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# INTERNATIONAL STANDARD

## NORME INTERNATIONALE



Waveguide type dielectric resonators - RD PREVIEW Part 1-4: General information and test conditions - Measurement method of complex relative permittivity for dielectric resonator materials at millimetre-wave frequency IEC 61338-1-4:2005

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Résonateurs diélectriques à modes guidés - 1-4-2005

Partie 1-4: Informations générales et conditions d'essais – Méthode de mesure de la permittivité relative complexe des matériaux des résonateurs diélectriques fonctionnant à des fréquences millimétriques





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## INTERNATIONAL ELECTROTECHNICAL COMMISSION

## WAVEGUIDE TYPE DIELECTRIC RESONATORS -

## Part 1-4: General information and test conditions – Measurement method of complex relative permittivity for dielectric resonator materials at millimetre-wave frequency

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International Standard IEC 61338-1-4 has been prepared by IEC Technical committee 49: Piezoelectric and dielectric devices for frequency control and selection.

This bilingual version (2014-02) corresponds to the monolingual English version, published in 2005-11.

The text of this standard is based on the following documents:

FDIS	Report on voting	
49/748/FDIS	49/751/RVD	

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

The French version of this standard has not been voted upon.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

IEC 61338 consists of the following parts, under the general title *Waveguide type dielectric resonators:* 

- Part 1: Generic specification
- Part 1-3: General information and test conditions Measurement method of complex relative permittivity for dielectric resonator materials at microwave frequency
- Part 1-4: General information and test conditions Measurement method of complex relative permittivity for dielectric resonator materials at millimetre-wave frequency
- Part 2: Guidelines for oscillator and filter applications
- Part 4: Sectional specification
- Part 4-1: Blank detail specification

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## WAVEGUIDE TYPE DIELECTRIC RESONATORS -

## Part 1-4: General information and test conditions – Measurement method of complex relative permittivity for dielectric resonator materials at millimetre-wave frequency

## **1** Scope and object

This part of IEC 61338 describes the measurement method of dielectric properties for dielectric resonator materials at millimetre-wave frequency.

This standard consists of two measurement methods: a) the dielectric rod resonator method excited by NRD-guide (Non-Radiative Dielectric waveguide) and b) the cut-off waveguide method excited by coaxial cables with small loops.

- a) The dielectric rod resonator method excited by NRD-guide is similar to the dielectric rod resonator method given in IEC 61338-1-3. This method has the following characteristics:
  - a complete and exact mathematical solution of complex permittivity is given by computer software;
  - the measurement error is less than 0,3 % for  $\mathcal{E}$  and less/than 0,05  $\times 10^{-4}$  for tan  $\delta$ ;
  - the applicable measuring ranges of complex permittivity for this method are as follows:
     frequency: 30 GHz < f < 100 GHz;</li>

relative permittivity: $2 < \mathcal{E}' \textcircled{30}$ loss factor:https://standards.iteh.ai/catalog/standards/sist/9893eb0d-4e18-4fb5-b5bd-<br/>200b003c80d/iec-61338-1-4-2005

- b) The cut-off waveguide method excited by coaxial cables with small loops uses a dielectric plate sample placed in a circular cylinder of the TE<sub>011</sub> mode. This method has the following characteristics:
  - fringe effect is corrected using the correction charts on the basis of rigorous analysis;
  - the measurement error is less than 0,5 % for  $\varepsilon'$  and less than 0,05×10<sup>-4</sup> for tan  $\delta$ ;
  - the *TCF* is measured with high accuracy;
  - the applicable measuring ranges of dielectric properties for this method are as follows:
     frequency: 30 GHz < f < 100 GHz;</li>

relative permittivity:  $2 < \varepsilon' < 30;$ loss factor:  $10^{-6} < \tan \delta < 10^{-2}.$ 

## 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 61338-1-3, Waveguide type dielectric resonators – Part 1-3: General information and test conditions – Measurement method of complex relative permittivity for dielectric resonator materials at microwave frequency

#### 3 Measurement parameter

The measuring parameters are defined as follows:

$$\dot{\varepsilon}_r = \varepsilon' - j\varepsilon'' = D/(\varepsilon_0 E) \tag{1}$$

$$\tan \,\delta = \varepsilon'' / \varepsilon' \tag{2}$$

$$TC\varepsilon = \frac{1}{\varepsilon_{\text{ref}}} \frac{\varepsilon_{\text{T}} - \varepsilon_{\text{ref}}}{T - T_{\text{ref}}} \times 10^6 \qquad (1 \times 10^{-6}/\text{K})$$
(3)

$$TCF = \frac{1}{f_{\text{ref}}} \frac{f_T - f_{\text{ref}}}{T - T_{\text{ref}}} \times 10^6 \qquad (1 \times 10^{-6} / \text{K})$$
(4)

where

D is the electric flux density; Ε is the electric field strength; is the permittivity in a vacuum;  $\mathcal{E}_0$ is the complex relative permittivity. D PREVIEW Ė, are the real and imaginary components of the complex relative permittivity  $\dot{\varepsilon}_r$ ;  $\varepsilon'$  and  $\varepsilon''$ (stanuarus.iten.ai) is the temperature coefficient of relative permittivity, and TCF being the TCε temperature coefficient of resonance frequency; are the real parts of the complex relative permittivity bat demonstrator T and  $\varepsilon_{\rm T}$  and  $\varepsilon_{\rm ref}$ reference temperature  $T_{ref}$  ( $T_{ref}$  = 20 °C to 25 °C), respectively; are the resonance frequency at temperature T and  $T_{ref}$ , respectively.  $f_{\rm T}$  and  $f_{\rm ref}$ 

The *TCF* is related to *TC* $\varepsilon$  by the following equation:

$$TCF = -\frac{1}{2}TC\varepsilon - \alpha \tag{5}$$

where  $\alpha$  is the coefficient of thermal expansion of the dielectric specimen.

It should be noted that this equation is satisfied when the 100 % of electro-magnetic energy in the measuring resonance mode is concentrated inside the dielectric specimen. In the actual case, *TCF* deviates by several  $10^{-6}$ /K from the calculated value, because some portion of electro-magnetic energy is stored outside the dielectric specimen.

## 4 Dielectric rod resonator method excited by NRD-guide

### 4.1 Measurement equipment and apparatus

The measurement equipment and apparatus are as follows:

#### a) Measurement equipment

Figure 1 shows a schematic diagram of the equipment required for millimetre wave measurement. For the measurement of dielectric properties, only the information on the amplitude of transmitted power is needed, that is, the information on the phase of the transmitted power is not required. A scalar network analyzer can be used for the measurement, but a vector network analyzer has an advantage in precision of the measurement data.



Figure 1a – Scalar network analyzer



Figure 1b – Vector network analyzer

Figure 1 – Schematic diagram of measurement equipment

### b) Measurement apparatus

Figure 2a shows a configuration of measuring apparatus of dielectric rod resonator method excited by NRD-guide. Figure 2b shows a cross-sectional view of the apparatus for measuring  $\varepsilon'$  and tan  $\delta$  of a dielectric specimen with height *h* and *d*. The dielectric specimen is placed at the centre of the apparatus between two parallel conducting plates, and coupled to input and output NRD-guides. There remains a small air gap  $\Delta h$  between the dielectric specimen and the upper conducting plate. For  $\Delta h < 50 \ \mu m$ , the air gap can be neglected for the calculation of  $\varepsilon'$  (see Annex A).







Figure 2b – Apparatus for  $\varepsilon'$  and tan  $\delta$  measurement



Figure 2c – Apparatus for *TCF* and *TC*  $\varepsilon$  measurement

Figure 2 – Measurement apparatus of dielectric rod resonator method excited by NRD-guide

Dimensions in millimetres

Figure 2c shows an apparatus for measuring the temperature coefficient of resonance frequency TCF or that of relative permittivity  $TC\varepsilon$ . For this measurement, the upper conducting plate should be contacted to the dielectric specimen.

strip for NRD-guide is designed to be smaller than height h of the dielectric specimen. The upper conducting plate is set gently to touch the top face of the specimen, so that an excessive pressure does not damage the surface of conducting plate.

As shown in Table 1, a diameter of the conducting plates in Figure 2b is determined by the diameter of dielectric specimen. In this measurement method, the  $\varepsilon'$  and tan  $\delta$  are calculated under the condition that the conducting plates have an infinitely large diameter. As actual conducting plates have a finite diameter, a part of electro-magnetic energy leaks outward the conducting plates. Although this leaky energy shifts the resonance frequency and decreases the unloaded Q, its contribution is negligibly small under the condition of d'/d > 5.

Table 2 shows the example of dimensions for dielectric strips of NRD-guide in Figure 2b. Dielectric strips of the NRD-guide are made of PTFE or cross-linked styrene copolymer.

Figure 3 shows a waveguide transducer that connects the measuring apparatus to the measurement equipment with WR-15 or WR-10 waveguides. Table 3 shows the dimensions of the waveguide transducers. As shown in Figure 2b, the end of the dielectric strip of the NRD-guide is sharpened in the transducer.

Diameter d' (standardsditedriodi)			
		d : diameter of dielectric specimen	
Material of conducting plate	<u>IEC 61338-</u>	Copper or silver is recommended	
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## Table 1 – Diameter of conducting plate

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### Table 2 – Dimension of dielectric strip of NRD-guide

Material	Measurement frequency range	Height h <sub>s</sub>	Width w <sub>s</sub>	
	GHz			
PTFE	55 to 65	2,25	2,50	
( & = 2,0)	75 to 80	1,80	1,90	
Cross-linked styrene	55 to 65	2,25	2,00	
Copolymer ( $\varepsilon$ ' = 2,5)	75 to 80	1,80	1,60	

Waveguide	Frequency range	h <sub>wg</sub>	<sup>₩</sup> wg	h <sub>s</sub>
	GHz	mm	mm	mm
WR-15	55 to 65	3,80	1,90	2,25
WR-10	75 to 80	2,54	1,27	1,80



Figure 3 – Waveguide transducer from NRD-guide to waveguide

## 4.2 Theory and calculation equations

## 4.2.1 Measurement of relative permittivity and loss factor

Figure 4 shows a configuration of the TE<sub>0m1</sub> mode resonator. The cylindrical dielectric specimen is short-circuited at both ends by the two parallel conducting plates. The values  $\varepsilon'$  and tan  $\delta$  of the dielectric resonator are calculated from the resonance frequency  $f_0$  and unloaded quality factor  $Q_u$  measured for the TE<sub>0m1</sub> resonance mode. It is recommended to use the TE<sub>011</sub>, TE<sub>021</sub> and TE<sub>031</sub> resonance modes for the materials with  $\varepsilon' = 2$  to 4, 4 to 20 and 20 to 30, respectively.

## 20d0b003c80d/iec-61338-1-4-2005

The resonance wavelength  $\lambda_0$  in free space and the guiding wavelength  $\lambda_g$  in the dielectric transmission line are given by the following equations:

$$\lambda_0 = \frac{c}{f_0}, \quad \lambda_g = 2h \tag{6}$$

where c is the velocity of light in a vacuum ( $c = 2,997.9 \times 10^8 \text{ m/s}$ ).

As described in 4.1b), the air gap  $\Delta h$  can be neglected for the calculation of  $\varepsilon'$  and tan  $\delta$  in the case of  $\Delta h < 50 \ \mu m$ . So, the height h is used in equation (6).



Figure 4 – Configuration of a cylindrical dielectric rod resonator short-circuited at both ends by two parallel conducting plates

The value  $v^2$  is calculated from  $\lambda_0$  and  $\lambda_g$ :

Using the value 
$$v^2$$
, the value  $u^2$  is calculated  $\frac{38-1-4:2005}{2000003c800}$   
https://standards.iteh.ai/catalog/standards/sist/9893eb0d-4e18-4fb5-b5bd-  
 $2000b003c800/iec-61338-1-4-2005$   
 $u\frac{J_0(u)}{J_1(u)} = -v\frac{K_0(v)}{K_1(v)}$ 
(8)

where  $J_n(u)$  is the Bessel function of the first kind and  $K_n(v)$  is the modified Bessel function of the second kind. For any value of v, the m-th solution u exists between  $u_{0m}$  and  $u_{1m}$ , where  $J_0(u_{0m}) = 0$  and  $J_1(u_{1m}) = 0$ . The first, second and third solution of m = 1, 2 and 3 are shown in Figure 5.



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The relative permittivity  $\varepsilon'$  is calculated by the following equation using the values  $v^2$  and  $u^2$ :

$$\mathcal{E}' = \left(\frac{\lambda_0}{\pi d}\right)^2 \left(u^2 + v^2\right) + 1 \tag{9}$$

By using the measured unloaded  $\mathit{Q}, ~\mathit{Q}_{\rm U},$  the loss factor tan  $\delta$  is calculated:

$$\tan \delta = \frac{A}{Q_{\rm u}} - BR_{\rm S} = \frac{A}{Q_{\rm u}} - \frac{B'}{\sqrt{\sigma_{\rm r}}}$$
(10)