International Standard 5629

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Paper and board — Determination of bending stiffness — Resonance method

Papier et carton — Détermination de la résistance à la flexion — Méthode par résonance

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Foreword

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Draft International Standards adopted by the technical committees are circulated to the member bodies for approval before their acceptance as International Standards by the ISO Council.

International Standard ISO 5629 was developed by Technical Committee ISO/TC 6, *Paper, board and pulps*, and was circulated to the member bodies in December 1982.

It has been approved by the member bodies of the following countries:

Belgium	India	Spain
Bulgaria	Italy	Sweden
Canada	Kenya	Switzerland
Czechoslovakia	Korea, Rep. of	Turkey
Egypt, Arab Rep. of	Netherlands	United Kingdom
Finland	Norway	USA
France	Poland	USSR
Germany, F.R.	Romania	Venezuela
Hungary	South Africa, Rep. of	

No member body expressed disapproval of the document.

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Paper and board — Determination of bending stiffness — Resonance method

0 Introduction

There are many methods of measuring the resistance of paper and board to bending forces. Because of the visco-elastic nature of the material, results will depend to a considerable extent on the test method used. In this method, the bending stiffness is determined under dynamic conditions (vibration) using only a relatively small bending angle.

1 Scope and field of application

This International Standard specifies a method for the determination of bending stiffness using the resonance method.

This method gives only a single result for each test piece tested, although in general, bending stiffness values may depend on which surface of the material forms the inner surface of the arc resulting from bending forces.

The bending stiffness of a very wide range of papers and boards can be measured using this method. However, for some materials, such as those listed below, the measured numerical values may not be strictly identical with bending stiffness as defined in clause 3:

a) multi-ply papers and boards in which the component plies can move separately during a test;

b) papers and boards with appreciable curl, especially if the axis of the curl is in the long direction of the test piece;

c) some soft papers with grammage below about 40 g/m^2.

This method is not applicable to corrugated boards.

2 References

ISO 186, *Paper and board — Sampling for testing.*

ISO 187, *Paper and board — Conditioning of samples.*

ISO 536, *Paper and board — Determination of grammage.*

3 Definition

For the purpose of this International Standard, the following definition applies:

bending stiffness S: The moment of the resistance per unit width that a paper or board offers to bending in the region of elastic deformation. It can be defined mathematically by the equation

$$S = \frac{EI}{b} \qquad \ldots (1)$$

where

E is the modulus of elasticity, i.e. Young's modulus;

I is the second moment of area (moment of inertia) of the cross-sectional area, about an axis through the centre of that area, in its plane, and perpendicular to the direction of bending;

b is the width of the test piece.

4 Principle

Determination, under standard conditions, of the vibrational resonant length of a test piece clamped at one end. Calculation of the bending stiffness from this value and the grammage of the material.

The vibrational resonant length of a material, in one direction, when it is subjected to vibration of a given frequency depends on its bending stiffness in that direction and its grammage.

5 Apparatus (see figure 1)

Usual laboratory apparatus and

5.1 Clamping system.

The clamp shall consist of two flat parallel metal plates which can be so adjusted as to give a separation and a clamping pressure such as to just permit the test piece to be pulled through the clamp. The clamp shall be so arranged that a test piece can project vertically in both directions through the clamp. The dimensions of the clamp are not critical, but the width shall be greater than the width of the test piece used. Generally a width of slightly more than 25 mm is satisfactory.

The upper edges of the clamp may be rounded, in which case the radius of curvature shall be less than 0,1 mm.

5.2 Means of vibrating the clamp[1] in a horizontal plane, perpendicularly to the plane of the paper, at a frequency of 25,0 ± 0,1 Hz with an amplitude of not greater than 0,2 mm.

5.3 Means of measuring the length of test piece projecting above the upper edge of the clamp.

5.4 Stroboscopic lamp, operating at the same frequency and in the same phase as the vibrator, for illuminating the top edge of the test piece.

5.5 Optional components.

5.5.1 Magnifier, which can be adjusted for viewing the top edge of the test piece.

5.5.2 Movable lower clamp, of non-critical construction, so arranged that with its aid the test piece can be pulled (and for some materials, pushed) through the vibrating clamp. This clamp may be suitably coupled to a measuring device, enabling the resonant length to be read directly from a scale.

6 Sampling

Select at least ten specimens from a sample taken in accordance with ISO 186, making sure that there are no folds, creases, visible cracks or other defects in the areas to be tested.

7 Conditioning

Condition the specimens in one of the atmospheres specified in ISO 187. Prepare the test pieces and carry out the tests in the same atmospheric conditions.

8 Preparation of the test piece

Cut at least one test piece of appropriate dimensions from each of the specimens in each of the desired test directions. If watermarks are present this should be stated in the test report.

The dimensions of the test piece are dependent on the type of material. In general terms, the width of the test piece shall be between 10 and 25 mm. The edges shall be clean, straight and parallel to within ±0,1 mm.

For papers of low grammage and for materials with a tendency to curl perpendicularly to the test piece length, a width of 10 or 15 mm is preferred. For materials of higher grammage a width of 25 mm is preferred.

The length of the test piece shall be sufficient to allow for the projecting length at resonance and the depth of the clamp, together with a sufficient length to permit handling of the test piece in the non-resonating area and attachment to the lower sample clamp (if fitted).

The resonating end of the test piece shall be clean, straight and perpendicular to the long edges of the test piece in both axes.

If procedure A (9.1) is to be used, the dimensions of each test piece shall be measured accurately.

9 Procedure

Two different procedures may be used. Procedure A is generally more accurate and permits the calculation of the coefficient of variation of bending stiffness, but takes more time than procedure B. Furthermore, the precision of the grammage values determined for the purpose of procedure A is less than the precision of normal grammage measurements.

In most cases, procedure B gives results that differ by less than 1 % from those given by procedure A. Greater differences may, however, be obtained where there are considerable variations in resonant lengths or grammage.

9.1 Procedure A

9.1.1 Grammage

Weigh each test piece to an accuracy of ±0,001 g. Mark each test piece so that the mass of the test piece and the resonant length can later be matched up.

NOTE — If the grammage of the sample shows wide variations, a rather more accurate final result may be obtained by carefully cutting the resonating length of the test piece along the upper edge of the clamp and redetermining the mass and area of each test piece.

9.1.2 Resonant length

Use one or more test pieces to determine an approximate resonant length.

1) A suitable instrument is available commercially. Details can be obtained from the secretariat of ISO/TC 6 (SCC) or from ISO Central Secretariat.

Fit the test piece so that a sufficient length protrudes above the upper edge of the clamp (see figure 1). Ensure that the test piece is fitted perpendicularly to the edge of the clamp. Adjust the clamping pressure so that the test piece can just be pulled through the clamp.

Set the clamp into vibration and carefully pull the test piece through the clamp until the free end of the test piece goes into resonance. This point is characterized by maximum amplitude of vibration and maximum sharpness of visibility of the vibrating end of the test piece under stroboscopic illumination (5.4).

Check that the edge of the test piece is still perpendicular to the edge of the clamp.

Measure as accurately as possible the length of test piece projecting past the edge of the clamp. This may be done by carefully cutting or marking the test piece at the clamp edge, removing it from the clamps and measuring its length using a steel rule or other suitable device. The accuracy of measurement of resonant length shall be within ± 0,25 mm or ± 0,5 %, whichever value is the greater in the particular instance concerned.

Alternatively, if the instrument is fitted with a lower clamp coupled to a measuring scale, this may be used. If this is done, the measurement should be carried out in accordance with the manufacturer's instructions, but it is essential that it should be checked that the test piece has not stretched significantly during the test and thus that the length indicated by the scale is identical with the length of test piece protruding past the clamp.

Check if necessary, by further decreasing the projecting length, that the resonance point has been reached and that resonance is occurring with only one antinode since it is possible to obtain resonance showing two antinodes and one node (see figure 2).

NOTE — For materials of very low grammage it may be necessary to release the clamping pressure while adjusting the projecting length to give resonance. For materials of higher grammage it may be possible to make the final adjustment of the projecting length to obtain resonance by both pulling and pushing the test piece through the clamp. However, the clamping pressure should be applied while the state of resonance is checked.

Using the initial values obtained for resonant length as a guide, carry out the procedure given above to determine the resonant length of ten test pieces in each of the desired directions.

9.2 Procedure B

9.2.1 Mean grammage

Determine the mean grammage of the sample in accordance with ISO 536.

9.2.2 Resonant length

Carry out the determination of resonant length as in 9.1.2.

10 Expression of results

See the annex for the derivation of the equations used.

10.1 Procedure A

The bending stiffness, S, expressed in newton metres, is given individually for each test piece by the equation

$$S = \frac{2 l^4 m}{10^6 A} \qquad \ldots (2)$$

where

l is the resonant length, in millimetres;

m is the appropriate mass, in grams, of the test piece;

A is the appropriate area, expressed in square millimetres, of the test piece.

NOTE — In practice it will often be more convenient to use the mathematically equivalent equation to derive the bending stiffness, S_1, expressed in millinewton metres

$$S_1 = 20 \left(\frac{l}{100} \right)^4 \times \frac{\varrho_A}{100}$$

where ϱ_A is the grammage, expressed in grams per square metre, derived from the equation

$$\varrho_A = \frac{m \times 10^6}{A}$$

The values of $l/100$ and $\varrho_A/100$ are usually within the range 0,1 to 10.

Calculate the mean bending stiffness from the individual values obtained, together with the standard deviation or coefficient of variation.

10.2 Procedure B

The bending stiffness, S, expressed in newton metres, is given individually for each test piece by the equation

$$S = \frac{2 l^4 \varrho_A}{10^{12}} \qquad \ldots (3)$$

where

l is the resonant length, in millimetres;

ϱ_A is the grammage, expressed in grams per square metre, of the material.

NOTE — In practice it will often be more convenient to use the mathematically equivalent equation to derive the bending stiffness, S_1, expressed in millinewton metres

$$S_1 = 20 \left(\frac{l}{100} \right)^4 \times \frac{\varrho_A}{100}$$

The values of $l/100$ and $\varrho_A/100$ are usually within the range 0,1 to 10.

Calculate the mean bending stiffness from the individual values obtained, together with the standard deviation or coefficient of variation.

10.3 Approximate results

For approximate work, and to provide a convenient check on reported values, it is frequently sufficient to determine the mean resonant length of the material and the mean grammage and thus, using the equations given above, to calculate a mean value for bending stiffness.

11 Precision

11.1 Repeatability

Under routine laboratory conditions the repeatability of procedure A or B is approximately 6 %.

The difference between the two single test values found on identical test material by one operator using the same instrument within a short time interval will exceed the repeatability on average not more than once in 20 instances in the normal and correct operation of the method.

11.2 Reproducibility

Insufficient information is available, at present, to quote details of the reproducibility of the method.

12 Test report

The test report shall include the following information:

a) a reference to this International Standard;

b) all information necessary for complete identification of the sample;

c) the standard conditioning atmosphere used;

d) a reference to the procedure used (A or B);

e) the number of tests carried out;

f) the test direction — for instance machine direction or cross direction;

g) the test width used;

h) the results obtained for resonant length;

j) the grammage of the material or the mass of the test piece, as appropriate;

k) the mean value of the bending stiffness, in micronewton metres, millinewton metres or newton metres as appropriate, to three significant figures;

m) the standard deviation, or coefficient of variation;

n) any operations not specified in this International Standard or in the International Standards to which reference is made, or regarded as optional, which might have affected the results.

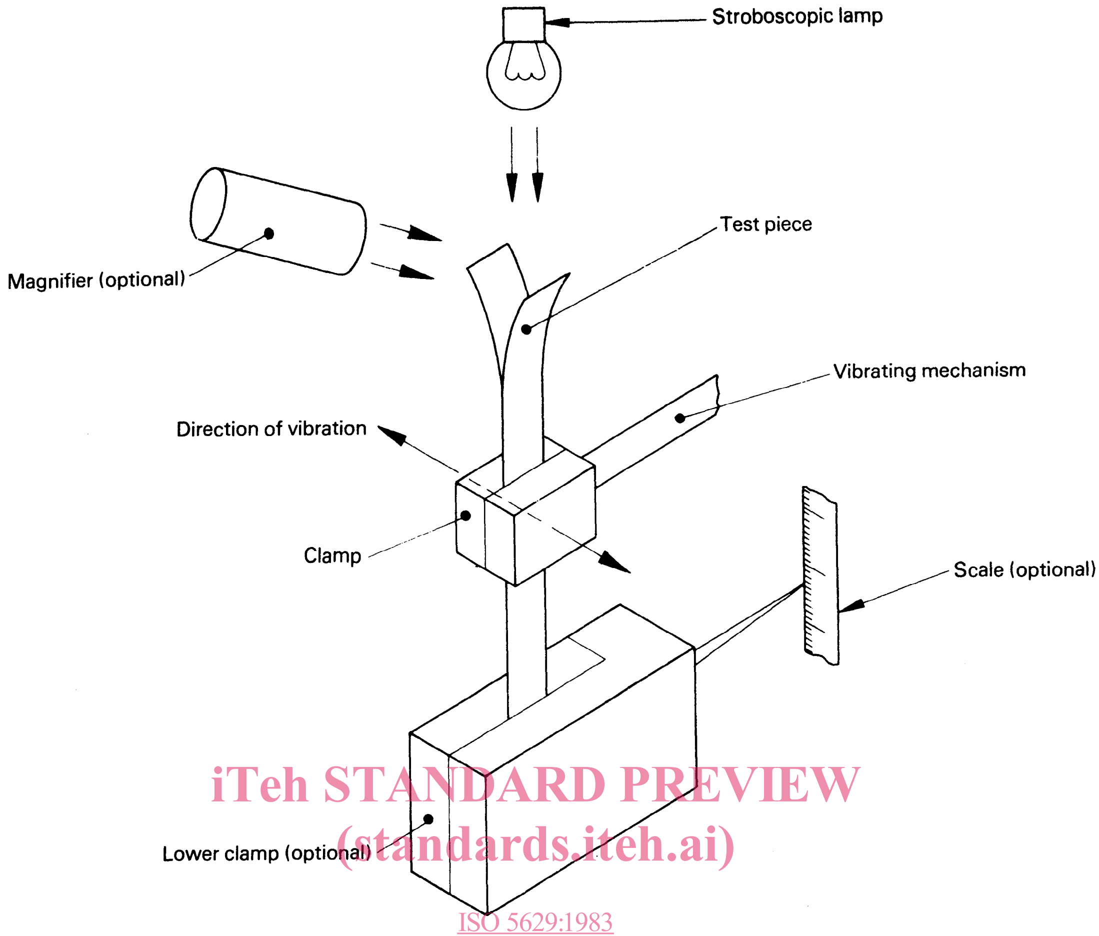

Figure 1 — General arrangement of apparatus

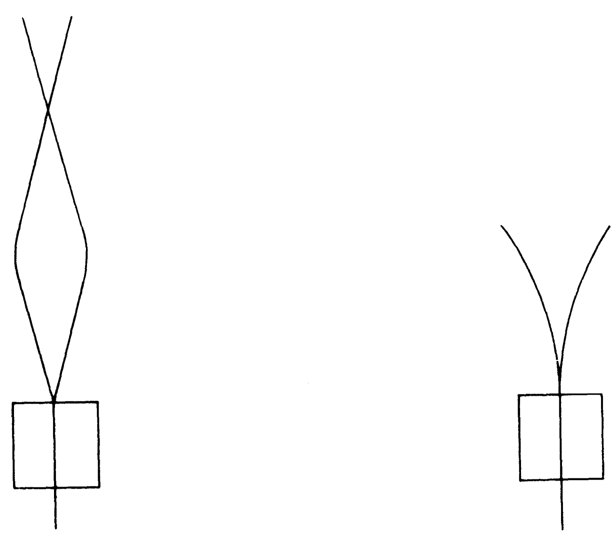

Figure 2 — Modes of test piece vibration

Annex

Theoretical basis of test method

The fundamental resonant frequency, f, in hertz, of a rectangular bar clamped at one end is given by the equation

$$f = \frac{1}{2\sqrt{3}} \times \frac{d}{l^2} \times \frac{k^2}{2\pi} \sqrt{\frac{E}{\varrho}} \qquad \ldots (4)$$

where

d is the thickness of the test piece;

E is the modulus of elasticity, i.e. Young's modulus;

k is a dimensionless constant, equal to 1,875 for the fundamental frequency of oscillation;

l is the length of test piece;

ϱ is the density of the material.

By definition (see clause 3) the bending stiffness, S, is given by the equation

$$S = \frac{EI}{b} \qquad \ldots (1)$$

where

I is the second moment of area (moment of inertia) of the cross-sectional area of the material, about an axis through the centre of that area, in its plane, and perpendicular to the direction of bending;

b is the width of the test piece.

Squaring equation (4) and substituting ϱ_A / d for ϱ, where ϱ_A is the grammage, gives the equation

$$f^2 = \frac{d^3}{12} \times \frac{1}{l^4} \times \frac{k^4}{4\pi^2} \times \frac{E}{\varrho_A}$$

Substituting I/b for $d^3/12$ and rearranging gives the equation

$$S = \frac{EI}{b} = 3{,}19\, \varrho_A\, l^4 f^2$$

Putting f = 25 Hz leads to equations (2) and (3) (see clause 10).