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Control charts — General guide and introduction

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Contents

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 7870 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Sub-Committee SC 4, *Statistical process control*.

Annex A of this International Standard is for information only.

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Control charts — General guide and introduction

1 Scope

This International Standard presents key elements and philosophy of the control chart approach, and identifies a wide variety of control charts, including those related to the Shewhart control chart and those with process acceptance or on-line predictive emphasis.

It presents an overview of the basic principles and concepts and illustrates the relationship among various control chart approaches to aid in the selection of the most appropriate standard for given circumstances.

It does not specify statistical control methods using control charts. These methods will be specified in ISO 7873 and ISO 7966, and in other future International Standards.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 3534-1:1993, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.*

ISO 3534-2:1993, *Statistics — Vocabulary and symbols — Part 2: Statistical quality control.*

ISO 7873:1993, *Control charts for arithmetic average with warning limits.*

ISO 7966:1993, *Acceptance control charts.*

ISO 8258:1991, *Shewhart control charts.*

3 Definitions

For the purposes of this International Standard, the definitions given in ISO 3534-1 and ISO 3534-2 apply.

4 General

Control charts are a fundamental tool of statistical quality control. They are a method for comparing information from samples representing the current state of a process against limits established after consideration of inherent process variability. Their primary use is to provide a means of evaluating whether a manufacturing service or administrative process is or is not in a "state of statistical control". While originally developed for industrial production and development applications, control chart methods are now widely used in a very broad range of service and support operations as well. In essence, control charts are a management tool to assist in determining when a process is stable or has changed and they are useful at management levels as well as for the operator to control at the workplace.

Inherent variability is present in all operations due to numerous, but usually minor, chance causes, so that the observed results from a stable process are not constant, and statistically valid limits are required to minimize erroneous decisions leading to over- or under-control.

A process is considered to be in a "state of statistical control" if there are no systematic shifts entering the process. In essence, when a process is "in control" it is possible to predict reliably the behaviour of that process, whereas when non-chance (or special) causes enter the system, the process is subject to the results of these causes and the outcome cannot be predicted without information about their presence and effect. A process that is found to be not in a "state of statistical control" requires intervention to bring it into such a state. For certain economic or natural phenomena there may be no known way to intervene and the control chart simply serves to identify a lack of control.

Control charts provide a simple graphical method for evaluating whether or not the process has attained, or continues in, a "state of statistical control". The determinations are made through comparison of the values or patterns of some statistical measure(s) for an ordered series of samples, or subgroups, with control limits. There is a variety of specific control charts, each designed for the types of decision to be made, the nature of the data, and the type of statistic used. The word "statistic" emphasizes that measurements are subject to inherent errors from such sources as obtaining the sample, or from the measurement process itself, and therefore represent a sample with inherent sampling variability.

A major virtue of the control chart is its ease of use and construction. It provides an indicator of the "state of statistical control" to the production or service operator, engineer, administrator and manager. However, the control chart serves only as part of the complete analysis procedure. It may suggest when an assignable cause has entered the process, but independent study is required to determine the nature of that cause and the corrective action needed.

5 Control charts for variables and attributes

Control charts may be used for either "variables" data or "attributes" data. Variables data represent observations obtained by measuring and recording the numerical magnitude of a characteristic for each of the units in the group under consideration. This involves reference to a continuous scale of some kind. Attributes data represent observations obtained by noting the presence (or absence) of some characteristic or attribute in each of the units in the group under consideration, and counting how many units do (or do not) possess the attribute, or how many such events occur in the unit, group, area, or volume in the sample.

In the case of variables data, two types of control chart are generally plotted. The first treats a measure of location such as the sample or subgroup average $\overline{X}$ or median. The second treats a measure of dispersion of observations within the sample or subgroup, such as the "range" (R or w), or the sample standard deviation (s). Both types of chart are required to constitute an effective variables control chart approach.

The chart for location is used to evaluate whether there is evidence of an actual shift in process level, while the chart for dispersion is used to evaluate whether there is evidence of a change in the magnitude of the within-sample or subgroup standard deviation. The control limits for the chart for location are a function of the within-sample or subgroup standard deviation. It is important to verify that this inherent variability parameter remains in control.

For most variables control charts, the normal distribution (see ISO 3534-1) is assumed. It is usual to plot averages of n measures from a subgroup because, except in extraordinary situations, averages tend to follow the normal distribution even when the distribution pattern of the individual observations is not normal, and because the contributions of random variations are reduced through the process of averaging. The reduction sharpens the ability to detect a signal that an assignable cause has occurred. While sample sizes of $n = 4$ or $n = 5$ are frequently selected for convenience, economical analysis may suggest more appropriate numbers. Amplification of these points may be found in the specific International Standards for various control charts.

In the case of control charts for attributes data, only one chart is plotted. The "p" chart (proportion of some specified classification), is based on the binomial distribution. The standard deviation (or standard error) for such a proportion is denoted s_p. Since

$$s_p = \sqrt{p(1-p)/n}$$

and therefore depends only on n and p, there is no need to plot a separate chart for s_p. Similarly, the "c" chart (count of events of a given classification) is based on the Poisson distribution. The standard deviation (or standard error) of the count is called s_c. Since $s_c = \sqrt{c}$, there again would be no value in plotting a separate chart for variability.

6 Control limits

Control limits are used as criteria for signalling the need for action, or for judging whether a set of data does or does not indicate a "state of statistical control". Sometimes a second set of limits called "warning limits" is also used, and the control limits are then sometimes called "action limits". Action may be in the form of

a) investigation of the source(s) of an "assignable cause";

b) making a process adjustment; or

c) stopping the process.

Rules for determining what constitutes exceeding the action or warning criteria are defined in the specific International Standards for control charts (see ISO 7873, ISO 7966 and ISO 8258), and take various forms such as points falling beyond the limits, runs, or patterns of observations within the limits.

7 Rational subgroups

A rational subgroup is a subgroup or sample, chosen for technical reasons, within which variations may be considered to be due only to non-assignable chance (or common) causes but between which there may be variations due to assignable (or special) causes whose presence is considered possible and important to detect. Technical reasons include issues of homogeneity, ability to sample, and economic considerations. One of the essential features of the control chart is the use of rational subgroups for the collection of data. The variability measured within reasonably homogeneous subgroups is used to determine the control limits, or to verify short-term stability, while longer-term stability is usually evaluated in terms of changes between subgroups. While a relatively short time span is a common basis for a rational subgroup on the basis of a limited length of potential exposure to assignable causes, other bases such as a relatively homogeneous sub-area or common conditions (e.g. work by a particular operator) may be appropriate. The same definition of a rational subgroup shall be used for data collection and for the determination of the control limits.

In most production applications, the rational subgroup represents data collected over a short period of time under essentially identical condition of material, tool setting, environmental conditions, etc. In service and office applications, the rational subgroups may be defined in terms of specific periods or logical groupings within tasks or assignments of some person or team. The variability encountered in these circumstances should represent that due only to chance (or common) causes. With longer time intervals, it can be expected that assignable (or special) causes may occur, such as a change in the source of material, a different variety of data to record, a readjusted tool setting, a new service environment, or a change of operator.

While it may be that such changes will not shift the process level, these causes represent potential variability above that due to chance causes. Thus the within-subgroup standard deviation (whether estimated from a set of subgroups, or known from past experience) serves as the basic measure of "random variability".

Note that the rational subgroup must be subject to all usual sources of chance (or common) variation if it is to have a meaningful value. For example, a series of repeat readings on a piece of material set in a testing instrument might fail to include the contribution of locating the material in the instrument or of obtaining the sample. If these aspects are inherent in a usual testing environment, the repeat readings would give an unrealistic, low estimate of inherent measurement variability. Thus almost any actual measure from the process would appear "out of control". However, if the subgroup is too large, so that variation due to assignable causes inflates the within-subgroup standard deviation, many assignable causes may occur without detection.

As indicated above, the standard deviation of the observed measures within each subgroup constitutes the basic measure of inherent variability for the control chart. When this is not already known, it is estimated by pooling the information collected from a sizable set of subgroups. It is recommended that the information from at least 20 subgroups be used. It is important to verify that the data collected during this base period are in a "state of statistical control" by plotting subgroup ranges or standard deviations on a control chart (i.e. data are in a state of statistical control with respect to within variation), and if they are not, to take the corrective action required to obtain valid base data.

Control limits are based on a multiple of σ_e, the standard error of the statistic being plotted, which in turn is derived from this within-subgroup standard deviation. The multiple of σ_e, the number of individual observations averaged (sample size), the use of supplementary rules (e.g. runs), and frequency of sampling and similar aspects are considered in the specific International Standards for control charts (see ISO 7873 and ISO 7966). If the sample range is used as a measure of variability, control limits are based on a multiple of $\overline{R}$, bypassing the estimate of the standard error σ_e.

8 Types of control chart

There are three major types of control chart (including cusum charts):

a) the Shewhart control chart, with several closely related variations (see ISO 8258);

b) the acceptance control chart (see ISO 7966);

c) the adaptive control chart.

The Shewhart control chart is used primarily to evaluate the "state of statistical control", although charts in this category are often used as a process acceptance tool, even though they are not specifically designed to relate to use criteria or process tolerance limits.

The acceptance control chart is intended specifically for this process acceptance role.

The adaptive control chart is used to regulate a process by anticipating trends and making adjustments beforehand based on predictions.

Some of the specific charts within these general types are described in clauses 9 to 11.

9 Shewhart and related control charts

9.1 General

Since the purpose of control limits is to offer a consistent procedure for reaching a decision about the "state of statistical control", Dr. W.A. Shewhart, who proposed the use of control charts for the "economic control of quality", selected limits derived on an empirical basis, but making use of knowledge of statistical considerations. Assumptions about the collection of the data, the exact form of the distribution of the data and other practical considerations such as the inability or lack of economic justification for minor assignable (or undiscovered) causes, made application of rigid theoretical probability values undesirable. It is necessary to define the central line of the chart before the control lines can be added. Dr. Shewhart recommended that the limits be set at $\pm 3\sigma_e$, i.e. three standard deviations (standard errors) of the statistical measure being plotted, based on the within-rational subgroup variability. Thus, when control charts for averages are applied, the limits are customarily set at $3\sigma_{\bar{X}}$. If it is assumed that the underlying distribution of the averages of the observed measures is normal, these limits would include 99,7 % of the averages plotted as long as the process is "in control" at the central value. That is, 0,3 % of the points (averages) plotted from an "in control" process will exceed the limits, thus erroneously signalling an "out of control" message. This is termed the alpha-risk ($\alpha = 0{,}003$), the risk of making a type I error of concluding that a process shift has occurred when, in fact, none has. In practice, however, if the distributions are not normal or, if some small departures from the specified central level are not economically worth worrying about, the probabilistic interpretations are inexact and serve only as helpful indicators of magnitude. For the "p" and "c" charts, the normal distribution is used to approximate the binomial and Poisson values. It is generally sufficient to use an agreed-upon decision criterion (the $3\sigma_e$ limits), and to recognize that for practical purposes there is a relatively small alpha-risk.

On the other hand, there is the question of the ability to detect shifts of some specified amount. For example, if it can be assumed that the individual observations have a normal distribution and the standard deviation of individual observations from the process is σ, then, if the process average were to shift from its target by 1σ, what would be the risk of failing to detect this shift (the type II error)? If the averages of four observations were being plotted, the type II risk would be 84,1 %, and if individual observations were being plotted, the risk would be 97,5 %. One reason that the control chart has been a practical tool in many applications is this lack of sensitivity to relatively small shifts in level that may be unimportant in a practical sense. Over-control frequently contributes more to lack of uniformity due to adding elements of non-random shifts to process variability than may be caused by the small shifts in level. A further discussion of this topic is given in clause 10.

Sometimes, if greater sensitivity to small changes in level is desired, warning limits, generally set at $\pm 2\sigma_e$, are used in addition to the control limits set at $\pm 3\sigma_e$, and additional decision rules based on runs are often established (see ISO 7873). This, however, increases the alpha-risk of erroneously calling a process "out of control". Other ways of increasing this type of sensitivity are through the use of control charts that accumulate data from several subgroups.

Various other decision criteria based on aspects of run theory are also used. For control chart work, a run is considered an uninterrupted sequence of occurrences of the same attribute or event in a series of observations, or a consecutive set of successively increasing (run "up") or successively decreasing (run "down") values, or a consecutive set of points above (or below) the central line.

Many of the control charts listed in this section were not developed by Dr. Shewhart, but are included because they are used principally to determine whether or not a process is in a "state of statistical control". The relationship to specification requirements generally is not an element in selecting their decision criteria.

Two general forms of the Shewhart control charts exist. The first is a control chart with no standard values specified. These charts use control limits based on the sample or subgroup data plotted on the chart. This form of control chart is used to determine whether the observed values of a series of samples vary among themselves by an amount greater than should be expected by chance alone. In essence, control charts based entirely on the data from the samples being evaluated are used to detect any lack of constancy of the cause system. This form of chart, particularly in the research and development stages, or in early pilot trials or initial production and service studies, is useful for determining whether a new process, product or service is reproducible, and whether test methods are repeatable.

The second form is a control chart whose control limits are based on adopted standard values applicable to the statistical measures plotted on the chart. This form of control chart is used to discover whether the observed measures for a sample value differ from the adopted standard values by an amount greater than should be expected by chance alone. The standard values may be based on

a) representative prior data (such as that obtained from experience using control charts with no standards specified);

b) an economic value based on consideration of needs of service and cost of production; or

c) desired or aimed-at value defined in a specification.

It must be noted that this form of control chart not only evaluates the constancy of the cause system, but also evaluates whether that cause system is properly located in terms of the adopted standard values.

9.2 Partial listing of Shewhart and related control charts (including cusum charts)

This listing is divided into two categories. The first includes charts using data obtained only from a single subgroup, while the second includes charts whose data points are accumulated from more than one subgroup.

9.2.1 Charts using data from only one rational subgroup for each plotted value

These charts are as follows.

a) $\bar{X}$ and R (average and range charts) [the median may replace $\bar{X}$ and s may replace R].

b) $\bar{X}$ and moving ranges [for individual moving range charts. See 9.2.2 a)].

c) p (percent or proportion chart).

d) np (number of affected units chart).

e) c (count chart).

f) $u = c/n$ (count per unit chart).

g) Q (quality score chart, a quality score being a weighted count).

h) D (demerit chart), a version of a Q chart in which demerits are used as the weight coefficients.

i) Multi-response chart:

   A control chart for evaluating a process in terms of the responses of two or more characteristics combined as a single statistic for the subgroup. When the variables or characteristics involved are independent of each other (uncorrelated), a x^2 statistic is usually plotted. When there is correlation between the characteristics, a T^2 statistic (see ISO 3534) is customary.

j) Trend chart:

   A control chart for evaluating the process level in terms of the deviation of the subgroup average from an expected trend in the process level. The trend may be determined empirically or by regression techniques.

9.2.2 Charts using data accumulated from more than one subgroup for each plotted value

These charts are as follows.

a) Moving average and moving range charts:

   Unweighted averaging of the latest n observations in which the current observation has replaced the oldest of the previous n observations. This type of chart is used to obtain the advantages of averaging to minimize random variability, particularly when only one observation per subgroup is available, but has the disadvantage of a non-weighted carry-over effect lasting throughout n points.

   In some situations, individual observations ($n = 1$) are plotted on an X chart and moving ranges (usually $n = 2$) on a range chart. These charts sacrifice the advantages of averaging in terms of minimizing random variability and normal distribution assumptions, but still provide visual assistance in evaluating data.

b) Exponentially weighted moving average (EWMA) charts, or exponentially smoothed or geometrically weighted charts:

   Individual observations or subgroup averages from the current and previous data sets are averaged, but those taken at earlier times are given progressively smaller weights. Because of the reinforcing carry-over effect, this chart is usually more sensitive to small shifts in level than the ordinary Shewhart $3\sigma_e$ control chart. The EWMA chart provides a ready estimate of the process average which is of particular value when the objective is to decide when to adjust a process setting and by how much.

c) Cusum, or cumulative sum charts:

   The cumulative sums of deviations of individual observations or subgroup averages from a reference value are plotted. Trends in the chart are identified by a decision mask. The most popular form is a (truncated) V mask. This chart, because of the reinforcing carry-over effect, is usually more sensitive to small shifts in level than the ordinary Shewhart $3\sigma_e$ control chart. The visual plot, when using the V mask, is often useful for locating the start of a shift in level.

10 Acceptance control charts

10.1 General

The acceptance control chart is a graphical method for the dual purpose of evaluating a process in terms of

a) whether or not it is in a "state of statistical control" with respect to within sample or subgroup variability; and

b) whether or not it can be expected to satisfy product or service requirements for the characteristic(s) being measured.

The emphasis of the acceptance control chart, in contrast to the usual control chart, is that the process usually does not need to remain in control about some single standard process level, but that as long as the within-subgroup variability remains in control, it can run at any level or levels within some zone of process levels established by empirical considerations. It is assumed that some assignable causes will create shifts in the process level which are small enough in relation to product or service requirements that it