Designation: D2244 – 23

Standard Practice for
Calculation of Color Tolerances and Color Differences from Instrumentally Measured Color Coordinates[1]

This standard is issued under the fixed designation D2244; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

This standard has been approved for use by agencies of the U.S. Department of Defense.

INTRODUCTION

This practice originally resulted from the consolidation of a number of separately published methods for the instrumental evaluation of color differences. As revised in 1979, it included four color spaces in which color-scale values could be measured by instruments, many of which were obsolete, and the color differences calculated by ten equations for different color scales. The sections on apparatus, calibration standards and methods, and measurement procedures served little purpose in the light of modern color-measurement technology. The revision published in 1993 omitted these sections, and limited the color spaces and color-difference equations considered, to the three most widely used in the paint and related coatings industry. A previous revision added two new color tolerance equations and put one of the color difference equations from the 1993 version in an informative appendix for historical purposes.

1. Scope

1.1 This practice covers the calculation, from instrumentally measured color coordinates based on daylight illumination, of color tolerances and small color differences between opaque specimens such as painted panels, plastic plaques, or textile swatches. Where it is suspected that the specimens may be metameric, that is, possess different spectral curves though visually alike in color, Practice D4086 should be used to verify instrumental results. The tolerances and differences determined by these procedures are expressed in terms of approximately uniform visual color perception in CIE 1976 CIELAB opponent-color space (**1**),[2] CMC tolerance units (**2**), CIE94 tolerance units (**3**), the DIN99o color difference formula given in DIN 6176 (**4**), or the CIEDE2000 color difference units (**5**).

1.2 For product specification, the purchaser and the seller shall agree upon the permissible color tolerance between test specimen and reference and the procedure for calculating the color tolerance. Each material and condition of use may require specific color tolerances because other appearance factors, (for example, specimen proximity, gloss, and texture), may affect the correlation between the magnitude of a measured color difference and its commercial acceptability.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.4 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[3]

D1729 Practice for Visual Appraisal of Colors and Color Differences of Diffusely-Illuminated Opaque Materials
D4086 Practice for Visual Evaluation of Metamerism
E284 Terminology of Appearance
E308 Practice for Computing the Colors of Objects by Using the CIE System
E805 Practice for Identification of Instrumental Methods of

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[1] This practice is under the jurisdiction of ASTM Committee E12 on Color and Appearance and is the direct responsibility of Subcommittee E12.04 on Color and Appearance Analysis.

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[2] The boldface numbers in parentheses refer to the list of references at the end of this standard.

[3] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

Color or Color-Difference Measurement of Materials

E1164 Practice for Obtaining Spectrometric Data for Object-Color Evaluation

2.2 *Other Standards:*

DIN 6176 English translation of DIN 6176:2018-10 Farbmetrische Bestimmung von Farbabstanden bei Korperfarben nach der DIN 99o-Formel, (Colorimetric determination of colour differences of object colours according to the DIN 99o formula)[4]

3. Terminology

3.1 Terms and definitions in Terminology E284 are applicable to this practice.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *colorimetric spectrometer, n*—spectrometer, one component of which is a dispersive element (such as a prism, grating or interference filter or wedge or tunable or discrete series of monochromatic sources), that is normally capable of producing as output colorimetric data (such as tristimulus values and derived color coordinates or indices of appearance attributes). Additionally, the colorimetric spectrometer may also be able to report the underlying spectral data from which the colorimetric data were derived.

3.2.2 *color tolerance equation, n*—a mathematical expression, derived from acceptability judgments, which distorts the metric of color space based on the coordinates in that color space, of a reference color, for the purpose of single number shade passing.

3.2.2.1 *Discussion*—The color tolerance equation computes a pass/fail value based on which of the pair of specimens is assigned the designation "standard." Thus, inter-changing the reference and test specimens will result in a change in the predicted level of acceptance between the specimens while the perceived difference is unchanged. A color difference equation quantifies distance in a color space using the metric of that space. Inter-changing the reference and test specimens does not change either the perceived or predicted color differences.

4. Summary of Practice

4.1 The differences in color between a reference and a test specimen are determined from measurements made by use of a spectral based or filter based colorimeter. Reflectance readings from spectral instruments are converted by computations to color-scale values in accordance with Practice E308, or these color-scale values may be read directly from instruments that automatically make the computations. Color-difference units are computed, from these color-scale values, and approximate the perceived color differences between the reference and the test specimen.

5. Significance and Use

5.1 The original CIE color scales based on tristimulus values X, Y, Z and chromaticity coordinates x, y are not uniform visually. Each subsequent color scale based on CIE values has had weighting factors applied to provide some degree of uniformity so that color differences in various regions of color space will be more nearly comparable. On the other hand, color differences obtained for the same specimens evaluated in different color-scale systems are not likely to be identical. To avoid confusion, color differences among specimens or the associated tolerances should be compared only when they are obtained for the same color-scale system. There is no simple factor that can be used to convert accurately color differences or color tolerances in one system to difference or tolerance units in another system for all colors of specimens.

5.2 Color differences calculated in ΔE_{00} units (**6**) are highly recommended for use with color-differences in the range of 0.0 to 5.0 ΔE^*_{ab} units. This color-difference equation is appropriate for and widely used in industrial and commercial applications including, but not limited to, automobiles, coatings, cosmetics, inks, packaging, paints, plastics, printing, security, and textiles.

5.3 Users of color tolerance equations have found that, in each system, summation of three, vector color-difference components into a single scalar value is very useful for determining whether a specimen color is within a specified tolerance from a standard. However, for control of color in production, it may be necessary to know not only the magnitude of the departure from standard but also the direction of this departure. It is possible to include information on the direction of a small color difference by listing the three instrumentally determined components of the color difference.

5.4 Selection of color tolerances based on instrumental values should be carefully correlated with a visual appraisal of the acceptability of differences in hue, lightness, and saturation obtained by using Practice D1729. The three tolerance equations given here have been tested extensively against such data for textiles and plastics and have been shown to agree with the visual evaluations to within the experimental uncertainty of the visual judgments. That implies that the equations themselves misclassify a color difference with a frequency no greater than that of the most experienced visual color matcher.

5.5 While color difference equations and color tolerance equations are routinely applied to a wide range of illuminants, they have been derived or optimized, or both, for use under daylight illumination. Good correlation with the visual judgments may not be obtained when the calculations are made with other illuminants. Use of a tolerance equation for other than daylight conditions will require visual confirmation of the level of metamerism in accordance with Practice D4086.

6. Description of Color-Difference and Color-Tolerance Equations

6.1 *CIE 1931 and 1964 Color Spaces*—The daylight colors of opaque specimens are represented by points in a space formed by three rectangular axes representing the lightness scale Y and chromaticity scales x and y, where:

$$x = \frac{X}{X+Y+Z} \quad (1)$$

$$y = \frac{Y}{X+Y+Z} \quad (2)$$

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[4] Available from TechStreet.com and Beuth Verlag GmbH, 10772, Berlin, Germany, http://www.beuth.de.

where X, Y, and Z are tristimulus values for either the 1931 CIE standard observer (2° observer) or the 1964 CIE standard observer (10° observer) and standard illuminant D_{65}, or other phase of daylight. These scales do not provide a perceptually uniform color space. Consequently, color differences are seldom if ever computed directly from differences in x, y, and Y.

6.2 *CIE 1976 L* a* b* Uniform Color Space and Color-Difference Equation* (**1, 7**)—This is an approximately uniform color space based on nonlinear expansion of the tristimulus values and taking differences to produce three opponent axes that approximate the percepts of lightness-darkness, redness-greenness and yellowness-blueness. It is produced by plotting in rectangular coordinates the quantities L^*, a^*, b^*, calculated as follows:

$$L^* = 116 f(Q_Y) - 16 \quad (3)$$

$$a^* = 500 [f(Q_X) - f(Q_Y)] \quad (4)$$

$$b^* = 200 [f(Q_Y) - f(Q_Z)] \quad (5)$$

where

$$Q_X = (X/X_n);\ Q_Y = (Y/Y_n);\ Q_Z = (Z/Z_n)$$

and

$$f(Q_i) = Q_i^{1/3} \quad if \quad Q_i > (6/29)^3$$

else

$$f(Q_i) = (841/108)Q_i + 4/29 \quad if \quad Q_i \leq (6/29)^3.$$

Here, i varies as X, Y, and Z.

The tristimulus values X_n, Y_n, Z_n define the color of the nominally white object-color stimulus. Usually, the white object-color stimulus is given by the spectral radiant power of one of the CIE standard illuminants, for example, C, D_{65} or another phase of daylight, reflected into the observer's eye by the perfect reflecting diffuser. Under these conditions, X_n, Y_n, Z_n are the tristimulus values of the standard illuminant with Y_n equal to 100.

6.2.1 The total color-difference ΔE_{ab}^* between two colors each given in terms of L^*, a^*, b^* is calculated as follows:

$$\Delta E^*_{ab} = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2} \quad (6)$$

NOTE 1—The color space defined above is called the CIE 1976 L^* a^* b^* space and the color-difference equation the CIE 1976 L^* a^* b^* color-difference formula. The abbreviation CIELAB (with all letters capitalized) is recommended.

6.2.2 The magnitude, ΔE^*_{ab}, gives no indication of the character of the difference since it does not indicate the relative quantity and direction of hue, chroma, and lightness differences.

6.2.3 The direction of the color difference is described by the magnitude and algebraic signs of the components ΔL^*, Δa^*, and Δb^*:

$$\Delta L^* = L^*_B - L^*_S \quad (7)$$

$$\Delta a^* = a^*_B - a^*_S \quad (8)$$

$$\Delta b^* = b^*_B - b^*_S \quad (9)$$

where L^*_S, a^*_S, and b^*_S refer to the reference or standard, and L^*_B, a^*_B , and b^*_B refer to the test specimen or batch. The signs of the components ΔL^*, Δa^*, and Δb^* have the following approximate meanings (**8**):

$$+\Delta L^* = \text{lighter} \quad (10)$$

$$-\Delta L^* = \text{darker} \quad (11)$$

$$+\Delta a^* = \text{redder (less green)} \quad (12)$$

$$-\Delta a^* = \text{greener (less red)} \quad (13)$$

$$+\Delta b^* = \text{yellow (less blue)} \quad (14)$$

$$-\Delta b^* = \text{bluer (less yellow)} \quad (15)$$

6.2.4 For judging the direction of the color difference between two colors, it is useful to calculate hue angles h_{ab} and CIE 1976 metric chroma C^*_{ab} according to the following pseudocode:

```
if $b^* = 0$ then                                                    (16)
    $h_{ab} = 90\, sign(a^*) [sign(a^*) - 1]$
else
    $h_{ab} = 180 - (180/\pi) arctan(a^*/b^*) - 90\, sign(b^*)$
end if.
```

Here *sign* is a function that returns the sign of the argument, and *arctan* is the inverse tangent function returning angles in units of radians. The units of h_{ab} calculated by the above are degrees counter-clockwise from the positive a^* axis. The function *sign* is expected to return a minus one for negative values of the argument, a zero when the argument is zero, and a positive one for positive values of the argument.

Differences in hue angle h_{ab} between the test specimen and reference can be correlated with differences in their visually perceived hue, except for very dark colors (**9**). Differences in chroma $\Delta C^*_{ab} = ([C^*_{ab}]_{batch} - [C^*_{ab}]_{standard})$ can similarly be correlated with differences in visually perceived chroma.

$$C^*_{ab} = \sqrt{(a^*)^2 + (b^*)^2} \quad (17)$$

6.2.5 For judging the relative contributions of lightness differences, chroma differences, and hue differences between two colors, it is useful to calculate the CIE 1976 Metric Hue Difference ΔH^*_{ab} between the colors as follows:

$$\Delta H^*_{ab} = s [2(C^*_{ab,B}\, C^*_{ab,S} - a^*_B\, a^*_S - b^*_B\, b^*_S)]^{0.5} \quad (18)$$

where

```
if $a^*_S\, b^*_B > a^*_B\, b^*_S$ then                                (19)
        $s = 1$
else
        $s = -1$
end if.
```

When ΔL^*_{ab} is calculated as in 6.2.3 and ΔC^*_{ab} is calculated as in 6.2.4, then

$$\Delta E^*_{ab} = [(\Delta L^*)^2 + (\Delta C^*)^2 + (\Delta H^*)^2]^{0.5} \quad (20)$$

contains terms showing the relative contributions of lightness differences ΔL^*_{ab}, chroma differences ΔC^*_{ab}, and hue differences ΔH^*_{ab}.

6.3 *CMC Color Tolerance Equation*—The Colour Measurement Committee of the Society of Dyers and Colourists undertook a task to improve upon the results of the JPC79 tolerance equation (**2**) developed at J & P Coats thread company in the United Kingdom. It was a combination of the

CIELAB equation and local optimization based on the position of the standard used to derive the FMC-2 equation. It was based on the more intuitive perceptual variables of lightness, chroma and hue instead of the lightness, redness/greenness and yellowness/blueness of the older equation. It is intended to be used as a single-number shade-passing equation. There should not be a need to break the equation down into perceptual components—the CIELAB components of the model do that already. Fig. 1(**10**) shows the CIELAB chromaticness plane (a^*, b^*) with a large number of CMC ellipsoids plotted on that plane. The figure clearly shows the change in area of the ellipses with increases in CIELAB metric chroma C^*_{ab} and with respect to changes in CIELAB metric hue angle h^*_{ab}. The CMC components and single number tolerances are computed as follows:

$$\Delta E_{CMC}(l{:}c) = \sqrt{\left(\frac{\Delta L^*}{l \cdot S_L}\right)^2 + \left(\frac{\Delta C^*}{c \cdot S_c}\right)^2 + \left(\frac{\Delta H^*}{S_H}\right)^2} \qquad (21)$$

The most common values for the lightness to chroma ratio *l:c* is (2:1) for textiles and plastics that are molded to simulate a woven material, implying that lightness differences carry half the importance of chroma and hue differences (**11**). The values (1:1), often assumed to represent a just perceptible difference, should be applied to materials that require very critical tolerances or have glossy surfaces. For specimens that are matte, randomly rough, or mildly textured, values intermediate between (1:1) and (2:1) can be used, with the value (1.3:1) being reported most frequently.

The color dependent functions are defined as:

$$S_L = \frac{0.040975 \cdot L^*}{(1 + 0.01765 \cdot L^*)} \qquad \text{for} \quad L^* \geq 16 \qquad (22)$$

$$S_L = 0.511, \qquad \text{for} \quad L^* < 16$$

$$S_C = \frac{0.0638 \cdot C^*}{(1 + 0.0131 \cdot C^*)} + 0.638$$

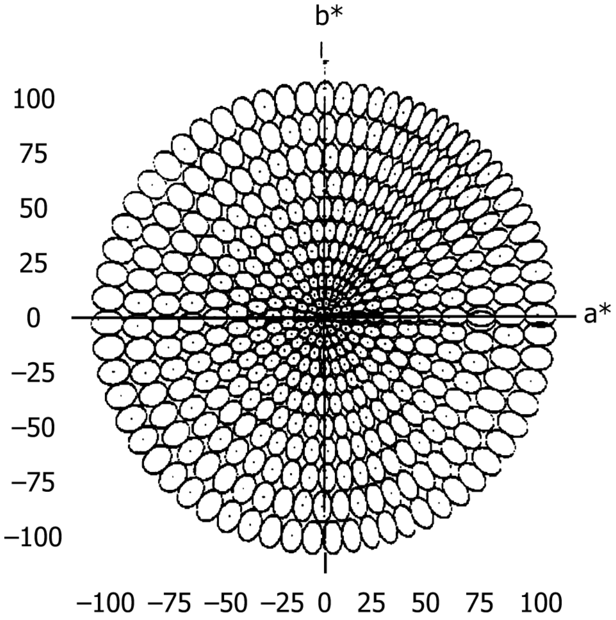

FIG. 1 CMC Ellipse Distribution in the CIELAB (*a, *b**) Plane**

$$S_H = S_C(T \cdot f + 1 - f)$$

where

$$f = \left\{\frac{(C^*)^4}{(C^*)^4 + 1900}\right\}^{\frac{1}{2}}$$

$$T = 0.56 + \left|0.2\cos(h + 168°)\right|, \qquad \text{if } 164° < h < 345°$$

else

$$T = 0.36 + \left|0.4\cos(h + 35°)\right|$$

All angles are given in degrees but will generally need to be converted to radians for processing on a digital computer. In Eq 22, the values of L^*, C^*, and h are taken to be those of the standard specimen.

The use of a commercial factor *cf* is no longer recommended. See Appendix X3.

6.4 *CIE94 Color Tolerance Equation* (**3**)—The development of this color tolerance equation was prompted by the success of the CMC tolerance equation. It was derived primarily from visual observations of automotive paints on steel panels. Like the CMC equation, it is based on the CIELAB color metric and uses the position of the standard in CIELAB color space to derive a set of analytical functions that modify the spacing of the CIELAB space in the region around the standard. Its weighting functions are much simpler than those of the CMC equation. CIE94 tolerances are computed as follows:

$$\Delta E^*_{94} = \left[\left(\frac{\Delta L^*}{k_L S_L}\right)^2 + \left(\frac{\Delta C^*}{k_C S_C}\right)^2 + \left(\frac{\Delta H^*}{k_H S_H}\right)^2\right]^{0.5}$$

Unlike many previous color difference equations, CIE94 comes with a well defined set of conditions under which the equation will provide optimum results and departures from this set of conditions will cause the agreement between the visually evaluated color-difference and the computed color-difference to be significantly poorer. Those conditions are given in Table 1. The parameters k_L, k_C, k_H are the parametric factors that can be used to compensate for texture and other specimen presentation effects. These should not be used to introduce a commercial factor into the equation. For more information on the use of commercial factors in color tolerance equations, see Appendix X3. All the k values default to 1 in the absence of specific information or agreement between parties. The parameters S_L, S_C, S_H are used to perform the local distortion of CIELAB color space, again based on the position of the standard specimen in that space. They are computed using the following equations:

TABLE 1 Basis Conditions for CIE94 Tolerance Equation

Attribute	Requirement
Illumination	D65 source
Specimen Illuminance	1000 lx
Observer	Normal color vision
Background	Uniform neutral gray $L^* = 50$
Viewing Mode	Object
Sample Size	>4° subtended visual angle
Sample Separation	Minimum possible
Size of Color Differences	0 to 5 CIELAB units
Sample Structure	Visually homogenous

$$S_L = 1 \tag{24}$$

$$S_C = 1 + 0.045 \cdot C^*$$

$$S_H = 1 + 0.015 \cdot C^*$$

In Eq 24, the value of C^* is taken to be that of the standard specimen.

6.5 *DIN99o Color Difference Equation*—The publication in 1996 of the paper by Rohner and Rich (**4**) prompted the German standards institute (DIN) to further develop and standardize a modified version as a new color difference formula that globally modeled color space using logarithms of the CIELAB coordinates rather than the linear and hyperbolic functions of CMC and CIE94. The equations derived and documented in standard DIN 6176 provides an axes rotation and the logarithmic expansion of the axes of the CIELAB formula to match perceptual color distance tolerances of a large number of color data sets, that have also been used to optimize the CIE94 and CIEDE2000 formula (**6**). With the advantage of using Euclidean distances in the new and expanded color space, the results of describing differences of color are even better for small color distances if compared to CIEDE2000 results (**6**), save the direct surrounding of the grey axes. Also, as neither the tristimulus values *XYZ* nor the CIELAB axes a^*, b^* are perceptual variables while the axes L^*, C^* and h^* are correlates of the perceptions of lightness, chroma, and hue, it seemed appropriate to scale the differences or distances in color space following the Weber-Fechner law of perception. In 2017, the coefficients of the original DIN99 were improved, resulting in the now recommended formulas DIN99o. The new coefficients resulted in a formula which is easy to use and has equivalent performance to CMC or CIE94, and computed color differences are based only on the Euclidean distance in the DIN99o space. The procedures for computing the DIN99o formula are listed as follows:

DIN reference condition parameters:

$$k_{CH} = k_E = 1 \tag{25}$$

DIN99o lightness:

$$L_{99o} = 303.67 \ \ln(1.0 + 0.0039\ L^*)/k_E \tag{26}$$

Auxiliary variable for redness:

$$e_o = a^* \cos(26\ °) + b^* \sin(26\ °) \tag{27}$$

Auxiliary variable for yellowness:

$$f_o = -0.83\ a^*\ \sin(26\ °) + 0.83\ b^*\ \cos(26\ °) \tag{28}$$

Auxiliary variable for chroma:

$$G_{99o} = (e_o{}^2 + f_o{}^2)^{0.5} \tag{29}$$

Auxiliary variable for hue angle: (in radians)

$$h_{eofo} = 0 \tag{30}$$

for $e_o = 0$ and $f_o = 0$

$$h_{eofo} = \arctan(f_o / e_o) \tag{31}$$

for $e_o > 0$ and $f_o \geq 0$

$$h_{eofo} = \pi/2 \tag{32}$$

for $e_o = 0$ and $f_o > 0$

$$h_{eofo} = \pi + \arctan(f_o / e_o) \tag{33}$$

for $e_o < 0$

$$h_{eofo} = 3\pi/2 \tag{34}$$

for $e_o = 0$ and $f_o < 0$

$$h_{eofo} = 2\pi + \arctan(f_o / e_o) \tag{35}$$

for $e_o > 0$ and $f_o < 0$

DIN hue angle: (in degrees)

$$h_{99o} = h_{eofo} 180°/\pi + 26° \tag{36}$$

for $h_{eofo} < 334°\pi/180°$

$$h_{99o} = (h_{eofo} - 2\ \pi) 180°/\pi + 26° \tag{37}$$

for $h_{eofo} \geq 334°\pi/180°$

DIN99o chroma:

$$C_{99o} = [\ln\ (1 + 0.075 G_o)]/(0.0435 k_{CH}\ k_E) \tag{38}$$

DIN99o redness-greenness:

$$a_{99o} = C_{99o} \cos(h_{99o}\ \pi / 180\ °) \tag{39}$$

DIN99o yellowness-blueness:

$$b_{99o} = C_{99o} \sin(h_{99o}\ \pi / 180\ °) \tag{40}$$

DIN99o lightness difference:

$$\Delta L_{99o} = L_{99oB} - L_{99oS} \tag{41}$$

DIN99o red-green difference:

$$\Delta a_{99o} = a_{99oB} - a_{99oS} \tag{42}$$

DIN99o yellow-blue difference:

$$\Delta b_{99o} = b_{99oB} - b_{99oS} \tag{43}$$

DIN99o color difference:

$$\Delta E_{99o} = [(\Delta L_{99o})^2 + (\Delta a_{99o})^2 + (\Delta b_{99o})^2]^{0.5} \tag{44}$$

or equivalently

DIN99o chroma difference:

$$\Delta C_{99o} = C_{99oB} - C_{99oS} \tag{45}$$

DIN99o hue difference:

$$\Delta H_{99o} = \frac{-(a_{99oB}\ b_{99oS} - a_{99oS}\ b_{99oB})}{[0.5\ (C_{99oB}\ C_{99oS} + a_{99oB}\ a_{99oS} + b_{99oB}\ b_{99oS})]^{0.5}} \tag{46}$$

DIN99o color difference:

$$\Delta E_{99o} = [(\Delta L_{99o})^2 + (\Delta C_{99o})^2 + (\Delta H_{99o})^2]^{0.5} \tag{47}$$

where subscript S refers to the product standard and subscript B refers to the current product batch or test sample. Here *arctan* is the inverse tangent function returning the function value into angles in units of radians. The units of h_{99o} calculated by the above are degrees counterclockwise for the range from 0 to 360 from the positive e_o axis.

The values of the parameters k_E and k_{CH} for most applications will be unity. These factors refer to a reference set of viewing conditions that are applicable to most color-difference industrial viewing practices and are essentially those of Table 1 of this standard. Changes in viewing conditions or changes in materials, such as textiles, can be taken into account by the appropriate selection of k_E and k_{CH} values. Values of k_E and

k_{CH} other than unity lead to changes in both color-difference and color coordinates. If it is intended to use *k* factors other than unity, their values shall be agreed upon and indicated in the measurement report.

6.6 *CIEDE2000 Color Difference Equation* (5)—The development of this color difference equation grew out of the research being performed to try to determine which of the two color tolerances equations, CMC or CIE94, was the better formula. In the process, the researchers came to the conclusion that neither formula was truly optimum. Therefore the CIE set up a new technical committee, TC 1-47, Hue & Lightness Dependant Correction to Industrial Colour Difference Equations, to recommend a new equation that addresses the short-comings in both color tolerance equations. One of the major weaknesses of the color tolerance equations was using the position of the reference color in CIELAB color space for computing the local distortion of CIELAB color space. When the identifications of the two specimens are reversed (calling the original test specimen the reference and the original reference now the test specimen) the computation results in a different computed color difference. This is contrary to what is observed. Visually, there is no change in the magnitude of the difference between the specimens simply by switching roles. By using the position of the arithmetic average color between the two specimens to compute the local distortions to CIELAB color space, the roles of the two specimens may be switched without changing the magnitude of the computed color-difference, in full agreement with the visual assessments. The report from CIE TC 1-47 has shown that CIEDE2000 outperforms both CMC and CIE94 across a wide array of specimens. The CIEDE2000 color differences are computed from the following equations:

$$L' = L^* \qquad a' = (1+G)\cdot a^* \qquad b' = b^* \tag{48}$$

$$C' = \sqrt{a'^2 + b'^2}$$

```
if $b' = 0$ then
    $h' = 90\, sign(a')\, [sign(a') - 1]$
else
    $h' = 180 - (180/\pi) arctan(a'/b') - 90\, sign(b')$
end if.
```

Here *sign* and *arctan* are functions that are defined in and are expected to return values as stated in 6.2.4.

$$G = 0.5\cdot\left(1 - \sqrt{\frac{\overline{C^*}^{\,7}}{\overline{C^*}^{\,7} + 25^7}}\right)$$

where $\overline{C^*}$ is the arithmetic mean of the CIELAB C^* values for the pair of specimens (standard and batch).

$$\Delta L' = L'_B - L'_S$$

$$\Delta C' = C'_B - C'_S$$

$$\Delta H' = s\left[2\left(C'_B C'_S - a'_B a'_S - b'_B b'_S\right)\right]^{0.5}$$

where

$$s = 1 \quad if \;\; a'_S b'_B > a'_B b'_S,\; else\; s = -1.$$

$$\Delta E_{00}^{\;2} = \left(\frac{\Delta L'}{k_L\cdot S_L}\right)^2 + \left(\frac{\Delta C'}{k_C\cdot S_C}\right)^2 + \left(\frac{\Delta H'}{k_H\cdot S_H}\right)^2 + R_T\cdot\left(\frac{\Delta C'\cdot\Delta H'}{k_C\cdot S_C\cdot k_H\cdot S_H}\right)$$

$$\Delta\, E_{00} = \sqrt{\Delta E_{00}^{\;2}}$$

The specimen or industry dependent parameters are k_L, k_C, k_H (all defaulting to unity in the absence of specific information or agreement between parties). S_L, S_C, S_H and R_T. The three *S* terms operate on the, assumed orthogonal, CIELAB coordinates and the R_T term computes a rotation of the color difference volume in the blue and purple-blue regions of the CIELAB diagram. The four color space terms are computed as follows:

$$S_L = 1 + \frac{0.015\cdot\left(\overline{L'} - 50\right)^2}{\sqrt{20 + \left(\overline{L'} - 50\right)^2}}$$

$$S_C = 1 + 0.045\cdot\overline{C'}$$

$$S_H = 1 + 0.015\cdot\overline{C'}\cdot T$$

$$R_T = -\,\sin\left(2\cdot\Delta\theta\right)\cdot R_C$$

$$R_C = 2\cdot\sqrt{\frac{\overline{C'}^{7}}{\overline{C'}^{7} + 25^7}}$$

$$\Delta\theta = 30\cdot\exp\left(-\left[\frac{\left(\overline{h'} - 275°\right)}{25}\right]^2\right)$$

$$T = 1 - 0.17\cdot\cos\left(\overline{h'} - 30°\right) + 0.24\cdot\cos\left(2\overline{h'}\right) + 0.32\cdot\cos\left(3\overline{h'} + 6°\right) - 0.20\cdot\cos\left(4\overline{h'} - 63°\right)$$

The following pseudocode (12) will calculate $\overline{h'}$ for substitution in the above equation:

```
$p = (h'_S + h'_B)/2$
$q = Abs(h'_S - h'_B)$
if $C'_S\, C'_B = 0$ then
    $\overline{h'} = 2p$
else if $q > 180$ then
    if $p < 180$ then
        $\overline{h'} = p + 180$
    else
        $\overline{h'} = p - 180$
    end if
else
    $\overline{h'} = p$
end if
```

Here *Abs* means the absolute value of the argument.

While not obvious from this listing, all displayed angles are assumed to be given in degrees, including $\Delta\theta$ and thus must generally be converted into radians for trigonometric analysis on digital computers.

6.6.1 Using the arithmetic average of the CIELAB color coordinates of the reference and test specimens to compute the local distortion of CIELAB color space introduces a new problem. Current color tolerance difference equations which base the distortion of CIELAB space on the position of the standard allows a user to predefine the acceptance volume. This is convenient for certain textile sorting applications and for graphical quality control charting. Such a predetermination is not possible with CIEDE2000. Nor is it possible or reasonable to plot groups of colors in terms of the modified space coordinates, L^*, a', b^* since the meaning of a' is determined uniquely for each pair of colors. Thus the equation is highly optimized for pairwise comparison of a product standard to a production test specimen but not for statistical process control.

7. Test Specimens

7.1 This practice does not cover specimen preparation techniques. Unless otherwise specified or agreed, prepare specimens in accordance with appropriate test methods and practices.

8. Procedure

8.1 Select appropriate geometric conditions for color measurement in accordance with Practice E805.

8.2 Operate the instrument in accordance with the manufacturer's instructions and the procedures given in Practice E1164.

8.3 When a colorimetric spectrometer is used, obtain the reflectance values of the reference specimen and test specimens, in turn, at a sufficient number of wavelength intervals to permit accurate calculation of CIE tristimulus values. See Practice E308.

8.4 Measure at least three portions of each specimen surface to obtain an indication of uniformity. Record the location where these measurements were made on the specimen.

9. Calculation

9.1 Calculate color-scale values L^*, a^*, b^*, and local tolerance weights (S_L, S_C, S_H) if not obtained automatically.

9.2 Calculate color differences ΔE^*_{ab},$\Delta\ E_{CMC}$ and their components, or ΔE_{94}, ΔE_{99}, or ΔE_{00}, if not obtained automatically, as described in 6.2 – 6.6, respectively.

10. Report

10.1 Report the following information:

10.1.1 Total color difference ΔE_{CMC}, ΔE_{94}, ΔE_{99}, or ΔE_{00} of each test specimen from its reference.

10.1.2 For CIELAB color differences, L^*, a^*, b^* for the reference, ΔL^*, Δa^*, Δb^* and if desired Δh_{ab}, ΔC^*_{ab}, and ΔH^*_{ab} for each specimen.

10.1.3 For other color tolerance or color difference metrics, only the CIELAB coordinates should be reported as the local distortions do not necessarily provide continuous, visually correlated parameters.

10.1.4 For non-uniform specimens, range of color-difference magnitudes obtained for different areas of the specimens.

10.1.5 Description or identification of the method of preparing the specimens.

10.1.6 Identification of the instrument used, by the manufacturer's name and model number.

10.1.7 The illuminant-observer combination and the color-difference equation used.

11. Precision and Bias

11.1 Since the precision and bias of a test method cannot be separated from the effect of the specimens and materials and since this practice does not address the issues related to the preparation and presentation of specimens, no definitive statement about precision and bias can be made. The next section, uses data from a commercial collaborative testing program to illustrate precision for one material. Because of the many trigonometric functions and power functions involved in computing the color space parameters, all computations should be carried out in IEEE floating point format to greatest number of bits of precision available on the computational system, usually known as double precision.

11.2 *The Collaborative Testing Services Color and Color Difference Collaborative Reference Program* (**13**) has surveyed the precision of color and color-difference measurements by sending out pairs of painted chips exhibiting small color differences on a quarterly basis since 1971. In a typical report (No. 111, February, 2000), 118 instruments were involved. Table 2 gives the mean color differences and their standard deviations for the groups of instruments considered separately in the intercomparison, together with the conditions of analysis and measurement.

11.2.1 *Reproducibility*—Based on the between-laboratory standard deviations, two color-difference results, obtained by operators in different laboratories measuring opaque, matte

TABLE 2 Precision of Calculated Color Differences Determined for Various Conditions of Measurement and Analysis

Measurement Conditions			ΔE Equation	No. of Instruments	Mean ΔE	Standard Deviation	R^{*A}
Geometry	Illuminant	Observer					
45°/0°	D_{65}	1964	CIELAB	54	1.05	0.07	0.21
45°/0°	D_{65}	1964	CMC(2:1)	54	0.55	0.03	0.09
Sphere[B]	D_{65}	1964	CIELAB	282	1.00	0.06	0.18
Sphere[B]	D_{65}	1964	CMC(2:1)	282	0.53	0.03	0.09

[A] R^* is the approximate inter-laboratory precision = 3.0 × standard deviation.

[B] Specular component included for integrating-sphere measurements.