



Standard Practice for Determination of Precision and Bias Data for Use in Test Methods for Petroleum Products, Liquid Fuels, and Lubricants¹

This standard is issued under the fixed designation D6300; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

INTRODUCTION

Both Research Report RR:D02-1007,² *Manual on Determining Precision Data for ASTM Methods on Petroleum Products and Lubricants*² and the ISO 4259, benefitted greatly from more than 50 years of collaboration between ASTM and the Institute of Petroleum (IP) in the UK. The more recent work was documented by the IP and has become ISO 4259.

ISO 4259 encompasses both the determination of precision and the application of such precision data. In effect, it combines the type of information in RR:D02-1007² regarding the determination of the precision estimates and the type of information in Practice D3244 for the utilization of test data. The following practice, intended to replace RR:D02-1007,² differs slightly from related portions of the ISO standard.

1. Scope*

1.1 This practice covers the necessary preparations and planning for the conduct of interlaboratory programs for the development of estimates of precision (determinability, repeatability, and reproducibility) and of bias (absolute and relative), and further presents the standard phraseology for incorporating such information into standard test methods.

1.2 This practice is generally limited to homogeneous petroleum products, liquid fuels, and lubricants with which serious sampling problems (such as heterogeneity or instability) do not normally arise.

1.3 This practice may not be suitable for products with sampling problems as described in 1.2, solid or semisolid products such as petroleum coke, industrial pitches, paraffin waxes, greases, or solid lubricants when the heterogeneous properties of the substances create sampling problems. In such instances, consult a trained statistician.

1.4 *This international standard was developed in accordance with internationally recognized principles on standard-*

ization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.

2. Referenced Documents

2.1 *ASTM Standards:*³

- D3244 Practice for Utilization of Test Data to Determine Conformance with Specifications
- D3606 Test Method for Determination of Benzene and Toluene in Spark Ignition Fuels by Gas Chromatography
- D6708 Practice for Statistical Assessment and Improvement of Expected Agreement Between Two Test Methods that Purport to Measure the Same Property of a Material
- D7915 Practice for Application of Generalized Extreme Studentized Deviate (GESD) Technique to Simultaneously Identify Multiple Outliers in a Data Set
- E29 Practice for Using Significant Digits in Test Data to Determine Conformance with Specifications
- E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods
- E456 Terminology Relating to Quality and Statistics
- E691 Practice for Conducting an Interlaboratory Study to Determine the Precision of a Test Method

¹ This practice is under the jurisdiction of ASTM Committee D02 on Petroleum Products, Liquid Fuels, and Lubricants and is the direct responsibility of Subcommittee D02.94 on Coordinating Subcommittee on Quality Assurance and Statistics. Current edition approved July 1, 2023. Published August 2023. Originally approved in 1998. Last previous edition approved in 2021 as D6300 – 21. DOI: 10.1520/D6300-23.

² Supporting data have been filed at ASTM International Headquarters and may be obtained by requesting Research Report RR:D02-1007. Contact ASTM Customer Service at service@astm.org.

³ For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

*A Summary of Changes section appears at the end of this standard

2.2 ISO Standards:

ISO 4259 Petroleum Products—Determination and Application of Precision Data in Relation to Methods of Test⁴

3. Terminology

3.1 Definitions:

3.1.1 *analysis of variance (ANOVA), n*—technique that enables the total variance of a method to be broken down into its component factors. **ISO 4259**

3.1.2 *bias, n*—the difference between the expectation of the test results and an accepted reference value.

3.1.2.1 *Discussion*—The term “expectation” is used in the context of statistics terminology, which implies it is a “statistical expectation.” **E177**

3.1.3 *between-method bias (relative bias), n*—a quantitative expression for the mathematical correction that can statistically improve the degree of agreement between the expected values of two test methods which purport to measure the same property. **D6708**

3.1.4 *degrees of freedom, n*—the divisor used in the calculation of variance, one less than the number of independent results.

3.1.4.1 *Discussion*—This definition applies strictly only in the simplest cases. Complete definitions are beyond the scope of this practice. **ISO 4259**

3.1.5 *determinability, n*—a quantitative measure of the variability associated with the same operator in a given laboratory obtaining successive determined values using the same apparatus for a series of operations leading to a single result; it is defined as the difference between two such single determined values that would be exceeded about 5 % of the time (one case in 20 in the long run) in the normal and correct operation of the test method.

3.1.5.1 *Discussion*—This definition implies that two determined values, obtained under determinability conditions, which differ by more than the determinability value should be considered suspect. If an operator obtains more than two determinations, then it would usually be satisfactory to check the most discordant determination against the mean of the remainder, using determinability as the critical difference (1).⁵

3.1.6 *mean square, n*—in analysis of variance, sum of squares divided by the degrees of freedom. **ISO 4259**

3.1.7 *normal distribution, n*—the distribution that has the probability function x , such that, if x is any real number, the probability density is

$$f(x) = (1/\sigma)(2\pi)^{-1/2}\exp[-(x - \mu)^2/2\sigma^2] \quad (1)$$

NOTE 1— μ is the true value and σ is the standard deviation of the normal distribution ($\sigma > 0$). **ISO 4259**

3.1.8 *outlier, n*—a result far enough in magnitude from other results to be considered not a part of the set. **RR:D02–1007²**

3.1.9 *precision, n*—the degree of agreement between two or more results on the same property of identical test material. In

this practice, precision statements are framed in terms of *repeatability* and *reproducibility* of the test method.

3.1.9.1 *Discussion*—The testing conditions represented by repeatability and reproducibility should reflect the normal extremes of variability under which the test is commonly used. Repeatability conditions are those showing the least variation; reproducibility, the usual maximum degree of variability. Refer to the definitions of each of these terms for greater detail. **RR:D02–1007²**

3.1.10 *random error, n*—the chance variation encountered in all test work despite the closest control of variables. **RR:D02–1007²**

3.1.11 *repeatability (a.k.a. Repeatability Limit), n*—a quantitative expression for the random error associated with the difference between two independent results obtained under repeatability conditions that would be exceeded about 5 % of the time (one case in 20 in the long run) in the normal and correct operation of the test method.

3.1.11.1 *Discussion*—Interpret as the limit value the absolute difference between two single test results obtained under repeatability conditions is expected to exceed with an approximate probability of 5 %.

3.1.11.2 *Discussion*—The difference is related to the repeatability standard deviation but it is not the standard deviation or its estimate. **RR:D02–1007²**

3.1.12 *repeatability conditions, n*—conditions where independent test results are obtained with the same method on identical test items in the same laboratory by the same operator using the same equipment within short intervals of time. **E177**

3.1.13 *reproducibility (a.k.a. Reproducibility Limit), n*—a quantitative expression for the random error associated with the difference between two independent results obtained under reproducibility conditions that would be exceeded about 5 % of the time (one case in 20 in the long run) in the normal and correct operation of the test method.

3.1.13.1 *Discussion*—Interpret as the limit value the absolute difference between two single test results obtained under reproducibility conditions is expected to exceed with an approximate a probability of 5 %.

3.1.13.2 *Discussion*—The difference is related to the reproducibility standard deviation but is not the standard deviation or its estimate. **RR:D02–1007²**

3.1.13.3 *Discussion*—In those cases where the normal use of the test method does not involve sending a sample to a testing laboratory, either because it is an in-line test method or because of serious sample instabilities or similar reasons, the precision test for obtaining reproducibility may allow for the use of apparatus from the participating laboratories at a common site (several common sites, if feasible). The statistical analysis is not affected thereby. However, the interpretation of the reproducibility value will be affected since the test data is collected under intermediate precision conditions as defined in Practice **E177**, and therefore, the precision statement shall, in this case, state the conditions to which the reproducibility value applies, and label this precision in a manner consistent with how the test data is obtained.

NOTE 2—The reproducibility precision outcome from 3.1.13.3 is a form

⁴ Available from American National Standards Institute (ANSI), 25 W. 43rd St., 4th Floor, New York, NY 10036, <http://www.ansi.org>.

⁵ The bold numbers in parentheses refers to the list of references at the end of this standard.

of Intermediate Precision as defined in Practice E177.

3.1.14 *reproducibility conditions, n*—conditions where independent test results are obtained with the same method on identical test items in different laboratories with different operators using different equipment.

NOTE 3—Different laboratory by necessity means a different operator, different equipment, and different location and under different supervisory control. **E177**

3.1.15 *standard deviation, n*—measure of the dispersion of a series of results around their mean, equal to the square root of the variance and estimated by the positive square root of the mean square. **ISO 4259**

3.1.16 *sum of squares, n—in analysis of variance*, sum of squares of the differences between a series of results and their mean. **ISO 4259**

3.1.17 *variance, n*—a measure of the dispersion of a series of accepted results about their average. It is equal to the sum of the squares of the deviation of each result from the average, divided by the number of degrees of freedom. **RR:D02-1007²**

3.1.18 *variance, between-laboratory, n*—that component of the overall variance due to the difference in the mean values obtained by different laboratories. **ISO 4259**

3.1.18.1 *Discussion*—When results obtained by more than one laboratory are compared, the scatter is usually wider than when the same number of tests are carried out by a single laboratory, and there is some variation between means obtained by different laboratories. Differences in operator technique, instrumentation, environment, and sample “as received” are among the factors that can affect the between laboratory variance. There is a corresponding definition for between-operator variance.

3.1.18.2 *Discussion*—The term “between-laboratory” is often shortened to “laboratory” when used to qualify representative parameters of the dispersion of the population of results, for example as “laboratory variance.”

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *determination, n*—the process of carrying out a series of operations specified in the test method whereby a single value is obtained.

3.2.2 *operator, n*—a person who carries out a particular test.

3.2.3 *probability density function, n*—function which yields the probability that the random variable takes on any one of its admissible values; here, we are interested only in the normal probability.

3.2.4 *result, n*—the final value obtained by following the complete set of instructions in the test method.

3.2.4.1 *Discussion*—It may be obtained from a single determination or from several determinations, depending on the instructions in the method. When rounding off results, the procedures described in Practice E29 shall be used.

4. Summary of Practice

4.1 A draft of the test method is prepared and a pilot program can be conducted to verify details of the procedure and to estimate roughly the precision of the test method.

4.1.1 If the responsible committee decides that an interlaboratory study for the test method is to take place at a later point in time, an interim repeatability is estimated by following the requirements in 6.2.1.

4.2 A plan is developed for the interlaboratory study using the number of participating laboratories to determine the number of samples needed to provide the necessary degrees of freedom. Samples are acquired and distributed. The interlaboratory study is then conducted on an agreed draft of the test method.

4.3 The data are summarized and analyzed. Any dependence of precision on the level of test result is removed by transformation. The resulting data are inspected for uniformity and for outliers. Any missing and rejected data are estimated. The transformation is confirmed. Finally, an analysis of variance is performed, followed by calculation of repeatability, reproducibility, and bias. When it forms a necessary part of the test procedure, the determinability is also calculated.

5. Significance and Use

5.1 ASTM test methods are frequently intended for use in the manufacture, selling, and buying of materials in accordance with specifications and therefore should provide such precision that when the test is properly performed by a competent operator, the results will be found satisfactory for judging the compliance of the material with the specification. Statements addressing precision and bias are required in ASTM test methods. These then give the user an idea of the precision of the resulting data and its relationship to an accepted reference material or source (if available). Statements addressing determinability are sometimes required as part of the test method procedure in order to provide early warning of a significant degradation of testing quality while processing any series of samples.

5.2 Repeatability and reproducibility are defined in the precision section of every Committee D02 test method. Determinability is defined above in Section 3. The relationship

TABLE 1 Sources of Variation

	Method	Apparatus	Operator	Laboratory	Time
Reproducibility	Complete (Result)	Different	Different	Different	Not Specified
Repeatability	Complete (Result)	Same	Same	Same	Almost same
Determinability	Incomplete (Part result)	Same	Same	Same	Almost same

among the three measures of precision can be tabulated in terms of their different sources of variation (see Table 1).

5.2.1 When used, determinability is a mandatory part of the Procedure section. It will allow operators to check their technique for the sequence of operations specified. It also ensures that a result based on the set of determined values is not subject to excessive variability from that source.

5.3 A bias statement furnishes guidelines on the relationship between a set of test results and a related set of accepted reference values. When the bias of a test method is known, a compensating adjustment can be incorporated in the test method.

5.4 This practice is intended for use by D02 subcommittees in determining precision estimates and bias statements to be used in D02 test methods. Its procedures correspond with ISO 4259 and are the basis for the Committee D02 computer software, *Calculation of Precision Data: Petroleum Test Methods*. The use of this practice replaces that of Research Report RR:D02-1007.²

5.5 Standard practices for the calculation of precision have been written by many committees with emphasis on their particular product area. One developed by Committee E11 on Statistics is Practice E691. Practice E691 and this practice differ as outlined in Table 2.

TABLE 2 Differences in Calculation of Precision in Practices D6300 and E691

Element	This Practice	Practice E691
<i>Number of replicates</i>	Two	Any number
<i>Precision is written for</i>	Test method	Each sample
<i>Outlier tests:</i> Within laboratories Between laboratories	Sequential Cochran test Hawkins test	Simultaneous <i>k</i> -value <i>h</i> -value
<i>Outliers</i>	Rejected, subject to subcommittee approval. Retesting not generally permitted.	Rejected if many laboratories or for cause such as blunder or not following method. Laboratory may retest sample having rejected data.
<i>Analysis of variance</i>	Two-way, applied globally to all the remaining data at once.	One-way, applied to each sample separately.
<i>Precision multiplier</i>	$t\sqrt{2}$, where <i>t</i> is the two-tailed Student's <i>t</i> for 95 % probability. Increases with decreasing laboratories × samples particularly below 12.	$2.8 = 1.96\sqrt{2}$ Constant.
<i>Variation of precision with level</i>	Minimized by data transformation. Equations for repeatability and reproducibility are generated in the retransformation process.	User may assess from individual sample precisions.

6. Stages in Planning of an Interlaboratory Test Program for the Determination of the Precision of a Test Method

6.1 The stages in planning an interlaboratory test program are: preparing a draft method of test (see 6.2), planning and executing a pilot program with at least two laboratories (optional but recommended for new test methods) (see 6.3), planning the interlaboratory program (see 6.4), and executing the interlaboratory program (see 6.5). The four stages are described in turn.

6.2 *Preparing a Draft Method of Test*—This shall contain all the necessary details for carrying out the test and reporting the results. Any condition which could alter the results shall be specified. The section on precision will be included at this stage only as a heading.

6.2.1 *Interim Repeatability Study*—If the responsible committee decides that an interlaboratory study for the test method is to take place at a later point in time, using this standard, an interim repeatability standard deviation is estimated by following the steps as outlined below. This interim repeatability standard deviation can be used to meet ASTM Form and Style Requirement A21.5.1. When the committee is ready to proceed with the ILS, continue with this practice from 6.3 onwards.

6.2.1.1 *Design*—The following minimum requirements shall be met:

(1) Three (3) samples, compositionally representative of the majority of materials within the design envelope of the test method, covering the low, medium, and high regions of the intended test method range.

(2) Twelve (12) replicates per sample, obtained under repeatability conditions in a single laboratory.

6.2.1.2 *Analysis*—Carry out the following analyses in the order presented:

(1) Perform GESD Outlier Rejection as per Practice D7915 for each sample.

(2) Calculate sample variance (*v*) and standard deviation (*s*) for each sample using non-rejected results.

(3) Perform the Hartley test for variance equality as follows:

calculate the ratio : $F_{max} = v_{max}/v_{min}$ where v_{max} and v_{min} are the largest and smallest variance obtained.

(4) If F_{max} is less than 4.85, estimate the interim repeatability standard deviation of the test method by taking the square root of the average variance calculated using individual variances from all samples as illustrated below using three samples:

Interim repeatability standard deviation = $[(v_1 + v_2 + v_3)/3]^{0.5}$, where v_1, v_2, v_3 are variances for each sample; it should be noted that if the number of non-outlying results used to calculate the variances are not the same, this equation provides an approximation only, but is suitable for the intended purpose.

(5) If F_{max} exceeds 4.85, list the averages and associated repeatability standard deviations for each sample separately.

(6) If F_{max} exceeds 4.85, and, v_{max} is associated with the sample with the lowest average, calculate the following ratio: $[10 s_{max}]/average_{sample}$, where s_{max} is $(v_{max})^{0.5}$, and $average_{sample}$ is the average of the sample. If this ratio is near

or exceeds 1, then it is likely that this sample is at or below the limit of quantitation of the test method. If this ratio is far below 1, it is likely this is a sample-specific effect. Method developers should investigate and take appropriate steps to revise the test method scope or improve the test method precision at the low limit prior to the conduct of a full ILS.

(7) If the sample set design meets the requirement in 6.4.2, the methodology in Appendix X2 can be used to estimate an interim repeatability function by treating the repeats per sample as results from ‘pseudo-laboratories’ without repeats.

NOTE 4—It is highly recommended that 6.2.1.2(7) be conducted under the guidance of a statistician familiar with the methodology in Appendix X2.

6.2.1.3 *Validation of Interim Repeatability Study by Another Laboratory*—It is highly recommended that the findings from the interim repeatability study be validated by conducting a similar study at another laboratory. If the findings from the validation study do not support the functional form (constant or per Appendix X2) of the interim repeatability study obtained by the initial laboratory, or, if the ratio:

$$\left[\frac{\text{interim repeatability standard deviation from lab A}}{\text{interim repeatability standard deviation from lab B}} \right]^2$$

exceeds 2.4, where the larger of the standard deviation value is in the numerator, that is, if the repeatability standard deviation for lab A is numerically larger than B; otherwise use the repeatability standard deviation for lab B in the numerator and the repeatability standard deviation for lab A in the denominator, it can be concluded that the findings from one laboratory cannot be validated by another laboratory. The method developer is advised to consult a statistician and subject matter experts to decide on which laboratory findings are to be used.

6.3 Planning and Executing a Pilot Program with at Least Two Laboratories:

6.3.1 A pilot program is recommended to be used with new test methods for the following reasons: (1) to verify the details in the operation of the test; (2) to find out how well operators can follow the instructions of the test method; (3) to check the precautions regarding sample handling and storage; and (4) to estimate roughly the precision of the test.

6.3.2 At least two samples are required, covering the range of results to which the test is intended to apply; however, include at least 12 laboratory-sample combinations. Test each sample twice by each laboratory under repeatability conditions. If any omissions or inaccuracies in the draft method are revealed, they shall now be corrected. Analyze the results for precision, bias, and determinability (if applicable) using this practice. If any are considered to be too large for the technical application, then consider alterations to the test method.

6.4 Planning the Interlaboratory Program:

6.4.1 There shall be at least six (6) participating laboratories, but it is recommended this number be increased to eight (8) or more in order to ensure the final precision is based on at least six (6) laboratories and to make the precision statement more representative of the qualified user population.

6.4.2 The number of samples shall be sufficient to cover the range of the property measured, and to give reliability to the

precision estimates. If any variation of precision with level was observed in the results of the pilot program, then at least six samples, spanning the range of the test method in a manner that ensures the leverage (h) of each sample (see Eq 2) is less than 0.5 shall be used in the interlaboratory program. In any case, it is necessary to obtain at least 30 degrees of freedom in both repeatability and reproducibility. For repeatability, this means obtaining a total of at least 30 pairs of results in the program. In the absence of pilot test program information to permit use of Fig. 1 (see 6.4.3) to determine the number of samples, the number of samples shall be greater than five, and chosen such that the number of laboratories times the number of samples is greater than or equal to 42.

Leverage calculation:

$$h_{ii} = \frac{1}{n} + \frac{(x_i - \bar{x})^2}{\sum_{k=1}^n (x_k - \bar{x})^2} \quad (2)$$

h_{ii} = leverage of sample i ,
 n = total number of planned samples,
 p_i = planned property level for sample i ,
 x_i = $\ln(p_i)$, and
 \bar{x} = grand average of all x_i .

6.4.3 For reproducibility, Fig. 1 gives the minimum number of samples required in terms of L , P , and Q , where L is the number of participating laboratories, and P and Q are the ratios of variance component estimates (see 8.3.1) obtained from the pilot program. Specifically, P is the ratio of the interaction component to the repeats component, and Q is the ratio of the laboratories component to the repeats component.

NOTE 5—Appendix X1 gives the derivation of the equation used. If Q is much larger than P , then 30 degrees of freedom cannot be achieved; the blank entries in Fig. 1 correspond to this situation or the approach of it (that is, when more than 20 samples are required). For these cases, there is likely to be a significant bias between laboratories. The program organizer shall be informed; further standardization of the test method may be necessary.

6.5 Executing the Interlaboratory Program:

6.5.1 One person shall oversee the entire program, from the distribution of the texts and samples to the final appraisal of the results. He or she shall be familiar with the test method, but should not personally take part in the actual running of the tests.

6.5.2 The text of the test method shall be distributed to all the laboratories in time to raise any queries before the tests begin. If any laboratory wants to practice the test method in advance, this shall be done with samples other than those used in the program.

6.5.3 The samples shall be accumulated, subdivided, and distributed by the organizer, who shall also keep a reserve of each sample for emergencies. It is most important that the individual laboratory portions be homogeneous. Instructions to each laboratory shall include the following:

6.5.3.1 *Testing Protocol*—The protocol to be used for testing of the ILS sample set shall be provided. Factors that may affect test method outcome but are not intended to be controlled in the normal execution of the test method shall not be intentionally removed nor controlled in the testing of the ILS

L = number of participating Laboratories component

P = interaction variance component/ repeats variance component

Q = Laboratories variance component/repeats variance

	L=6	L=7
	Q: 0 1 2 3 4 5 6 7 8 9	Q: 0 1 2 3 4 5 6 7 8 9
	P: 0 3 1 4 11 2 5 7 3 5 7 14 4 5 6 10 5 6 6 8 15 6 6 6 8 11 7 6 6 7 10 15 8 6 6 7 9 12 9 6 6 7 8 10 15	P: 0 4 1 5 2 6 11 3 6 9 4 7 8 16 5 7 8 12 6 7 8 11 19 7 7 8 10 15 8 7 8 9 13 9 7 8 9 11 17
L=8	L=9	L=10
Q: 0 1 2 3 4 5 6 7 8 9	Q: 0 1 2 3 4 5 6 7 8 9	Q: 0 1 2 3 4 5 6 7 8 9
P: 0 3 1 3 5 2 4 5 9 3 4 5 7 14 4 4 4 6 9 20 5 4 4 5 7 11 6 4 4 5 6 8 13 7 4 4 5 6 7 10 16 8 4 5 5 6 6 8 11 18 9 4 5 5 5 6 7 9 13	P: 0 2 1 3 4 2 3 4 7 3 3 4 5 9 4 4 4 5 6 11 5 4 4 5 6 7 12 6 4 4 4 5 6 9 14 7 4 4 4 5 6 7 10 15 8 4 4 4 5 5 6 8 10 16 9 4 4 4 5 5 6 7 8 11 18	P: 1 2 8 1 3 4 11 2 3 4 5 12 3 3 3 4 6 13 4 3 4 4 5 7 14 5 3 4 4 5 6 8 14 6 3 4 4 4 5 6 9 14 7 3 4 4 4 5 6 7 9 14 8 3 4 4 4 5 5 6 7 10 14 9 4 4 4 4 4 5 6 6 8 10
L=11	L=12	L=13
Q: 0 1 2 3 4 5 6 7 8 9	Q: 0 1 2 3 4 5 6 7 8 9	Q: 0 1 2 3 4 5 6 7 8 9
P: 0 2 4 1 2 3 5 2 3 3 3 7 3 3 3 4 5 8 4 3 3 4 4 6 8 18 5 3 3 4 4 5 6 9 15 6 3 3 3 4 4 5 6 9 14 7 3 3 3 4 4 5 5 7 9 13 8 3 3 3 4 4 4 5 6 7 9 9 3 3 3 4 4 4 5 5 6 7	P: 0 2 4 1 2 3 5 2 2 3 4 6 14 3 3 3 3 4 6 11 4 3 3 3 4 5 6 9 5 3 3 3 4 4 5 6 9 16 6 3 3 3 3 4 4 5 6 9 13 7 3 3 3 3 4 4 5 5 6 8 8 3 3 3 3 4 4 4 5 5 6 9 3 3 3 3 3 4 4 4 5 6	P: 0 2 3 1 2 3 4 12 2 2 3 3 4 8 3 2 3 3 4 5 7 14 4 3 3 3 3 4 5 7 10 5 3 3 3 3 4 4 5 6 9 15 6 3 3 3 3 3 4 4 5 6 8 7 3 2 2 3 3 3 4 4 5 6 8 3 3 3 3 3 3 4 4 5 5 9 3 3 3 3 3 3 4 4 4 5
L=14	L=15	L=16
Q: 0 1 2 3 4 5 6 7 8 9	Q: 0 1 2 3 4 5 6 7 8 9	Q: 0 1 2 3 4 5 6 7 8 9
P: 0 2 3 1 2 2 3 7 2 2 2 3 4 6 12 3 2 2 3 3 4 5 8 18 4 2 3 3 3 3 4 5 7 11 5 2 3 3 3 3 4 4 5 6 8 6 3 3 3 3 3 3 4 4 5 6 7 3 3 3 3 3 3 3 4 4 5 8 3 3 3 3 3 3 3 4 4 4 9 3 3 3 3 3 3 3 3 4 4	P: 0 2 2 13 1 2 2 3 5 19 2 2 2 3 3 4 7 3 2 2 3 3 3 4 6 9 4 2 2 3 3 4 4 5 7 10 5 2 2 3 3 3 3 4 4 5 6 6 2 2 3 3 3 3 3 4 4 5 7 2 2 3 3 3 3 3 3 4 4 8 2 2 3 3 3 3 3 3 3 4 9 2 2 3 3 3 3 3 3 3 3	P: 0 2 5 1 2 2 3 4 8 2 2 2 2 3 4 5 9 3 2 2 2 3 3 4 4 6 9 4 2 2 2 3 3 3 4 4 5 6 5 2 2 2 3 3 3 3 4 4 5 6 2 2 2 2 3 3 3 3 4 4 7 2 2 2 2 3 3 3 3 3 4 8 2 2 2 2 3 3 3 3 3 3 9 2 2 2 2 3 3 3 3 3 3

FIG. 1 Determination of Number of Samples Required (see 6.4.3)

samples, unless explicitly permitted by the sponsoring subcommittee of the ILS for special studies where certain factors are controlled intentionally as part of the testing protocol to meet the intended ILS study objectives. To remove, control, or set limits on factors that are not intended to be controlled in the normal execution of the test method in the conduct of an ILS that is intended for the precision evaluation of the test method executed under normal operating conditions will result in

overly optimistic precision. Precision statements thus generated will likely be unattainable by majority of users in the normal execution of the test method.

6.5.3.2 The agreed draft method of test;

6.5.3.3 Material Safety Data Sheets, where applicable, and the handling and storage requirements for the samples;

6.5.3.4 The order in which the samples are to be tested (a different random order for each laboratory);

6.5.3.5 The statement that two test results are to be obtained in the shortest practical period of time on each sample by the same operator with the same apparatus. For statistical reasons it is imperative that the two results are obtained independently of each other, that is, that the second result is not biased by knowledge of the first. If this is regarded as impossible to achieve with the operator concerned, then the pairs of results shall be obtained in a blind fashion, but ensuring that they are carried out in a short period of time (preferably the same day). The term *blind fashion* means that the operator does not know that the sample is a replicate of any previous run.

6.5.3.6 The period of time during which repeated results are to be obtained and the period of time during which all the samples are to be tested;

6.5.3.7 A blank form for reporting the results. For each sample, there shall be space for the date of testing, the two results, and any unusual occurrences. The unit of accuracy for reporting the results shall be specified. This should be, if possible, more digits reported than will be used in the final test method, in order to avoid having rounding unduly affect the estimated precision values.

6.5.3.8 When it is required to estimate the determinability, the report form must include space for each of the determined values as well as the test results.

6.5.3.9 A statement that the test shall be carried out under normal conditions, using operators with good experience but not exceptional knowledge; and that the duration of the test shall be the same as normal.

6.5.4 The pilot program operators may take part in the interlaboratory program. If their extra experience in testing a few more samples produces a noticeable effect, it will serve as a warning that the test method is not satisfactory. They shall be identified in the report of the results so that any such effect may be noted.

6.5.5 It can not be overemphasized that the statement of precision in the test method is to apply to test results obtained by running the agreed procedure exactly as written. Therefore, the test method must not be significantly altered after its precision statement is written.

7. Inspection of Interlaboratory Results for Uniformity and for Outliers

7.1 Introduction:

7.1.1 This section specifies procedures for examining the results reported in a statistically designed interlaboratory program (see Section 6) to establish:

7.1.1.1 The independence or dependence of precision and the level of results;

7.1.1.2 The uniformity of precision from laboratory to laboratory, and to detect the presence of outliers.

NOTE 6—The procedures are described in mathematical terms based on the notation of Annex A1 and illustrated with reference to the example data (calculation of bromine number) set out in Annex A2. Throughout this section (and Section 8), the procedures to be used are first specified and then illustrated by a worked example using data given in Annex A2.

NOTE 7—It is assumed throughout this section that all the deviations are either from a single normal distribution or capable of being transformed into such a distribution (see 7.2). Other cases (which are rare) would require different treatment that is beyond the scope of this practice. Also, see (2) for a statistical test of normality.

7.2 Transformation of Data:

7.2.1 In many test methods the precision depends on the level of the test result, and thus the variability of the reported results is different from sample to sample. The method of analysis outlined in this practice requires that this shall not be so and the position is rectified, if necessary, by a transformation.

7.2.1.1 Prior to commencement of analysis to determine if transformation is necessary, it is a good practice to examine information gathered from ILS participants to determine compliance with agreed upon ILS protocol and method of test. As part of this examination, the raw data as reported should be inspected for existence of extreme or outlandish values that are visually obvious. Exclusion of extreme or outlandish results from transformation analysis is recommended if assignable causes can be found in order to help ensure test data dependability, transformation reliability, and subsequent computation efficiency. If assignable causes cannot be found, exclusion of extreme or outlandish results from transformation analysis should be confirmed for each sample using a formal statistical test such as the General Extreme Studentized Deviation (GESD) multi-outlier technique (see Practice D7915) or other technically equivalent techniques at the 99 % confidence level on the difference and average (or sum) of the two replicate results as submitted by each ILS participant for each sample as follows:

- (1) Compute the difference of the two replicates submitted by each participant for the sample;
- (2) Perform GESD on the differences from all participants for the sample;
- (3) For each difference that is identified as outlier, reject the result that is farthest from the median of all results for that sample;
- (4) Compute the average (or sum) of the two replicates for each participant for the sample; for participants who submitted only a single result, or, if one of the submitted replicates is rejected in (3), use the remaining result as the average (or $2 \times$ the remaining result as sum) for the participant;
- (5) Perform GESD on the averages (or sums) from all participants for the sample;
- (6) Reject all results identified as outliers in (5); and
- (7) Continue execution of the remainder of this practice using the retained results.

It is recommended that such statistical tests be conducted under the guidance of a statistician.

7.2.2 The laboratories' standard deviations D_j , and the repeats standard deviations d_j (see Annex A1) are calculated and plotted separately against the sample means m_j . If the points so plotted may be considered as lying about a pair of lines parallel to the m -axis, then no transformation is necessary. If, however, the plotted points describe non-horizontal straight lines or curves of the form $D = f_1(m)$ and $d = f_2(m)$, then a transformation will be necessary.

7.2.3 The relationships $D = f_1(m)$ and $d = f_2(m)$ will not in general be identical. It is frequently the case, however, that the ratios $u_j = \frac{d_j}{D_j}$ are approximately the same for all m_j , in which case f_1 is approximately proportional to f_2 and a single transformation will be adequate for both repeatability and

reproducibility. The statistical procedures of this practice are greatly facilitated when a single transformation can be used. For this reason, unless the u_j clearly vary with property level, the two relationships are combined into a single dependency relationship $D = f(m)$ (where D now includes d) by including a dummy variable T . This will take account of the difference between the relationships, if one exists, and will provide a means of testing for this difference (see A4.1).

7.2.4 In the event that the ratios u_j do vary with level (mean, m_j), as confirmed with a regression of u_j on m_j , or $\log(u_j)$ on $\log(m_j)$, follow the instructions in Annex A5. Otherwise, continue with 7.2.5.

7.2.5 The single relationship $D = f(m)$ is best estimated by weighted linear regression analysis. Strictly speaking, an iteratively weighted regression should be used, but in most cases even an unweighted regression will give a satisfactory approximation. The derivation of weights is described in A4.2, and the computational procedure for the regression analysis is described in A4.3. Typical forms of dependence $D = f(m)$ are given in A3.1. These are all expressed in terms of at most two (2) transformation parameters, B and B_0 .

7.2.6 The typical forms of dependence, the transformations they give rise to, and the regressions to be performed in order to estimate the transformation parameters B , are all summarized in A3.2. This includes statistical tests for the significance of the regression (that is, is the relationship $D = f(m)$ parallel to the m -axis), and for the difference between the repeatability and reproducibility relationships, based at the 5 % significance level. If such a difference is found to exist, follow the procedures in Annex A5.

7.2.7 If it has been shown at the 5 % significance level that there is a significant regression of the form $D = f(m)$, then the appropriate transformation $y = F(x)$, where x is the reported result, is given by the equation

$$F(x) = K \int \frac{dx}{f(x)} \quad (3)$$

where $K =$ a constant. In that event, all results shall be transformed accordingly and the remainder of the analysis carried out in terms of the transformed results. Typical transformations are given in A3.1.

7.2.8 The choice of transformation is difficult to make the subject of formalized rules. Qualified statistical assistance may be required in particular cases. The presence of outliers may affect judgement as to the type of transformation required, if any (see 7.7).

7.2.9 *Worked Example:*

7.2.9.1 Table 3 lists the values of m , D , and d for the eight samples in the example given in Annex A2, correct to three significant digits. Corresponding degrees of freedom are in parentheses. Inspection of the values in Table 3 shows that both D and d increase with m , the rate of increase diminishing as m increases. A plot of these figures on log-log paper (that is, a

graph of $\log D$ and $\log d$ against $\log m$) shows that the points may reasonably be considered as lying about two straight lines (see Fig. A4.1 in Annex A4). From the example calculations given in A4.4, the gradients of these lines are shown to be the same, with an estimated value of 0.638. Bearing in mind the errors in this estimated value, the gradient may for convenience be taken as 2/3.

$$\int x^{-\frac{2}{3}} dx = 3x^{\frac{1}{3}} \quad (4)$$

7.2.9.2 Hence, the same transformation is appropriate both for repeatability and reproducibility, and is given by the equation. Since the constant multiplier may be ignored, the transformation thus reduces to that of taking the cube roots of the reported bromine numbers. This yields the transformed data shown in Table A1.3, in which the cube roots are quoted correct to three decimal places.

7.3 *Tests for Outliers:*

7.3.1 The reported data or, if it has been decided that a transformation is necessary, the transformed results shall be inspected for outliers. These are the values which are so different from the remainder that it can only be concluded that they have arisen from some fault in the application of the test method or from testing a wrong sample. Many possible tests may be used and the associated significance levels varied, but those that are specified in the following subsections have been found to be appropriate in this practice. These outlier tests all assume a normal distribution of errors.

7.3.1.1 The total percentage of outliers rejected, as defined by 100x (no. of rejected results/no. of reported results), shall be reported explicitly to the ILS Program Manager for approval by the sponsoring subcommittee and main committee.

7.3.2 *Uniformity of Repeatability*—The first outlier test is concerned with detecting a discordant result in a pair of repeat results. This test (3) involves calculating the e_{ij}^2 over all the laboratory/sample combinations. Cochran’s criterion at the 1 % significance level is then used to test the ratio of the largest of these values over their sum (see A1.5). If its value exceeds the value given in Table A2.2, corresponding to one degree of freedom, n being the number of pairs available for comparison, then the member of the pair farthest from the sample mean shall be rejected and the process repeated, reducing n by 1, until no more rejections are called for. In certain cases, specifically when the number of digits used in reporting results leads to a large number of repeat ties, this test can lead to large proportion of rejections. If this is so, consideration should be given to cease this rejection test and retain some or all of the rejected results. A decision based on judgement in consultation with a statistician will be necessary in this case.

7.3.3 *Worked Example*—In the case of the example given in Annex A2, the absolute differences (ranges) between transformed repeat results, that is, of the pairs of numbers in Table

TABLE 3 Computed from Bromine Example Showing Dependence of Precision on Level

Sample Number	3	8	1	4	5	6	2	7
m	0.756	1.22	2.15	3.64	10.9	48.2	65.4	114
D	0.0669 (14)	0.159 (9)	0.729 (8)	0.211 (11)	0.291 (9)	1.50 (9)	2.22 (9)	2.93 (9)
d	0.0500 (9)	0.0572 (9)	0.127 (9)	0.116 (9)	0.0943 (9)	0.527 (9)	0.818 (9)	0.935 (9)

A1.3, in units of the third decimal place, are shown in Table 4. The largest range is 0.078 for Laboratory G on Sample 3. The sum of squares of all the ranges is

$$0.042^2 + 0.021^2 + \dots + 0.026^2 + 0^2 = 0.0439.$$

Thus, the ratio to be compared with Cochran’s criterion is

$$\frac{0.078^2}{0.0439} = 0.138 \quad (5)$$

where 0.138 is the result obtained by electronic calculation of unrounded factors in the expression. There are 72 ranges and as, from Table A2.2, the criterion for 80 ranges is 0.1709, this ratio is not significant.

7.3.4 Uniformity of Reproducibility:

7.3.4.1 The following outlier tests are concerned with establishing uniformity in the reproducibility estimate, and are designed to detect either a discordant pair of results from a laboratory on a particular sample or a discordant set of results from a laboratory on all samples. For both purposes, the Hawkins’ test (4) is appropriate.

7.3.4.2 This involves forming for each sample, and finally for the overall laboratory averages (see 7.6), the ratio of the largest absolute deviation of laboratory mean from sample (or overall) mean to the square root of certain sums of squares (A1.6).

7.3.4.3 The ratio corresponding to the largest absolute deviation shall be compared with the critical 1 % values given in Table A1.5, where *n* is the number of laboratory/sample cells in the sample (or the number of overall laboratory means) concerned and where *v* is the degrees of freedom for the sum of squares which is additional to that corresponding to the sample in question. In the test for laboratory/sample cells *v* will refer to other samples, but will be zero in the test for overall laboratory averages.

7.3.4.4 If a significant value is encountered for individual samples the corresponding extreme values shall be omitted and the process repeated. If any extreme values are found in the laboratory totals, then all the results from that laboratory shall be rejected.

7.3.4.5 If the test leads to large proportion of rejections, consideration should be given to cease this rejection test and retain some or all of the rejected results. A decision based on judgement in consultation with a statistician will be necessary in this case.

7.3.5 Worked Example:

7.3.5.1 The application of Hawkins’ test to cell means within samples is shown below.

TABLE 4 Absolute Differences Between Transformed Repeat Results: Bromine Example

Laboratory	Sample							
	1	2	3	4	5	6	7	8
A	42	21	7	13	7	10	8	0
B	23	12	12	0	7	9	3	0
C	0	6	0	0	7	8	4	0
D	14	6	0	13	0	8	9	32
E	65	4	0	0	14	5	7	28
F	23	20	34	29	20	30	43	0
G	62	4	78	0	0	16	18	56
H	44	20	29	44	0	27	4	32
J	0	59	0	40	0	30	26	0

7.3.5.2 The first step is to calculate the deviations of cell means from respective sample means over the whole array. These are shown in Table 5, in units of the third decimal place. The sum of squares of the deviations are then calculated for each sample. These are also shown in Table 5 in units of the third decimal place.

7.3.5.3 The cell to be tested is the one with the most extreme deviation. This was obtained by Laboratory D from Sample 1. The appropriate Hawkins’ test ratio is therefore:

$$B^* = \frac{0.314}{\sqrt{0.117+0.015+\dots+0.017}} = 0.7281 \quad (6)$$

7.3.5.4 The critical value, corresponding to *n* = 9 cells in sample 1 and *v* = 56 extra degrees of freedom from the other samples is interpolated from Table A1.5 as 0.3729. The test value is greater than the critical value, and so the results from Laboratory D on Sample 1 are rejected.

7.3.5.5 As there has been a rejection, the mean value, deviations, and sum of squares are recalculated for Sample 1, and the procedure is repeated. The next cell to be tested will be that obtained by Laboratory F from Sample 2. The Hawkins’ test ratio for this cell is:

$$B^* = \frac{0.097}{\sqrt{0.006+0.015+\dots+0.017}} = 0.3542 \quad (7)$$

7.3.5.6 The critical value corresponding to *n* = 9 cells in Sample 2 and *v* = 55 extra degrees of freedom is interpolated from Table A1.5 as 0.3756. As the test ratio is less than the critical value there will be no further rejections.

7.4 Rejection of Complete Data from a Sample:

7.4.1 The laboratories standard deviation and repeats standard deviation shall be examined for any outlying samples. If a transformation has been carried out or any rejection made, new standard deviations shall be calculated.

7.4.2 If the standard deviation for any sample is excessively large, it shall be examined with a view to rejecting the results from that sample.

7.4.3 Cochran’s criterion at the 1 % level can be used when the standard deviations are based on the same number of degrees of freedom. This involves calculating the ratio of the largest of the corresponding sums of squares (laboratories or repeats, as appropriate) to their total (see A1.5). If the ratio exceeds the critical value given in Table A2.2, with *n* as the number of samples and *v* the degrees of freedom, then all the results from the sample in question shall be rejected. In such an

TABLE 5 Deviations of Cell Means from Respective Sample Means: Transformed Bromine Example

Laboratory	Sample							
	1	2	3	4	5	6	7	8
A	20	8	14	15	10	48	6	3
B	75	7	20	9	10	47	6	3
C	64	35	3	20	30	4	22	25
D	314	33	18	42	7	39	80	50
E	32	32	30	9	7	18	18	39
F	75	97	31	20	30	8	74	53
G	10	34	32	20	20	61	9	62
H	42	13	4	42	13	21	8	50
J	1	28	22	29	14	8	10	53
Sum of Squares	117	15	4	6	3	11	13	17

event, care should be taken that the extreme standard deviation is not due to the application of an inappropriate transformation (see 7.1), or undetected outliers.

7.4.4 There is no optimal test when standard deviations are based on different degrees of freedom. However, the ratio of the largest variance to that pooled from the remaining samples follows an *F*-distribution with ν_1 and ν_2 degrees of freedom (see A1.7). Here ν_1 is the degrees of freedom of the variance in question and ν_2 is the degrees of freedom from the remaining samples. If the ratio is greater than the critical value given in A2.6, corresponding to a significance level of $0.01/S$ where S is the number of samples, then results from the sample in question shall be rejected.

7.4.5 Worked Example:

7.4.5.1 The standard deviations of the transformed results, after the rejection of the pair of results by Laboratory D on Sample 1, are given in Table 6 in ascending order of sample mean, correct to three significant digits. Corresponding degrees of freedom are in parentheses.

7.4.5.2 Inspection shows that there is no outlying sample among these. It will be noted that the standard deviations are now independent of the sample means, which was the purpose of transforming the results.

7.4.5.3 The values in Table 7, taken from a test program on bromine numbers over 100, will illustrate the case of a sample rejection.

7.4.5.4 It is clear, by inspection, that the laboratories standard deviation of Sample 93 at 15.76 is far greater than the others. It is noted that the repeats standard deviation in this sample is correspondingly large.

7.4.5.5 Since laboratory degrees of freedom are not the same over all samples, the variance ratio test is used. The variance pooled from all samples, excluding Sample 93, is the sum of the sums of squares divided by the total degrees of freedom, that is

$$\frac{(8 \times 5.10^2 + 9 \times 4.20^2 + \dots + 8 \times 3.85^2)}{(8+9+\dots+8)} = 19.96 \quad (8)$$

7.4.5.6 The variance ratio is then calculated as

$$\frac{15.26^2}{19.96} = 11.66 \quad (9)$$

where 11.66 is the result obtained by electronic calculation without rounding the factors in the expression.

7.4.5.7 From Table A1.8 the critical value corresponding to a significance level of $0.01/8 = 0.00125$, on 8 and 63 degrees of freedom, is approximately 4. The test ratio greatly exceeds this and results from Sample 93 shall therefore be rejected.

7.4.5.8 Turning to repeats standard deviations, it is noted that degrees of freedom are identical for each sample and that Cochran's test can therefore be applied. Cochran's criterion

will be the ratio of the largest sum of squares (Sample 93) to the sum of all the sums of squares, that is

$$2.97^2 / (1.13^2 + 0.99^2 + \dots + 1.36^2) = 0.510 \quad (10)$$

This is greater than the critical value of 0.352 corresponding to $n = 8$ and $\nu = 8$ (see Table A2.2), and confirms that results from Sample 93 shall be rejected.

7.5 Estimating Missing or Rejected Values:

7.5.1 One of the Two Repeat Values Missing or Rejected—If one of a pair of repeats (Y_{ij1} or Y_{ij2}) is missing or rejected, this shall be considered to have the same value as the other repeat in accordance with the least squares method.

7.5.2 Both Repeat Values Missing or Rejected:

7.5.2.1 If both the repeat values are missing, estimates of a_{ij} ($= Y_{ij1} + Y_{ij2}$) shall be made by forming the laboratories \times samples interaction sum of squares (see Eq 18), including the missing values of the totals of the laboratories/samples pairs of results as unknown variables. Any laboratory or sample from which all the results were rejected shall be ignored and new values of L and S used. The estimates of the missing or rejected values shall be those that minimize the interaction sum of squares.

7.5.2.2 If the value of single pair sum a_{ij} has to be estimated, the estimate is given by the equation:

$$a_{ij} = \frac{1}{(L-1)(S'-1)} (LL_1 + S'S_1 - T_1) \quad (11)$$

where:

- L_1 = total of remaining pairs in the i th laboratory,
- S_1 = total of remaining pairs in the j th sample,
- S' = S - number of samples rejected in 7.4, and
- T_1 = total of all pairs except a_{ij} .

7.5.2.3 If more estimates are to be made, the technique of successive approximation can be used. In this, each pair sum is estimated in turn from Eq 11, using L_1 , S_1 , and T_1 values, which contain the latest estimates of the other missing pairs. Initial values for estimates can be based on the appropriate sample mean, and the process usually converges to the required level of accuracy within three complete iterations (5).

7.5.3 Worked Example:

7.5.3.1 The two results from Laboratory D on Sample 1 were rejected (see 7.3.4) and thus a_{41} has to be estimated.

Total of remaining results in Laboratory 4 = 36.354
 Total of remaining results in Sample 1 = 19.845
 Total of all the results except a_{41} = 348.358
 Also $S' = 8$ and $L = 9$.

Hence, the estimate of a_{41} is given by

$$a_{ij} = \frac{1}{(9-1)(8-1)} [(9 \times 36.354) + (8 \times 19.845) - 348.358] \quad (12)$$

Therefore,

TABLE 6 Standard Deviations of Transformed Results: Bromine Example

Sample number	3	8	1	4	5	6	2	7
<i>m</i>	0.9100	1.066	1.240	1.538	2.217	3.639	4.028	4.851
<i>D</i>	0.0278	0.0473	0.0354	0.0297	0.0197	0.0378	0.0450	0.0416
	(14)	(9)	(13)	(11)	(9)	(9)	(9)	(9)
<i>d</i>	0.0214	0.0182	0.028	0.0164	0.0063	0.0132	0.0166	0.0130
	(9)	(9)	(8)	(9)	(9)	(9)	(9)	(9)

TABLE 7 Example Statistics Indicating Need to Reject an Entire Sample

Sample number	90	89	93	92	91	94	95	96
<i>m</i>	96.1	99.8	119.3	125.4	126.0	139.9	139.4	159.5
<i>D</i>	5.10	4.20	15.26	4.40	4.09	4.87	4.74	3.85
	(8)	(9)	(8)	(11)	(10)	(8)	(9)	(8)
<i>d</i>	1.13	0.99	2.97	0.91	0.73	1.32	1.12	1.36
	(8)	(8)	(8)	(8)	(8)	(8)	(8)	(8)

$$a_{ij} = \frac{137.588}{56} = 2.457 \quad (13)$$

7.6 Rejection Test for Outlying Laboratories:

7.6.1 At this stage, one further rejection test remains to be carried out. This determines whether it is necessary to reject the complete set of results from any particular laboratory. It could not be carried out at an earlier stage, except in the case where no individual results or pairs are missing or rejected. The procedure again consists of Hawkins' test (see 7.3.4), applied to the laboratory averages over all samples, with any estimated results included. If any laboratories are rejected on all samples, new estimates shall be calculated for any remaining missing values (see 7.5).

7.6.2 Worked Example:

7.6.2.1 The procedure on the laboratory averages shown in Table 8 follows exactly that specified in 7.3.4. The deviations of laboratory averages from the overall mean are given in Table 9 in units of the third decimal place, together with the sum of squares. Hawkins' test ratio is therefore:

$$B^* = 0.026/\sqrt{0.00222} = 0.5518 \quad (14)$$

Comparison with the value tabulated in Table A1.5, for *n* = 9 and *v* = 0, shows that this ratio is not significant and therefore no complete laboratory rejections are necessary.

7.7 Confirmation of Selected Transformation:

7.7.1 At this stage it is necessary to check that the rejections carried out have not invalidated the transformation used. If necessary, the procedure from 7.2 shall be repeated with the outliers replaced, and if a new transformation is selected, outlier tests shall be reapplied with the replacement values reestimated, based on the new transformation.

7.7.2 Worked Example:

7.7.2.1 It was not considered necessary in this case to repeat the calculations from 7.2 with the outlying pair deleted.

8. Analysis of Variance and Calculation of Precision Estimates

8.1 After the data have been inspected for uniformity, a transformation has been performed, if necessary, and any outliers have been rejected (see Section 7), an analysis of variance shall be carried out. First an analysis of variance table shall be constructed, and finally the precision estimates derived.

8.2 Analysis of Variance:

8.2.1 Forming the Sums of Squares for the Laboratories × Samples Interaction Sum of Squares—The estimated values, if any, shall be put in the array and an approximate analysis of variance performed.

$$M = \text{mean correction} = T^2/2L'S' \quad (15)$$

where:

L' = *L* – number of laboratories rejected in 7.6 – number of laboratories with no remaining results after rejections in 7.3.4,

S' = total of remaining pairs in the *j*th sample, and

T = the total of all replicate test results.

$$\text{Samples sum of squares} = \left[\sum_{j=1}^{S'} (g_j^2/2L') \right] - M \quad (16)$$

where *g_j* is the sum of sample *j* test results.

$$\text{Laboratories sum of squares} = \left[\sum_{i=1}^{L'} (h_i^2/2S') \right] - M \quad (17)$$

where *h_i* is the sum of laboratory *i* test results.

$$\text{Pairs sum of squares} = (1/2) \left[\sum_{i=1}^{L'} \sum_{j=1}^{S'} a_{ij}^2 \right] - M \quad (18)$$

I = Laboratories × samples interaction sum of squares
 = (pairs sum of squares) – (laboratories sum of squares)
 – (sample sum of squares)

Ignoring any pairs in which there are estimated values, repeats sum of squares,

$$E = (1/2) \sum_{i=1}^{L'} \sum_{j=1}^{S'} e_{ij}^2 \quad (19)$$

The purpose of performing this approximate analysis of variance is to obtain the minimized laboratories × samples interaction sum of squares, *I*. This is then used as indicated in 8.2.2, to obtain the laboratories sum of squares. If there were no estimated values, the above analysis of variance is exact and paragraph 8.2.2 shall be disregarded.

8.2.1.1 Worked Example:

$$\text{Mean correction} = \frac{350.815^2}{144} \quad (20)$$

$$= 854.6605$$

TABLE 8 Averages of All Transformed Results from Each Laboratory

Laboratory	A	B	C	D	E	F	G	H	J	Grand Average
Average	2.437	2.439	2.424	2.426 ^A	2.444	2.458	2.410	2.428	2.462	2.436

^A Including estimated value.

TABLE 9 Absolute Deviations of Laboratory Averages from Grand Average × 1000

Laboratory	A	B	C	D	E	F	G	H	J	Sum of Squares
Deviation	1	3	12	10	8	22	26	8	26	2.22

where 854.6605 is the result obtained by electronic calculation without rounding the factors in the expression.

$$\begin{aligned} \text{Samples sum of squares} & \quad (21) \\ &= \frac{22.302^2 + 72.512^2 + \dots + 19.192^2}{18} - 854.6605 \end{aligned}$$

$$\begin{aligned} \text{Laboratories sum of squares} & \quad (22) \\ &= \frac{38.992^2 + 39.020^2 + \dots + 39.387^2}{16} \end{aligned}$$

$$\begin{aligned} \text{Pairs sum of squares} &= (1/2) (2.520^2 + 8.041^2 + \dots + 2.238^2) \\ &- 854.6605 \quad (23) \end{aligned}$$

$$\begin{aligned} \text{Repeats sum of squares} &= (1/2) (0.042^2 + 0.021^2 + \dots + 0^2) \\ &= 0.0219 \quad (24) \end{aligned}$$

Table 10 can then be derived.

8.2.2 Forming the Sum of Squares for the Exact Analysis of Variance:

8.2.2.1 In this subsection, all the estimated pairs are disregarded and new values of g_j are calculated. The following sums of squares for the exact analysis of variance (6) are formed.

$$\text{Uncorrected sample sum of squares} = \sum_{j=1}^{S'} \frac{g_j^2}{S_j} \quad (25)$$

where:

$$S_j = 2(L' - \text{number of missing pairs in that sample}).$$

$$\text{Uncorrected pairs sum of squares} = (1/2) \sum_{i=1}^{L'} \sum_{j=1}^{S'} a_{ij}^2 \quad (26)$$

The laboratories sum of squares is equal to (pairs sum of squares) – (samples sum of squares) – (the minimized laboratories × samples interaction sum of squares)

$$= (1/2) \left[\sum_{i=1}^{L'} \sum_{j=1}^{S'} a_{ij}^2 \right] - \left[\sum_{j=1}^{S'} \frac{g_j^2}{S_j} \right] - I \quad (27)$$

8.2.2.2 Worked Example:

TABLE 10 Sums of Squares: Bromine Example

Sources of Variation	Sum of Squares
Samples	293.5409
Laboratories	0.0356
Laboratories × samples interaction	0.1143
Pairs	293.6908
Repeats	0.0219

$$\text{Uncorrected samples sum of squares} \quad (28)$$

$$= \frac{19.845^2}{16} + \frac{72.512^2}{18} + \dots + \frac{19.192^2}{18}$$

$$= 1145.1834$$

$$\text{Uncorrected pairs sum of squares} = \frac{2.520^2}{2} + \frac{8.041^2}{2} + \dots + \frac{2.238^2}{2} \quad (29)$$

$$= 1145.3329$$

$$\text{Therefore, laboratories sum of squares} \quad (30)$$

$$= 1145.3329 - 1145.1834 + 0.1143$$

$$= 0.0352$$

8.2.3 Degrees of Freedom:

8.2.3.1 The degrees of freedom for the laboratories are $(L' - 1)$. The degrees of freedom for laboratories × samples interaction are $(L' - 1)(S' - 1)$ for a complete array and are reduced by one for each pair which is estimated. The degrees of freedom for repeats are $(L'S')$ and are reduced by one for each pair in which one or both values are estimated.

8.2.3.2 *Worked Example*—There are eight samples and nine laboratories in this example. As no complete laboratories or samples were rejected, then $S' = 8$ and $L' = 9$.

$$\text{Laboratories degrees of freedom} = L - 1 = 8.$$

Laboratories × samples interaction degrees of freedom if there had been no estimates, would have been $(9 - 1)(8 - 1) = 56$. But one pair was estimated, hence laboratories × samples interaction degrees of freedom = 55. Repeats degrees of freedom would have been 72 if there had been no estimates. In this case one pair was estimated, hence repeats degrees of freedom = 71.

8.2.4 Mean Squares and Analysis of Variance:

8.2.4.1 The mean square in each case is the sum of squares divided by the corresponding degrees of freedom. This leads to the analysis of variance shown in Table 11. The ratio M_I/M_{LS} is distributed as F with the corresponding laboratories and interaction degrees of freedom (see A1.7). If this ratio exceeds the 5 % critical value given in Table A1.6, then serious bias

TABLE 11 Analysis of Variance Table

Sources of Variation	Degrees of Freedom	Sum of Squares	Mean Square
Laboratories	$L' - 1$	Laboratories sum of squares	M_L
Laboratories × samples	$(L' - 1)(S' - 1) - \text{number of estimated pairs}$	I	M_{LS}
Repeats	$L'S' - \text{number of pairs in which one or both values are estimated}$	E	M_r

between the laboratories is implied and the program organizer shall be informed (see 6.5); further standardization of the test method may be necessary, for example, by using a certified reference material.

8.2.4.2 *Worked Example*—The analysis of variance is shown in Table 12. The ratio $M_L/M_{LS} = 0.0044/0.002078$ has a value 2.117. This is greater than the 5 % critical value obtained from Table A1.6, indicating bias between laboratories.

8.3 *Expectation of Mean Squares and Calculation of Precision Estimates:*

8.3.1 *Expectation of Mean Squares with No Estimated Values*—For a complete array with no estimated values, the expectations of mean squares are

$$\begin{aligned} \text{Laboratories: } & \sigma_o^2 + 2\sigma_1^2 + 2S'\sigma_2^2 \\ \text{Laboratories } \times \text{ samples: } & \sigma_o^2 + 2\sigma_1^2 \\ \text{Repeats: } & \sigma_o^2 \end{aligned}$$

where:

σ_1^2 = the component of variance due to interaction between laboratories and samples, and
 σ_2^2 = the component of variance due to differences between laboratories.

8.3.2 *Expectation of Mean Squares with Estimated Values:*

8.3.2.1 The coefficients of σ_1^2 and σ_2^2 in the expectation of mean squares are altered in the cases where there are estimated values. The expectations of mean squares then become

$$\begin{aligned} \text{Laboratories: } & \alpha\sigma_o^2 + 2\sigma_1^2 + \beta\sigma_2^2 \\ \text{Laboratories } \times \text{ samples: } & \gamma\sigma_o^2 + 2\sigma_1^2 \\ \text{Repeats: } & \sigma_o^2 \end{aligned}$$

where:

$$\beta = 2 \frac{K - S'}{L' - 1} \tag{31}$$

where:

K = the number of laboratory \times sample cells containing at least one result, and α and γ are computed as in 8.3.2.5

8.3.2.2 If there are no cells with only a single estimated result, then $\alpha = \gamma = 1$.

8.3.2.3 If there are no empty cells (that is, every lab has tested every sample at least once, and $K = L \times S'$), then α and γ are both one plus the proportion of cells with only a single result.

8.3.2.4 If there are both empty cells and cells with only one result, then, for each lab, compute the proportion of samples tested for which there is only one result, p_i , and the sum of these proportions over all labs, P . For each sample, compute the proportion of labs that have tested the sample for which there is only one result on it, q_j , and the sum of these

proportions over samples, Q . Compute the total number of cells with only one result, W , and the proportion of these among all nonempty cells, W/K . Then

$$\alpha = 1 + \frac{P - W/K}{L' - 1} \tag{32}$$

and

$$\gamma = 1 + \frac{W - P - Q + W/K}{K - L' - S' + 1} \tag{33}$$

NOTE 8—These subsections are based upon the assumptions that both samples and laboratories are random effects.

8.3.2.5 *Worked Example*—For the example, which has eight samples and nine laboratories, one cell is empty (Laboratory D on Sample 1), so $K = 71$ and

$$\beta = 2 \frac{71 - 8}{(9 - 1)} = 15.75 \tag{34}$$

None of the nonempty cells has only one result, so $\alpha = \gamma = 1$. To make the example more interesting, assume that only one result remains from Laboratory A on Sample 1. Then $W = 1$, $p_1 = 1/8$, $p_2 = p_3 = \dots = p_9 = 0$, and $P = 0.125$. We compute $q_1 = 1/8$ (we don't count Laboratory D in the denominator), $q_2 = q_3 = \dots = q_8 = 0$, and $Q = 0.125$. Consequently,

$$\alpha = 1 + \frac{0.125 - 1/71}{9 - 1} = 1.014 \tag{35}$$

and

$$\gamma = 1 + \frac{1 - 0.125 - 0.125 + 1/71}{55} = 1.014 \tag{36}$$

8.3.3 *Calculation of Precision Estimates:*

8.3.3.1 *Repeatability*—The repeatability variance is twice the mean square for repeats. The repeatability estimate is the product of the repeatability standard deviation and the “ t -value” with appropriate degrees of freedom (see Table A2.3) corresponding to a two-sided probability of 95 %. Round calculated estimates of repeatability in accordance with Practice E29, specifically paragraph 7.6 of that practice. Note that if a transformation $y = f(x)$ has been used, then

$$r(x) \approx \left| \frac{dx}{dy} \right| r(y) \tag{37}$$

where $r(x)$, $r(y)$ are the corresponding repeatability functions (see Table A3.1). A similar relationship applies to the reproducibility functions $R(x)$, $R(y)$.

8.3.3.2 *Worked Example:*

$$\text{Repeatability variance} = 2\sigma_o^2 \tag{38}$$

$$= 0.000616$$

$$\text{Repeatability of } y = t_{71} \sqrt{0.000616}$$

$$= 1.994 \times 0.0248$$

$$= 0.0495$$

$$\text{Repeatability of } x = 3x^{2/3} \times 0.0495$$

$$= 0.148x^{2/3}$$

8.3.3.3 *Reproducibility*—Reproducibility variance = $2(\sigma_o^2 + \sigma_1^2 + \sigma_2^2)$ and can be calculated using Eq 39.

TABLE 12 Analysis of Variance Table: Transformed Benzene Example

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F
Laboratories	0.0352	8	0.004400	2.117
Laboratories \times samples	0.1143	55	0.002078	
Repeats	0.0219	71	0.000308	...