TECHNICAL REPORT

ISO/TR 7066-1

First edition
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Assessment of uncertainty in calibration and use of flow measurement devices —

Part 1:
Linear calibration relationships

Évaluation de l'incertitude dans l'étalonnage et l'utilisation des appareils de mesure du débit —

Partie 1: Relations d'étalonnage linéaires

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare International Standards. In exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

— type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;

— type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;

— type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 7066-1, which is a Technical Report of type 1, was prepared by Technical Committee ISO/TC 30, *Measurement of fluid flow in closed conduits*, Subcommittee SC 9, *Uncertainties in flow measurement*.

This document is being issued as a type 1 Technical Report because no consensus could be reached between ISO TC 30/SC 9 and ISO TAG 4, *Metrology*, concerning the harmonization of this document with the *Guide to the expression of uncertainty in measurement*, which is a basic document in the ISO/IEC Directives. A future revision of this Technical Report will align it with the *Guide*.

This first edition as a Technical Report cancels and replaces the first edition as an International Standard (ISO 7066-1:1988), which has been technically revised.

ISO/TR 7066 consists of the following parts, under the general title *Assessment of uncertainty in calibration and use of flow measurement devices*:

— *Part 1: Linear calibration relationships*

— *Part 2: Non-linear calibration relationships*

Annex A forms an integral part of this part of ISO/TR 7066. Annexes B and C are for information only.

Introduction

One of the first International Standards to specifically address the subject of uncertainty in measurement was ISO 5168, *Measurement of fluid flow — Estimation of uncertainty of a flow-rate measurement,* published in 1978. The extensive use of ISO 5168 in practical applications identified many improvements to its methods; these were incorporated into a draft revision of this International Standard, which in 1990 received an overwhelming vote in favour of its publication. ISO 7066-1, *Assessment of uncertainty in the calibration and use of flow measurement devices — Part 1: Linear calibration relationships,* published in 1989, was drawn up according to the principles outlined in ISO 5168:1978. The draft revision of ISO 7066-1 is consistent with both the draft revision of ISO 5168 and with ISO 7066-2:1988.

However, the draft revisions of both ISO/TR 5168 and ISO/TR 7066-1 were withheld from publication for a number of years since, despite lengthy discussions, no consensus could be reached with the draft version of a document under development by a Working Group of ISO Technical Advisory Group 4, *Metrology* ISO TAG 4/WG 3). The TAG 4 document, *Guide to the expression of uncertainty in measurement* (GUM), was published in late 1993 as a basic document in the ISO/IEC Directives. At a meeting of the ISO Technical Management Board in May 1995 it was decided to publish the revisions of ISO 5168 and ISO 7066-1 as Technical Reports.

This document is published as a type 1 Technical Report instead of an International Standard because it is not consistent with the GUM. A future revision of this part of ISO/TR 7066 will align the two documents.

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Assessment of uncertainty in calibration and use of flow measurement devices —

Part 1:
Linear calibration relationships

1 Scope

1.1 This part of ISO/TR 7066 describes the procedures to be used in deriving the calibration curve for any method of measuring flowrate in closed conduits or open channels, and of assessing the uncertainty associated with such calibrations. Procedures are also given for estimation of the uncertainty arising in measurements obtained with the use of the resultant graph, and for calculation of the uncertainty in the mean of a number of measurements of the same flowrate.

1.2 Only linear relationships are considered in this part of ISO/TR 7066; the uncertainty in non-linear relationships forms the subject of ISO/TR 7066-2. This part of ISO/TR 7066 is applicable, therefore, only if

a) the relationship between the two variables is itself linear,

or

one or both variables can be transformed in such a manner as to create a linear relationship between them, as, for instance by the use of logarithms,

or

the total range can be subdivided in such a way that within each subdivision the relationship between the two variables can be regarded as being linear; and if

b) systematic deviations from the fitted line are negligible compared with the uncertainty associated with the individual observations forming the graph.

NOTE — Examples of the application of the principles contained in this part of ISO/TR 7066 are given in annexes B and C.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this part of ISO/TR 7066. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this part of ISO/TR 7066 are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 772:1996, *Hydrometric determinations — Vocabulary and symbols.*

ISO 1100-2:—[1], *Liquid flow measurement in open channels — Part 2: Determination of the stage-discharge relationship.*

ISO 4006:1991, *Measurement of fluid flow in closed conduits — Vocabulary and symbols.*

ISO/TR 5168:—[2] *Measurement of fluid flow — Evaluation of uncertainties.*

ISO 7066-2:1988, *Assessment of uncertainty in the calibration and use of flow measurement devices — Part 2: Non-linear calibration relationships.*

3 Definitions and symbols

For the purposes of this part of ISO/TR 7066, the definitions and symbols given in ISO 772 and ISO 4006 and the following definitions and symbols apply.

3.1 Definitions

3.1.1 calibration graph: Curve drawn through the points obtained by plotting some index of the response of a flow meter against some function of the flowrate.

3.1.2 confidence limits: Upper and lower limits about an observed or calculated value within which the true value is expected to lie with a specified probability, assuming a negligible uncorrected systematic error.

3.1.3 correlation coefficient: Indicator of the degree of relationship between two variables.

NOTE — Such a relationship may be causal or may operate through the agency of a third variable, but a decision on this point cannot be made on statistical grounds alone.

3.1.4 covariance: First product moment measured about the variate means, i.e.

$$\mathrm{Cov}(x, y) = \left[\sum (x_i - \bar{x})(y_i - \bar{y})\right] / (n-1)$$

3.1.5 error of measurement: Collective term meaning the difference between the measured value and the true value.

It includes both systematic and random components.

3.1.6 error, random: That component of the error of measurement which varies unpredictably from measurement to measurement.

NOTE — No correction is possible for this type of error, the cause of which may be known or unknown.

3.1.7 error, systematic: That component of the error of measurement which remains constant or varies predictably from measurement to measurement.

NOTE — The cause of this type of error may be known or unknown.

1) To be published. (Revision of ISO 1100-2:1982)

2) To be published.

3.1.8 error, spurious: Error which invalidates a measurement.

Such errors generally have a single cause, such as instrument malfunction or the misrecording of one or more digits of the measurement value.

3.1.9 function: Mathematical formula expressing the relationship between two or more variables.

3.1.10 line of best fit: Line drawn through a series of points in such a way as to minimize the variance of the points about the line.

3.1.11 residual: Difference between an observed value and the corresponding value calculated from the regression equation.

3.1.12 sample [experimental] standard deviation: Measure of the dispersion about the mean of a series of n values of a measurand, defined by the formula:

$$s(x) = \left[\sum (x_i - \bar{x})^2 / (n-1)\right]^{1/2}$$

NOTE — If the n measurements are regarded as a sample of the underlying population, then the formula below provides a sample estimate of the population standard deviation.

$$\sigma = \left[\sum (x_i - \mu)^2 / n\right]^{1/2}$$

3.1.13 systematic error limit: That component of the total uncertainty associated with the systematic error.

Its value cannot be reduced by taking many measurements.

3.1.14 uncertainty, random: Estimate characterizing the range of values within which it is asserted with a given degree of confidence that the true value of the measurand may be expected to lie.

Its magnitude in terms of mean values may be reduced by taking many measurements.

3.1.15 variance: Measure of dispersion based on the mean squared deviation from the arithmetic mean, defined as

$$\mathrm{Var}(x) = \sum (x_i - \bar{x})^2 / (n-1)$$

3.2 Symbols

NOTE — Symbols used in the open channel and closed conduit examples of annexes B and C where these differ from, or are in addition to, those listed below are included at the beginning of the respective annexes.

a	Intercept of the calibration curve on the ordinate
b	Gradient or slope of the calibration curve
c	Coefficient in a weighted least-squares equation
Cov( )	Covariance of variables in brackets
e_R( )	Random uncertainty of variable in brackets
e_S( )	Systematic error limits of variable in brackets
ln	Natural logarithm
n	Number of measurements used in deriving the calibration curve
Q	Flowrate
r	Correlation coefficient

$s(\,)$ Experimental standard deviation of variable in brackets

s_R Standard deviation (standard error) of points about best-fitting straight line

t "Student's" t (as obtained from ISO 5168 or from any set of statistical tables)

w_i ith weighting factor, in weighted least-squares

x Independent variable; variable subject to the smallest error

y Dependent variable; variable subject to the greatest error

U Total or overall uncertainty

U_{ADD} Uncertainty using the additive model; provides between approximately 95 % and 99 % coverage

$$U_{ADD} = e_S + e_R$$

U_{RSS} Uncertainty using the root-sum-square model; provides approximately 95 % coverage

$$U_{RSS} = (e_S{}^2 + e_R{}^2)^{1/2}$$

γ Ratio of the standard deviation of the independent, or x, variable to that of the dependent, or y, variable

Δ Difference between an observed and a calculated value

μ Population mean

σ Population standard deviation

θ Influence coefficient

NOTE — In a number of International Standards, the random uncertainty e_R and systematic error limits e_S are denoted by the symbols U_r and U_s or B respectively.

Subscripts and superscripts

NOTE — In the following, the summation sign Σ is used to represent

$$\sum_{i=1}^{n}$$

unless otherwise noted; a bar above a symbol (¯) denotes the mean value of that quantity; a circumflex (^) denotes the value of the variable predicted by the equation of the fitted curve.

i ith value of a variable

ij ith value of the jth category

4 General

4.1 With the majority of calibrations considered in this part of ISO 7066, the relationship between the variables is of a functional nature and is defined by some form of mathematical expression. Any departure of the observed values from this relationship can then be attributed to errors of measurement of one kind or another, which may affect either or both variables and which may be random or systematic or a combination of the two.

4.2 The role of the calibration procedure is thus twofold: to assess the form of the underlying mathematical relationship and to provide an estimate of the uncertainty of the fitted line.

4.3 From a practical viewpoint there will exist pairs of values (x, y) for which the random uncertainties and systematic error limits in x and y will have been estimated by one of the methods given in clause 5. The choice of the procedure to be used in the calculation of the coefficients and uncertainty of the calibration equation will depend on the relative magnitudes of the random components $e_R(x)$ and $e_R(y)$.

4.4 Where the error in one or the other of the two variables can be assumed to be negligible, the methods set out in clauses 8, 9 and 11 shall be used, the underlying equation being taken to be of the form

$$y = a + bx \qquad \ldots (1)$$

where

x is that variable with the smaller error;

a and b are coefficients of the fitted line to be determined.

Where both variables are subject to error and x is the variable with the smaller error, the methods described in clauses 8 and 9 can still be used if the x variable can be set to predetermined values during the calibration. This approach is known as the Berkson method.

4.5 A special case arises where y is effectively constant and independent of x, i.e. where the fitted line is parallel to the x-axis. In these cases, the methods specified in clause 10 shall be used in estimating the uncertainty.

4.6 To provide the information needed in selecting the fitting procedure to be used, a preliminary study of the data is essential. In particular, this should be directed towards establishing the uncertainties and systematic error limits in x and y and the adequacy of the linearity assumption. Where the relationship is known to be curvilinear, some attention should be given to the possibility of converting it to a linear form, thus simplifying the subsequent manipulation of the data.

5 Random uncertainties and systematic error limits in individual measurements

5.1 In determining the random uncertainties and systematic error limits in the two variables, there are no alternatives to the procedures given in ISO/TR 5168. As a first step in the estimation process, a table for each variable should be prepared indicating the various sources of error. These should include the errors in any basic measurements which have to be made and should list the random and systematic elements separately.

5.2 For variate values determined by direct measurement, the random uncertainty at a fixed value of the measurand x can be found by calculating the experimental standard deviation from a series of n measurements, using the formula

$$s = \left[\sum (x_i - \bar{x})^2 / (n-1)\right]^{1/2} \qquad \ldots (2)$$

or, alternatively,

$$s = \left\{\left[n\sum x_i^2 - \left(\sum x_i\right)^2\right] \Big/ \left[n(n-1)\right]\right\}^{1/2} \qquad \ldots (3)$$

and then substituting into

$$e_R(x) = ts(x) \qquad \ldots (4)$$

5.3 In carrying out the above calculations, it should be remembered that the result obtained may vary depending on the magnitude of y at which x is measured. Similarly, the uncertainty in y, which can be found by substituting y for x in the above formulae, may also vary with the value of x at which it is measured. Since such variations will dictate the method to be used in the subsequent fitting of the calibration curve, it is essential that the estimation of uncertainty be carried out at a sufficient number of points to enable the extent of any problem to be accurately assessed.

5.4 Where the variate values are obtained as the sum or difference of a number of independent component measurements, the uncertainty shall be obtained by calculating the overall standard deviation from the formula:

$$s(x) = \left[\sum s(x_i)^2 \right]^{1/2} \qquad \text{. . . (5)}$$

followed by substitution into equation (4).

In other cases, where the variables are derived from more complex functions of the constituent elements such as products or quotients, or where the elements are correlated, the overall standard deviation shall be determined by the methods given in annex A. The uncertainty may then again be obtained by substituting into equation (4).

5.5 The evaluation of the systematic error, which is somewhat more difficult, is described in ISO/TR 5168. Even when all known sources have been identified and allowed for, there will still remain a number of unidentified errors. In these cases any assessment will depend on a subjective judgement based on such evidence, e.g. past calibrations, previous history, etc., as is available.

5.6 Where the variate values are based on the sum of a number of elemental components, some difficulty may be experienced in determining the overall systematic error limits, due to the fact that, in a majority of cases, the sign of the components is unknown. In these instances the errors shall be combined using the root-sum-square procedure as defined by

$$e_S = \left(\sum_i e_{S,i}^2 \right)^{1/2} \qquad \text{. . . (6)}$$

Where more complex funtions are involved, the systematic error limit, e_S, shall be found using the method given in annex A, replacing the variance terms by the corresponding $e_{S,i}^2$ terms.

5.7 The estimation process can be regarded as complete once all the sources of error have been identified and evaluated and the individual elements combined to give an overall assessment of the random uncertainty and systematic error limits for each variable.

6 Linearity of calibration graph

6.1 An initial investigation is also desirable to establish whether a linear calibration curve will provide an adequate and unbiased fit to the observed measurements. Of the methods available, the most effective are those based on a visual study of the deviations of the measurements from the fitted line. An approximation to this line can be obtained using Bartlett's method, as described in 6.2 to 6.5.

6.2 As a first step, the data should be ranked in ascending order in either the x or y direction, and the general means of the two variables found from the equations

$$\bar{x} = \sum x_i / n; \quad \bar{y} = \sum y_i / n \qquad \text{. . . (7)}$$

The data should now be divided into three equal and mutually exclusive groups and the means of the two end groups calculated as before. Denoting these by $\bar{x}_1, \bar{y}_1$ and $\bar{x}_3, \bar{y}_3$, respectively, the slope b of the approximate line can be found as

$$b = (\bar{y}_3 - \bar{y}_1) / (\bar{x}_3 - \bar{x}_1) \qquad \text{. . . (8)}$$