluating the quality of the test record and the quality of the resultant fit.

1.2 Data quality metrics are applied that evaluate the level of noise and the digital resolution of the data to determine if the test record is adequate for a linear regression analysis. Fit quality metrics use analysis of residuals in the vicinity of the fit range to determine if the test record is adequately linear and the fit range is sufficiently large.

1.3 For test records that meet the data and fit quality metrics, the practice determines a repeatable slope without the need for operator input that is independent of operator judgment. For test records that fail one or more of the quality metrics, it is recommended that the analyst evaluate the fit to determine if it is acceptable.

1.4 This practice represents a general purpose approach that is applicable for any test standard or method in which a linear fit is desired. It is intended that this practice can be called upon by standard test methods when slope must be determined.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.6 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the*

Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.

2. Referenced Documents

2.1 *ASTM Standards:*²

E1942 [Guide for Evaluating Data Acquisition Systems Used in Cyclic Fatigue and Fracture Mechanics Testing](#)

E2443 [Guide for Verifying Computer-Generated Test Results Through The Use Of Standard Data Sets](#)

2.2 *ASTM Data Set:*³

E3076-DS1(2018) [File 01 Benchmark Data Set to Evaluate Computer Implementation of the Algorithm in Standard E3076.](#)

3. Terminology

3.1 *Definitions:*

3.1.1 *digital resolution, n*—the precision of stored information resulting from a discrete digital representation of analog data.¹

3.1.2 *linear region, n*—a region of the test record where the underlying physics indicate that the dependent variable would increase in proportion to the independent variable if there were no noise in the test record.

3.1.3 *normalized, n*—data with global minima of 0 and global maxima of 1 in x and y .

3.1.4 *residual, n*—the difference between the linear fit and the original test record y -values at a given x -value.

3.1.5 *test record, n*—the basic raw data from a data set

4. Summary of Practice

4.1 This practice is intended for the analysis of test records where the y variable increases roughly in proportion to the x variable over a region early in the test record.

² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

³ This data set is available for download from ASTM at <https://www.astm.org/get-involved/technical-committees/adhoc-e08.html>

¹ This practice is under the jurisdiction of ASTM Committee E08 on Fatigue and Fracture and is the direct responsibility of Subcommittee E08.03 on Advanced Apparatus and Techniques.

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4.2 The linear-fitting algorithm utilizes analysis of residuals coupled with computational effort to numerically determine the most linear region in a given test record.⁴ The analysis of residuals methodology is more robust than conventional least-squares fitting techniques based on a correlation coefficient because the latter is not useful when the underlying physics indicate that the behavior is linear.

4.3 The test record is offset and normalized so that quality metrics can be applied that are independent of scale and engineering units.

4.4 Metrics on noise level and digital resolution in the normalized data set are applied to evaluate the quality of the test record.

4.5 The test record is evaluated to determine the most linear region using a tangency point approach to truncate the test record. This approach assumes that there is a linear region early in the test record and that the slope decreases after the linear region.

4.6 Analysis of residuals is used to evaluate the quality of the resulting fit.

4.7 The linear-fitting method presented in this practice is objective and fully automated for the case where the test record passes all quality metrics. If a test record fails one or more quality metrics, the analyst should examine the test record and the fit to determine whether it is acceptable based on the application or calling standard.

5. Significance and Use

5.1 It is often necessary to determine the slope of a linear region within a test record, and for standardization purposes, it

⁴ S. M. Graham and M. A. Adler, "Determining the Slope and Quality of Fit for the Linear Part of a Test Record," *J Testing and Evaluation*, v 39, n 2, pp. 260–268, March 2011.

is desirable to have a method for determining the slope that is not subjective. There are numerous ASTM standard test methods where the test procedure or analysis requires that slope be determined, but the procedures for doing so are not well defined. Ideally, if multiple laboratories analyze the same data for determination of slope, they should produce the same result. The objective of this standard practice is to eliminate the linear-fit as a source of variability in test results.

6. Procedure

6.1 The procedural description of this standard practice is assisted through demonstration of an example linear-fit made to a tensile test record. The test record is entered into arrays for force and extension where the dependent variable (y) is force and the independent variable (x) is extension, and the indices start from 1.

6.1.1 This test record is designated as File 01 and is available from ASTM International.³ The test record is plotted in Fig. 1. See Guide E2443.

6.2 Limit the range of the test record that will be searched for the most linear region by determining if the test record has a local maximum or a knee above which the slope shows a significant decrease. This behavior is common in many types of materials testing. A tangent construction is used to find the local maximum or knee of the record and establish the upper end of the data used in the search. The process is illustrated in Fig. 2 and described in the following sub-sections.

6.2.1 Shift the test record such that the starting x and y values are zeroed by subtracting the x -value for the first point ($xshift$) from the x -values for all points in the test record and subtracting the y -value for the first point ($yshift$) from the y -values for all points in the test record. Record the $xshift$ and $yshift$ values so that the final intercept and fit range from linear regression can be corrected. For this example, $xshift = 5.792E-5$ and $yshift = 0.1914$.

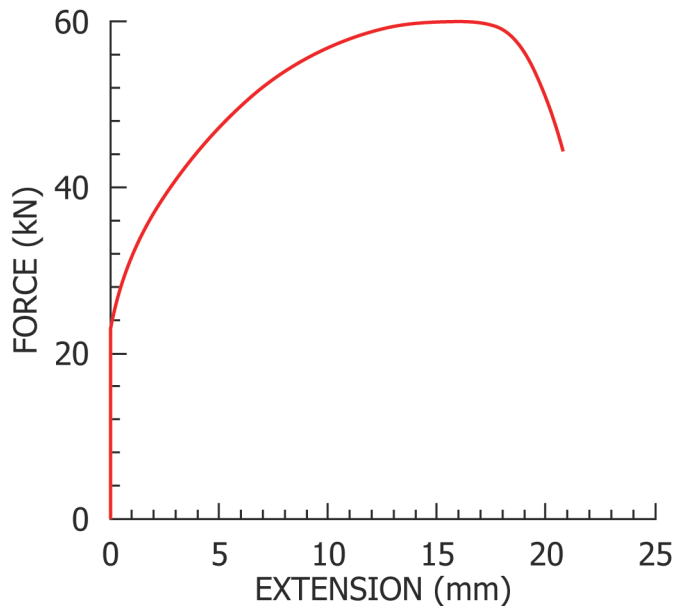


FIG. 1 Plot of Original Test Record File 01

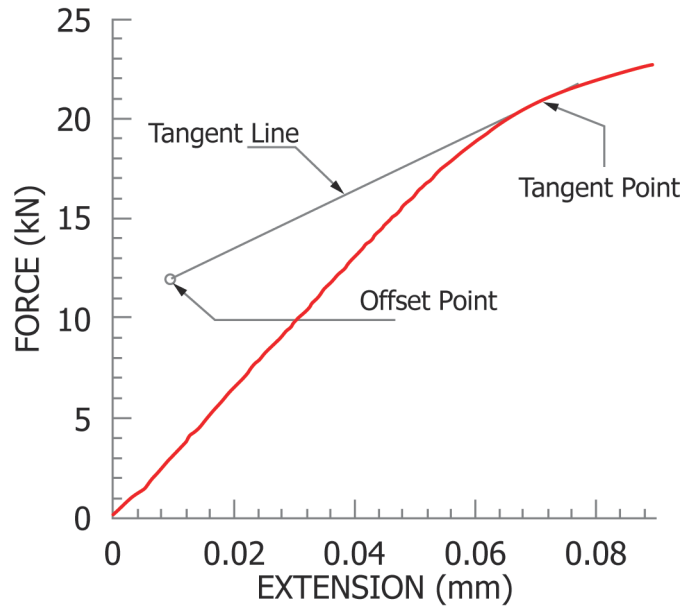


FIG. 2 Tangent Construction Used to Truncate Data Set

6.2.2 Find the point in the *shifted test record* where the *y*-value just exceeds 5 % of the maximum *y*-value and designate the *x* and *y* values (x_I, y_I). For this example, $x_I = 0.01016$ and $y_I = 3.012$.

6.2.3 Create an offset point with the same *x*-value and a *y*-value equal to y_I plus 15 % of the maximum *y*-value, (x_{offset}, y_{offset}) = ($x_I, y_I + 0.15 * \max(y)$). For this example, $y_{offset} = 11.98$.

6.2.4 Starting with the first point where the *y*-value is greater than y_{offset} , find the point in the test record where the line from that point to the offset point has the largest slope and record this as the tangent point ($x_{tangent}, y_{tangent}$). For this

example, the tangency point has an index of 220 with coordinates (0.06989, 20.60).

6.3 Truncate all data beyond the tangent point, and normalize all *y*-values by dividing by the tangency-point *y*-value, and normalize all *x*-values by dividing by the tangency point *x*-value. The resultant data is now normalized and has a range from 0 to 1 in both *x* and *y*, as shown in Fig. 3. This facilitates the development of fit criteria that are independent of the data maxima and the units of measure.

6.4 *First Data Quality Metric*—Check for excessive noise in the data, which is the first metric on data quality. The allowable

<https://standards.iteh.ai/catalog/standards/sist/d0c53239-7474-4b2d-ab0b-6366f0b4508e/astm-e3076-18e1>

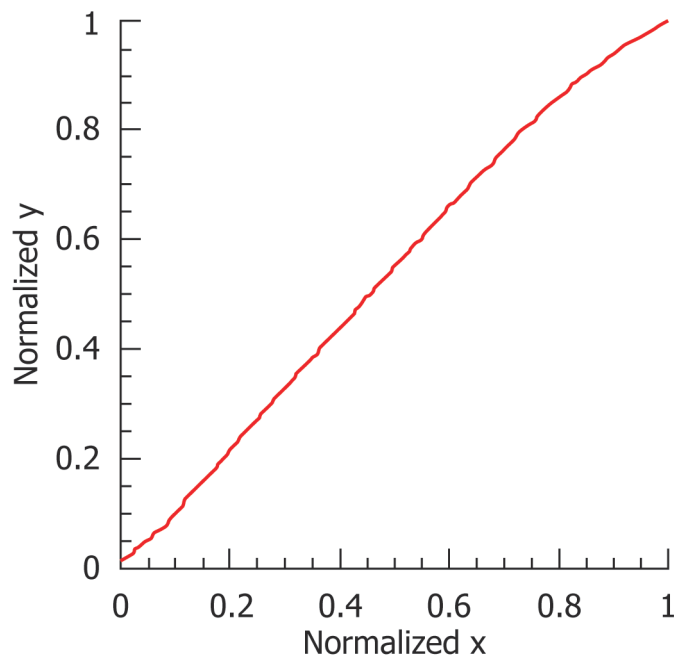


FIG. 3 Shifted, Truncated and Normalized Test Record