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NORME INTERNATIONALE

Electric power engineering – Modal components in three-phase a.c. systems – Quantities and transformations

Energie électrique – Composantes modales dans les systèmes a.c. triphasés – Grandeurs et transformations

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ELECTRIC POWER ENGINEERING – MODAL COMPONENTS IN THREE-PHASE AC SYSTEMS – QUANTITIES AND TRANSFORMATIONS

FOREWORD

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International Standard IEC 62428 has been prepared by IEC technical committee 25: Quantities and units.

The text of this standard is based on the following documents:

FDIS	Report on voting
25/382/FDIS	25/390/RVD

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

The committee has decided that the contents of this publication will remain unchanged until the maintenance result date indicated on the IEC web site under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed;
- withdrawn;
- replaced by a revised edition; or
- amended.

ELECTRIC POWER ENGINEERING – MODAL COMPONENTS IN THREE-PHASE AC SYSTEMS – QUANTITIES AND TRANSFORMATIONS

1 Scope

This International Standard deals with transformations from original quantities into modal quantities for the widely used three-phase a.c. systems in the field of electric power engineering.

The examination of operating conditions and transient phenomena in three-phase a.c. systems becomes more difficult by the resistive, inductive or capacitive coupling between the phase elements and line conductors. Calculation and description of these phenomena in three-phase a.c. systems are easier if the quantities of the coupled phase elements and line conductors are transformed into modal quantities. The calculation becomes very easy if the transformation leads to decoupled modal systems. The original impedance and admittance matrices are transformed to modal impedance and admittance matrices. In the case of decoupling of the modal quantities, the modal impedance and admittance matrices become diagonal matrices.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-141, *International Electrotechnical Vocabulary (IEV) – Part 141: Polyphase systems and circuits*

3 Terms, definitions, quantities and concepts

3.1 General

Quantities in this standard are usually time-dependent. These quantities are for instance electric currents, voltages, linked fluxes, current linkages, electric and magnetic fluxes.

For quantities the general letter symbol g in case of real instantaneous values, $\underline{g}$ in case of complex instantaneous values and $\underline{G}$ in case of phasors (complex r.m.s. values) are used.

NOTE Complex quantities in this standard are underlined. Conjugated complex quantities are indicated by an additional asterisk (*). Matrices and column vectors are printed in bold face type, italic.

3.2 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC 60050-141 and the following apply.

3.2.1
original quantities
quantities g or $\underline{G}$ of a three-phase a.c. system

NOTE Subscripts 1, 2, 3 are generally used in this standard; additional letters may be put, for instance L1, L2, L3 as established in IEC 60909, IEC 60865 and IEC 61660.

3.2.2
modal components
quantities g_{M}, $\underline{g}_{\mathrm{M}}$ or $\underline{G}_{\mathrm{M}}$ found by a transformation from the original quantities according to Clause 3

NOTE Additional subscripts 1, 2, 3 are used.

3.2.3
column vector of quantities
column matrix containing the three original quantities or modal components of a three-phase a.c. system

NOTE Column vectors are described by $\boldsymbol{g}$ or $\underline{\boldsymbol{g}}_{\mathrm{M}}$ and $\underline{\boldsymbol{G}}$ or $\underline{\boldsymbol{G}}_{\mathrm{M}}$, respectively.

3.2.4
modal transformation
matrix equation $\underline{\boldsymbol{T}}\ \underline{\boldsymbol{g}}_{\mathrm{M}} = \underline{\boldsymbol{g}}$ for a column vector $\underline{\boldsymbol{g}}_{\mathrm{M}}$ containing the three unknown modal quantities, where $\underline{\boldsymbol{g}}$ is a column vector containing the three given original quantities and $\underline{\boldsymbol{T}}$ is a 3×3 transformation matrix

NOTE The transformation can be power-variant or power-invariant, see Tables 1 and 2.

3.2.5
inverse modal transformation
solution $\underline{\boldsymbol{g}}_{\mathrm{M}} = \underline{\boldsymbol{T}}^{-1}\ \underline{\boldsymbol{g}}$ of the modal transformation that expresses a column vector $\underline{\boldsymbol{g}}_{\mathrm{M}}$ containing the three modal quantities as a matrix product of the inverse transformation matrix $\underline{\boldsymbol{T}}^{-1}$ by a column vector $\underline{\boldsymbol{g}}$ containing the three original quantities

3.2.6
transformation into symmetrical components
Fortescue transformation
linear modal transformation with constant complex coefficients, the solution of which converts the three original phasors of a three-phase a.c. system into the reference phasors of three symmetric three-phase a.c. systems — the so-called symmetrical components — , the first system being a positive-sequence system, the second system being a negative-sequence system and the third system being a zero-sequence system

NOTE 1 The transformation into symmetrical components is used for example for the description of asymmetric steady-state conditions in three-phase a.c. systems.

NOTE 2 See Tables 1 and 2.

3.2.7
transformation into space phasor components
linear modal transformation with constant or angle-dependent coefficients, the solution of which replaces the instantaneous original quantities of a three-phase a.c. system by the complex space phasor in a rotating or a non-rotating frame of reference, its conjugate complex value and the real zero-sequence component

NOTE 1 The term “space vector” is also used for “space phasor”.

NOTE 2 The space phasor transformation is used for example for the description of transients in three-phase a.c. systems and machines.

NOTE 3 See Tables 1 and 2.

3.2.8
transformation into αβ0 components
Clarke transformation
linear modal transformation with constant real coefficients, the solution of which replaces the instantaneous original quantities of a three-phase a.c. system by the real part and the

imaginary part of a complex space phasor in a non-rotating frame of reference and a real zero-sequence component or replaces the three original phasors of the three-phase a.c. system by two phasors (α and β phasor) and a zero-sequence phasor

NOTE 1 The power-variant form of the space phasor is given by $\underline{g}_{\text{s}} = g_\alpha + \text{j}\,g_\beta$ and the power-invariant form is given by $\underline{g}_{\text{s}} = \frac{1}{\sqrt{2}}(g_\alpha + \text{j}\,g_\beta)$.

NOTE 2 The αβ0 transformation is used for example for the description of asymmetric transients in three-phase a.c. systems.

NOTE 3 See tables 1 and 2.

3.2.9
transformation into dq0 components
Park transformation
linear modal transformation with coefficients sinusoidally depending on the angle of rotation, the solution of which replaces the instantaneous original quantities of a three-phase a.c. system by the real part and the imaginary part of a complex space phasor in a rotating frame of reference and a real zero-sequence component

NOTE 1 The power-variant form of the space phasor is given by $\underline{g}_{\text{r}} = g_{\text{d}} + \text{j}\,g_{\text{q}}$ and the power-invariant form is given by $\underline{g}_{\text{r}} = \frac{1}{\sqrt{2}}(g_{\text{d}} + \text{j}\,g_{\text{q}})$.

NOTE 2 The dq0 transformation is normally used for the description of transients in synchronous machines.

NOTE 3 See Tables 1 and 2.

4 Modal transformation

4.1 General

The original quantities g_1, g_2, g_3 and the modal components $\underline{g}_{\text{M1}}, \underline{g}_{\text{M2}}, \underline{g}_{\text{M3}}$ are related to each other by the following transformation equations:

$$\begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} = \begin{pmatrix} \underline{t}_{11} & \underline{t}_{12} & \underline{t}_{13} \\ \underline{t}_{21} & \underline{t}_{22} & \underline{t}_{23} \\ \underline{t}_{31} & \underline{t}_{32} & \underline{t}_{33} \end{pmatrix} \begin{pmatrix} \underline{g}_{\text{M1}} \\ \underline{g}_{\text{M2}} \\ \underline{g}_{\text{M3}} \end{pmatrix} \tag{1}$$

or in a shortened form:

$$\boldsymbol{g} = \underline{\boldsymbol{T}}\,\underline{\boldsymbol{g}}_{\text{M}} \tag{2}$$

The coefficients $\underline{t}_{ik}$ of the transformation matrix $\underline{\boldsymbol{T}}$ can all be real or some of them can be complex. It is necessary that the transformation matrix $\underline{\boldsymbol{T}}$ is non-singular, so that the inverse relationship of Equation (2) is valid.

$$\underline{\boldsymbol{g}}_{\text{M}} = \underline{\boldsymbol{T}}^{-1}\,\boldsymbol{g} \tag{3}$$

If the original quantities are sinusoidal quantities of the same frequency, it is possible to express them as phasors and to write the transformation Equations (2) and (3) in an analogue form with constant coefficients:

$$\begin{pmatrix} \underline{G}_1 \\ \underline{G}_2 \\ \underline{G}_3 \end{pmatrix} = \begin{pmatrix} \underline{t}_{11} & \underline{t}_{12} & \underline{t}_{13} \\ \underline{t}_{21} & \underline{t}_{22} & \underline{t}_{23} \\ \underline{t}_{31} & \underline{t}_{32} & \underline{t}_{33} \end{pmatrix} \begin{pmatrix} \underline{G}_{\mathrm{M1}} \\ \underline{G}_{\mathrm{M2}} \\ \underline{G}_{\mathrm{M3}} \end{pmatrix} \tag{4}$$

$$\underline{\boldsymbol{G}} = \underline{\boldsymbol{T}}\,\underline{\boldsymbol{G}}_{\mathrm{M}} \tag{5}$$

$$\underline{\boldsymbol{G}}_{\mathrm{M}} = \underline{\boldsymbol{T}}^{-1}\,\underline{\boldsymbol{G}} \tag{6}$$

4.2 Power in modal components

Transformation relations are used either in the power-variant form as given in Table 1 or in the power-invariant form as given in Table 2.

For the power-invariant form of transformation, the power calculated with the three modal components is equal to the power calculated from the original quantities of a three-phase a.c. system with three line conductors and a neutral conductor, where u_1, u_2 and u_3 are the line-to-neutral voltages and i_1, i_2 and i_3 are the currents of the line conductors at a given location of the network. In a three-phase a.c. system with only three line conductors, u_1, u_2 and u_3 are the voltages between the line conductors and a virtual star point at a given location of the network.

The instantaneous power p expressed in terms of the original quantities is defined by:

$$p = u_1 i_1^* + u_2 i_2^* + u_3 i_3^* = \begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix} \begin{pmatrix} i_1^* \\ i_2^* \\ i_3^* \end{pmatrix} = \boldsymbol{u}^{\mathrm{T}} \boldsymbol{i}^* \tag{7}$$

NOTE The asterisks denote formally the complex conjugate of the currents i_1, i_2, i_3. If these are real, i_1^*, i_2^*, i_3^* are identical to i_1, i_2, i_3.

If the relationship between the original quantities and the modal components given in Equation (2) is introduced for the voltages as well as for the currents:

$$\boldsymbol{u} = \underline{\boldsymbol{T}}\,\underline{\boldsymbol{u}}_{\mathrm{M}} \text{ and } \boldsymbol{i} = \underline{\boldsymbol{T}}\,\underline{\boldsymbol{i}}_{\mathrm{M}} \tag{8}$$

taking into account

$$\boldsymbol{u}^{\mathrm{T}} = \left(\underline{\boldsymbol{T}}\,\underline{\boldsymbol{u}}_{\mathrm{M}}\right)^{\mathrm{T}} = \underline{\boldsymbol{u}}_{\mathrm{M}}^{\mathrm{T}}\,\underline{\boldsymbol{T}}^{\mathrm{T}}, \tag{9}$$

the power p expressed in terms of modal components is found as:

$$p = \underline{\boldsymbol{u}}_{\mathrm{M}}^{\mathrm{T}}\,\underline{\boldsymbol{T}}^{\mathrm{T}}\,\underline{\boldsymbol{T}}^{*}\,\underline{\boldsymbol{i}}_{\mathrm{M}}^{*}. \tag{10}$$

For the power-variant case where $\underline{\boldsymbol{T}}^{\mathrm{T}}\underline{\boldsymbol{T}}^{*}$ is not equal to the unity matrix an example is given at the end of this section. In case of

$$\underline{\boldsymbol{T}}^{\mathrm{T}}\,\underline{\boldsymbol{T}}^{*} = \boldsymbol{E} \tag{11}$$

with the matrix $\boldsymbol{E}$ being the unity matrix of third order, Equation (10) changes to

$$p = \underline{\boldsymbol{u}}_{\mathrm{M}}^{\mathrm{T}} \underline{\boldsymbol{i}}_{\mathrm{M}}^{*} = \begin{pmatrix} \underline{u}_{\mathrm{M1}} & \underline{u}_{\mathrm{M2}} & \underline{u}_{\mathrm{M3}} \end{pmatrix} \begin{pmatrix} \underline{i}_{\mathrm{M1}}^{*} \\ \underline{i}_{\mathrm{M2}}^{*} \\ \underline{i}_{\mathrm{M3}}^{*} \end{pmatrix} = \underline{u}_{\mathrm{M1}} \underline{i}_{\mathrm{M1}}^{*} + \underline{u}_{\mathrm{M2}} \underline{i}_{\mathrm{M2}}^{*} + \underline{u}_{\mathrm{M3}} \underline{i}_{\mathrm{M3}}^{*} . \tag{12}$$

The condition $\underline{\boldsymbol{T}}^{\mathrm{T}} \underline{\boldsymbol{T}}^{*} = \boldsymbol{E}$ or $\underline{\boldsymbol{T}}^{-1} = \underline{\boldsymbol{T}}^{\mathrm{T}*}$ means that the transformation matrix $\underline{\boldsymbol{T}}$ is a unitary matrix.

Because the Equations (7) and (12) have identical structure, the transformation relationship with a unitary matrix is called the power invariant form of transformation.

In connection with Table 2, the following examples can be given:

$$p_{\alpha\beta 0} = u_{\alpha} i_{\alpha} + u_{\beta} i_{\beta} + u_{0} i_{0}$$

$$p_{\mathrm{dq0}} = u_{\mathrm{d}} i_{\mathrm{d}} + u_{\mathrm{q}} i_{\mathrm{q}} + u_{0} i_{0}$$

$$p_{\mathrm{ss}^{*}0} = \underline{u}_{\mathrm{s}} \underline{i}_{\mathrm{s}}^{*} + \underline{u}_{\mathrm{s}}^{*} \underline{i}_{\mathrm{s}} + u_{0} i_{0} = 2\mathrm{Re}\left\{ \underline{u}_{\mathrm{s}} \underline{i}_{\mathrm{s}}^{*} \right\} + u_{0} i_{0}$$

$$p_{\mathrm{rr}^{*}0} = \underline{u}_{\mathrm{r}} \underline{i}_{\mathrm{r}}^{*} + \underline{u}_{\mathrm{r}}^{*} \underline{i}_{\mathrm{r}} + u_{0} i_{0} = 2\mathrm{Re}\left\{ \underline{u}_{\mathrm{r}} \underline{i}_{\mathrm{r}}^{*} \right\} + u_{0} i_{0}$$

In case of three-phase systems of voltages and currents the complex power is given in original phasor quantities as follows:

$$\underline{S} = \underline{U}_{1} \underline{I}_{1}^{*} + \underline{U}_{2} \underline{I}_{2}^{*} + \underline{U}_{3} \underline{I}_{3}^{*} = \begin{pmatrix} \underline{U}_{1} & \underline{U}_{2} & \underline{U}_{3} \end{pmatrix} \begin{pmatrix} \underline{I}_{1}^{*} \\ \underline{I}_{2}^{*} \\ \underline{I}_{3}^{*} \end{pmatrix} = \underline{\boldsymbol{U}}^{\mathrm{T}} \underline{\boldsymbol{I}}^{*} \tag{13}$$

Substituting the modal components by

$$\underline{\boldsymbol{U}}^{\mathrm{T}} = \left(\underline{\boldsymbol{T}} \, \underline{\boldsymbol{U}}_{\mathrm{M}} \right)^{\mathrm{T}} = \underline{\boldsymbol{U}}_{\mathrm{M}}^{\mathrm{T}} \underline{\boldsymbol{T}}^{\mathrm{T}} \text{ and } \underline{\boldsymbol{I}}^{*} = \underline{\boldsymbol{T}}^{*} \underline{\boldsymbol{I}}_{\mathrm{M}}^{*}$$

the complex apparent power is found as:

$$\underline{S} = \underline{\boldsymbol{U}}_{\mathrm{M}}^{\mathrm{T}} \underline{\boldsymbol{T}}^{\mathrm{T}} \underline{\boldsymbol{T}}^{*} \underline{\boldsymbol{I}}_{\mathrm{M}}^{*} \tag{14}$$

In case of power invariance, the condition $\underline{\boldsymbol{T}}^{\mathrm{T}} \underline{\boldsymbol{T}}^{*} = \boldsymbol{E}$ must also be valid. Then Equation (14) leads to the following power invariant expression:

$$\underline{S} = \underline{\boldsymbol{U}}_{\mathrm{M}}^{\mathrm{T}} \underline{\boldsymbol{I}}_{\mathrm{M}}^{*} = \underline{U}_{\mathrm{M1}} \underline{I}_{\mathrm{M1}}^{*} + \underline{U}_{\mathrm{M2}} \underline{I}_{\mathrm{M2}}^{*} + \underline{U}_{\mathrm{M3}} \underline{I}_{\mathrm{M3}}^{*} = \begin{pmatrix} \underline{U}_{\mathrm{M1}} & \underline{U}_{\mathrm{M2}} & \underline{U}_{\mathrm{M3}} \end{pmatrix} \begin{pmatrix} \underline{I}_{\mathrm{M1}}^{*} \\ \underline{I}_{\mathrm{M2}}^{*} \\ \underline{I}_{\mathrm{M3}}^{*} \end{pmatrix} \tag{15}$$

The power-variant forms of transformation matrices are given in the Tables 3 and 5. They are also known as reference-component-invariant transformations, because, under balanced symmetrical conditions, the reference component (the first component) of the modal components is equal to the reference component of the original quantities or its complex phasors, respectively. This is not the case for transformations in a rotating frame.

EXAMPLE According to Table 2 for the power-invariant form of the transformation matrix $\underline{\boldsymbol{T}}$ it follows:

$$\underline{\boldsymbol{T}} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ \underline{a}^2 & \underline{a} & 1 \\ \underline{a} & \underline{a}^2 & 1 \end{pmatrix}, \quad \underline{\boldsymbol{T}}^{\mathrm{T}} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & 1 & 1 \end{pmatrix}, \quad \underline{\boldsymbol{T}}^{\mathrm{T}*} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & 1 & 1 \end{pmatrix},$$

showing that $\underline{\boldsymbol{T}}^{-1} = \underline{\boldsymbol{T}}^{\mathrm{T}*}$ or $\underline{\boldsymbol{T}}^{\mathrm{T}}\underline{\boldsymbol{T}}^{*} = \boldsymbol{E}$, fulfilling the condition for power invariance.

If the transformation matrix $\underline{\boldsymbol{T}}$ from Table 1 for the power-variant transformation is used, then the following results are found:

$$\underline{\boldsymbol{T}} = \begin{pmatrix} 1 & 1 & 1 \\ \underline{a}^2 & \underline{a} & 1 \\ \underline{a} & \underline{a}^2 & 1 \end{pmatrix}, \quad \underline{\boldsymbol{T}}^{\mathrm{T}} = \begin{pmatrix} 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & 1 & 1 \end{pmatrix}, \quad \underline{\boldsymbol{T}}^{\mathrm{T}*} = \begin{pmatrix} 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & 1 & 1 \end{pmatrix}.$$

$\underline{\boldsymbol{T}}^{-1}$ from Table 1 is equal to $\frac{1}{3}\underline{\boldsymbol{T}}^{\mathrm{T}*}$, so that $\underline{\boldsymbol{T}}^{\mathrm{T}}\underline{\boldsymbol{T}}^{*} = 3 \cdot \boldsymbol{E}$.

4.3 Established transformations

The most widely used transformation matrices $\underline{\boldsymbol{T}}$ and their inverse matrices $\underline{\boldsymbol{T}}^{-1}$ are given in the Tables 1 and 2, whereby Table 1 contains the power-variant (reference-component-invariant) form and Table 2 the power-invariant form of transformation matrices. The subscripts for the components are chosen to be equal in both cases of Tables 1 and 2, 3 and 4, 5 and 6.

The Tables 3 to 6 give the relations between the different types of modal components.

Table 1 – Power-variant form of modal components and transformation matrices

Modal components	Component: First / Second / Third	Subscript: M1 / M2 / M3	$\underline{\boldsymbol{T}}$ [a]	$\underline{\boldsymbol{T}}^{-1}$
symmetrical components (Fortescue components)	positive-sequence negative-sequence zero-sequence	(1) (2) (0) [b]	$\begin{pmatrix} 1 & 1 & 1 \\ \underline{a}^2 & \underline{a} & 1 \\ \underline{a} & \underline{a}^2 & 1 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & 1 & 1 \end{pmatrix}$
αβ0 components, non-rotating frame (Clarke components)	α β zero-sequence	α β 0	$\begin{pmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{pmatrix}$	$\frac{2}{3}\begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$
dq0 components, rotating frame (Park components)	direct-axis quadrature-axis zero-sequence	d q 0	$\begin{pmatrix} c_1 & -s_1 & 1 \\ c_2 & -s_2 & 1 \\ c_3 & -s_3 & 1 \end{pmatrix}$ [c]	$\frac{2}{3}\begin{pmatrix} c_1 & c_2 & c_3 \\ -s_1 & -s_2 & -s_3 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ [c]
space phasor components, non-rotating frame	space phasor conjugated complex space phasor zero-sequence	s s* 0	$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 \\ \underline{a}^2 & \underline{a} & 1 \\ \underline{a} & \underline{a}^2 & 1 \end{pmatrix}$	$\frac{2}{3}\begin{pmatrix} 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$
space phasor components, rotating frame	space phasor conjugated complex space phasor zero-sequence	r r* 0	$\frac{1}{2}\begin{pmatrix} e^{j\vartheta} & e^{-j\vartheta} & 2 \\ \underline{a}^2 e^{j\vartheta} & \underline{a}\ e^{-j\vartheta} & 2 \\ \underline{a}\ e^{j\vartheta} & \underline{a}^2 e^{-j\vartheta} & 2 \end{pmatrix}$ [c]	$\frac{2}{3}\begin{pmatrix} e^{-j\vartheta} & \underline{a}\ e^{-j\vartheta} & \underline{a}^2 e^{-j\vartheta} \\ e^{j\vartheta} & \underline{a}^2 e^{j\vartheta} & \underline{a}\ e^{j\vartheta} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ [c]

[a] All the transformation matrices $\underline{\boldsymbol{T}}$ given here fulfil the following conditions:

$\underline{t}_{11} + \underline{t}_{21} + \underline{t}_{31} = 0$, $\underline{t}_{12} + \underline{t}_{22} + \underline{t}_{32} = 0$, $\underline{t}_{13} = \underline{t}_{23} = \underline{t}_{33}$.

[b] The IEC Standards 60909, 60865 and 61660 have introduced the subscripts (1), (2), (0) for the power-variant form of the symmetrical components, to avoid confusion, if the subscripts 1, 2, 3 instead of L1, L2, L3 are used.

[c] $c_1 = \cos\vartheta$, $c_2 = \cos(\vartheta - \frac{2\pi}{3})$, $c_3 = \cos(\vartheta + \frac{2\pi}{3})$, $s_1 = \sin\vartheta$, $s_2 = \sin(\vartheta - \frac{2\pi}{3})$, $s_3 = \sin(\vartheta + \frac{2\pi}{3})$, $\underline{a} = e^{j2\pi/3}$, $\underline{a}^2 = \underline{a}^*$, $1 + \underline{a} + \underline{a}^2 = 0$.

In case of synchronous machines ϑ is given by $\vartheta = \int \Omega(t)\,dt$, where Ω is the instantaneous angle velocity of the rotor.

Table 2 – Power-invariant form of modal components and transformation matrices

Modal components	Component: First Second Third	Subscript: M1 M2 M3	$\underline{\boldsymbol{T}}$ [a]	$\underline{\boldsymbol{T}}^{-1}$
symmetrical components (Fortescue components)	positive-sequence negative-sequence zero-sequence	(1) (2) (0) [b]	$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ \underline{a}^2 & \underline{a} & 1 \\ \underline{a} & \underline{a}^2 & 1 \end{pmatrix}$	$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & 1 & 1 \end{pmatrix}$
$\alpha\beta 0$ components, non-rotating frame (Clarke components)	α β zero-sequence	α β 0	$\sqrt{\frac{2}{3}}\begin{pmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$	$\sqrt{\frac{2}{3}}\begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$
dq0 components, rotating frame (Park components)	direct-axis quadrature-axis zero-sequence	d q 0	$\sqrt{\frac{2}{3}}\begin{pmatrix} c_1 & -s_1 & \frac{1}{\sqrt{2}} \\ c_2 & -s_2 & \frac{1}{\sqrt{2}} \\ c_3 & -s_3 & \frac{1}{\sqrt{2}} \end{pmatrix}$ [c]	$\sqrt{\frac{2}{3}}\begin{pmatrix} c_1 & c_2 & c_3 \\ -s_1 & -s_2 & -s_3 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ [c]
space phasor components, non-rotating frame	space phasor conjugated complex space phasor zero-sequence	s s* 0	$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ \underline{a}^2 & \underline{a} & 1 \\ \underline{a} & \underline{a}^2 & 1 \end{pmatrix}$	$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & 1 & 1 \end{pmatrix}$
space phasor components, rotating frame	space phasor conjugated complex space phasor zero-sequence	r r* 0	$\frac{1}{\sqrt{3}}\begin{pmatrix} e^{j\vartheta} & e^{-j\vartheta} & 2 \\ \underline{a}^2 e^{j\vartheta} & \underline{a}\, e^{-j\vartheta} & 2 \\ \underline{a}\, e^{j\vartheta} & \underline{a}^2 e^{-j\vartheta} & 2 \end{pmatrix}$ [c]	$\frac{1}{\sqrt{3}}\begin{pmatrix} e^{-j\vartheta} & \underline{a}\ e^{-j\vartheta} & \underline{a}^2 e^{-j\vartheta} \\ e^{j\vartheta} & \underline{a}^2 e^{j\vartheta} & \underline{a}\ e^{j\vartheta} \\ 1 & 1 & 1 \end{pmatrix}$ [c]

[a] All the transformation matrices $\underline{\boldsymbol{T}}$ given here fulfil the following conditions:

$$\underline{t}_{11} + \underline{t}_{21} + \underline{t}_{31} = 0, \quad \underline{t}_{12} + \underline{t}_{22} + \underline{t}_{32} = 0, \quad \underline{t}_{13} = \underline{t}_{23} = \underline{t}_{33}.$$

[b] The IEC Standards 60909, 60865 and 61660 have introduced the subscripts (1), (2), (0) for the power-variant form of the symmetrical components, to avoid confusion, if the subscripts 1, 2, 3 instead of L1, L2, L3 are used.

[c] $c_1 = \cos\vartheta$, $c_2 = \cos(\vartheta - \frac{2\pi}{3})$, $c_3 = \cos(\vartheta + \frac{2\pi}{3})$, $s_1 = \sin\vartheta$, $s_2 = \sin(\vartheta - \frac{2\pi}{3})$, $s_3 = \sin(\vartheta + \frac{2\pi}{3})$, $\underline{a} = e^{j\,2\pi/3}$, $\underline{a}^2 = \underline{a}^*$, $1 + \underline{a} + \underline{a}^2 = 0$.

In case of synchronous machines ϑ is given by $\vartheta = \int \Omega(t)\,dt$, where Ω is the instantaneous angle velocity of the rotor.

Tables 3 and 4 contain the relations of the $\alpha\beta 0$ components and the dq0 components with the space phasor components in the power-variant and the power-invariant form.

Table 3 – Clark, Park and space phasor components – modal transformations in the power-variant form

	$\bullet\begin{pmatrix} g_1 & g_2 & g_3 \end{pmatrix}^T$	$\bullet\begin{pmatrix} g_\alpha & g_\beta & g_0 \end{pmatrix}^T$	$\bullet\begin{pmatrix} g_d & g_q & g_0 \end{pmatrix}^T$	$\bullet\begin{pmatrix} \underline{g}_s & \underline{g}_s^* & \underline{g}_0 \end{pmatrix}^T$	$\bullet\begin{pmatrix} \underline{g}_r & \underline{g}_r^* & \underline{g}_0 \end{pmatrix}^T$
$\begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} =$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{pmatrix}$	$\begin{pmatrix} c_1 & -s_1 & 1 \\ c_2 & -s_2 & 1 \\ c_3 & -s_3 & 1 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} 1 & 1 & 2 \\ \underline{a}^2 & \underline{a} & 2 \\ \underline{a} & \underline{a}^2 & 2 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} e^{j\vartheta} & e^{-j\vartheta} & 2 \\ \underline{a}^2 e^{j\vartheta} & \underline{a}e^{-j\vartheta} & 2 \\ \underline{a}e^{j\vartheta} & \underline{a}^2 e^{-j\vartheta} & 2 \end{pmatrix}$
$\begin{pmatrix} g_\alpha \\ g_\beta \\ g_0 \end{pmatrix} =$	$\frac{2}{3}\begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ -j & j & 0 \\ 0 & 0 & 2 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} e^{j\vartheta} & e^{-j\vartheta} & 0 \\ -je^{j\vartheta} & je^{-j\vartheta} & 0 \\ 0 & 0 & 2 \end{pmatrix}$
$\begin{pmatrix} g_d \\ g_q \\ g_0 \end{pmatrix} =$	$\frac{2}{3}\begin{pmatrix} c_1 & c_2 & c_3 \\ -s_1 & -s_2 & -s_3 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} e^{-j\vartheta} & e^{j\vartheta} & 0 \\ -je^{-j\vartheta} & je^{j\vartheta} & 0 \\ 0 & 0 & 2 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ -j & j & 0 \\ 0 & 0 & 2 \end{pmatrix}$
$\begin{pmatrix} \underline{g}_s \\ \underline{g}_s^* \\ \underline{g}_0 \end{pmatrix} =$	$\frac{2}{3}\begin{pmatrix} 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} 1 & j & 0 \\ 1 & -j & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} e^{j\vartheta} & je^{j\vartheta} & 0 \\ e^{-j\vartheta} & -je^{-j\vartheta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} e^{j\vartheta} & 0 & 0 \\ 0 & e^{-j\vartheta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$
$\begin{pmatrix} \underline{g}_r \\ \underline{g}_r^* \\ \underline{g}_0 \end{pmatrix} =$	$\frac{2}{3}\begin{pmatrix} e^{-j\vartheta} & \underline{a}e^{-\vartheta} & \underline{a}^2 e^{-j\vartheta} \\ e^{j\vartheta} & \underline{a}^2 e^{j\vartheta} & \underline{a}e^{j\vartheta} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} e^{-j\vartheta} & je^{-j\vartheta} & 0 \\ e^{j\vartheta} & -je^{j\vartheta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & j & 0 \\ 1 & -j & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} e^{-j\vartheta} & 0 & 0 \\ 0 & e^{j\vartheta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$c_1 = \cos\vartheta$, $c_2 = \cos(\vartheta - \frac{2\pi}{3})$, $c_3 = \cos(\vartheta + \frac{2\pi}{3})$, $s_1 = \sin\vartheta$, $s_2 = \sin(\vartheta - \frac{2\pi}{3})$, $s_3 = \sin(\vartheta + \frac{2\pi}{3})$.

In case of synchronous machines ϑ is given by $\vartheta = \int \Omega(t)\,dt$, where Ω is the instantaneous angle velocity of the rotor.