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Acceptance control charts

Cartes de contrôle pour acceptation

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Contents

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 7966 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Sub-Committee SC 4, *Statistical process control*.

Annex A forms an integral part of this International Standard. Annex B is for information only.

Introduction

An acceptance control chart combines consideration of control implications with elements of acceptance sampling. It is an appropriate tool for helping to make decisions with respect to process acceptance. The bases for the decisions may be defined in terms of

a) whether or not a designated percentage of units of a product or service derived from that process will satisfy specification requirements;

b) whether or not the process has shifted beyond some allowable zone of process level locations.

A difference from most acceptance sampling approaches is the emphasis on process acceptability rather than on product disposition decisions.

A difference from usual control chart approaches is that the process usually does not need to be in control about some single standard process level, but that as long as the within-subgroup variability remains in control, it can (for the purpose of acceptance) run at any level or levels within some zone of process levels which would be acceptable in terms of tolerance requirements. Thus, it is assumed that some assignable causes will create shifts in the process levels which are small enough in relation to requirements that it would be uneconomical to attempt to control them too tightly for the purpose of mere acceptance.

The use of an acceptance control chart does not, however, rule out the possibility of identifying and removing assignable causes for the purpose of continuing process improvement.

A check on the inherent stability of the process is required. Therefore, variables are monitored using Shewhart-type range or sample standard deviation control charts to confirm that the variability inherent within rational subroups remains in a steady state. Supplementary examinations of the distribution of the encountered process levels form an additional source of control information. A preliminary Shewhart control chart study should be conducted to verify the validity of using an acceptance control chart.

Acceptance control charts

1 Scope

This International Standard gives guidance on the uses of acceptance control charts and establishes general procedures for determining sample sizes, action limits and decision criteria. Examples are included to illustrate a variety of circumstances in which this technique has advantages and to provide details of the determination of the sample size, the action limits and the decision criteria.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 3534-1:1993, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.*

ISO 3534-2:1993, *Statistics — Vocabulary and symbols — Part 2: Statistical quality control.*

ISO 8258:1991, *Shewhart control charts.*

3 Definitions

For the purposes of this International Standard, the definitions given in ISO 3534-1 and ISO 3534-2 apply.

An acceptable process would be a process which is represented by a Shewhart control chart (see ISO 8258) with a central line within the acceptable process zone (see figure 1). Ideally the average value $\overline{X}$ of such a control chart would be at the target value.

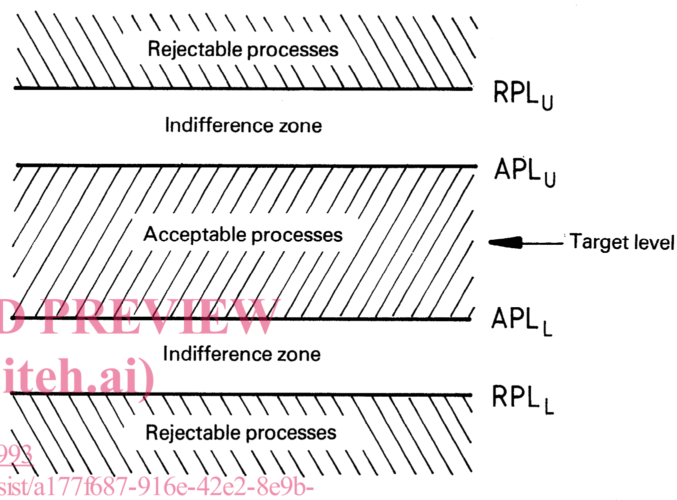


Figure 1 — Two-sided specification limits: Upper and lower APL and RPL lines in relation to processes of acceptable, rejectable, and indifference (borderline) quality

4 Symbols and abbreviations

USL	upper specification limit
LSL	lower specification limit
ACL	acceptance control limits
APL	acceptable process level
RPL	rejectable process level or non-acceptable process zone
n	subgroup sample size
p_0	acceptable proportion nonconforming items
p_1	rejectable proportion nonconforming items
P_a	probability of acceptance
T	target value, i.e. optimum value of the characteristic

$\overline{X}$ — average value of the variable X plotted on a control chart

z — variable that has a normal distribution with zero mean and unit standard deviation

$z_{p'}$ — normal deviate that is exceeded by $100p'$ % of the deviate in a specified direction (similarly for z_{α}, z_{β}, etc.)

α — risk of not accepting a process centred at the APL

β — risk of not rejecting a process centred at the RPL

μ — process mean

σ — standard deviation corresponding to the inherent process variability

σ_w — within-subgroup standard deviation

$\sigma_{\overline{X}}$ — standard deviation of the subgroup average corresponding to the inherent process variability: $\sigma_{\overline{X}} = \sigma / \sqrt{n}$.

5 Description of acceptance control chart practice

In the pursuit of an acceptable product or service, there often is room for some latitude in the ability to centre a process around its target level. The contribution to overall variation of such location factors is additional to the inherent random variability of individual elements around a given process level. In most cases, some shifts in process level must be expected and can be tolerated. These shifts usually result from an assignable cause that cannot be eliminated because of engineering or economic considerations. They often enter the system at infrequent or irregular intervals, but can rarely be treated as random components of variance.

There are several seemingly different approaches to treating these location factors contributing variation beyond that of inherent variability. At one extreme is the approach in which all variability that results in deviations from the target value must be minimized. Supporters of such an approach seek to improve the capability to maintain a process within tighter tolerance limits so that there is greater potential for process or product quality improvement.

At the other extreme is the approach that if tolerance limits are satisfied, it not only may be uneconomic and wasteful of resources to tightly control the process, but it is very likely to be counterproductive to improving the capability of reducing variability. This often is the result of the introduction of pressures which encourage "tampering" with the process (overcontrol) by people qualified to work on control aspects but not product or process quality improvement programmes.

The acceptance control chart is a useful tool for covering this wide range of approaches in a logical and simple manner. It distinguishes between the inherent variability components randomly occurring throughout the process and the additional location factors which contribute at less frequent intervals.

When shifts appear, the process may then stabilize at a new level until the next such event occurs. Between such disturbances, the process runs in control with respect to inherent variability.

An illustration of this situation is a process using large uniform batches of raw material. The within-batch variability could be considered to be the inherent variability. When a new batch of material is introduced, its deviation from the target may differ from that of the previous batch. The between-batch variation component enters the system at discrete intervals.

An example of this within- and between-batch variation might very well occur in a situation where a blanking die is blanking a machine part. The purpose of the chart is to determine when the die has worn to a point where it must be repaired or reworked. The rate of wear is dependent upon the hardness of the successive batches of material and is therefore not readily predictable. It will be seen that the use of an acceptance control chart makes it possible to judge the appropriate time to service the blanking die.

The acceptance control chart is based on the Shewhart control chart but is set up so that the process can shift outside of control limits if the specifications are sufficiently wide, or be confined to narrower limits if the inherent variability of the process is comparatively large or a large fraction of the total tolerance spread.

What is required is protection against a process that has shifted so far from the target value that it will yield some predetermined undesirable percentage of items falling outside the specification limits, or exhibits an excessive degree of process level shift.

When a chart of the average value of data sets from a process is plotted, in sequence of the production, one notices a continual variation in average values. In a central zone (acceptable process, figure 1), there is product that is indisputably acceptable. Data in the outer zones (figure 1) represent a process that is producing product that is indisputably not acceptable.

Between the inner and the outer zones are zones where the product being produced is acceptable but there is an indication that the process should be watched and as the outer zone is approached corrective action may be taken. These criteria are the basic concepts for the acceptance control chart. The description in this International Standard is designed to provide practices for the establishment of appropriate action lines for one- and two-sided specification situations.

Since it is impossible to have a single dividing line that can sharply distinguish a good from an unsatisfactory quality level, one must define a process level that represents a process that should be accepted almost always $(1-\alpha)$. This is called the acceptable process level (APL), and it marks the outer boundary of the acceptable process zone located about the target value (see figure 1).

Any process centred closer to the target value than the APL will have a risk smaller than α of not being accepted. So the closer the process is to target, the smaller the likelihood that a satisfactory process will not be accepted.

It is also necessary to define the process level that represents processes that should almost never be accepted $(1-\beta)$. This undesirable process level is labelled the rejectable process level (RPL). Any process located further away from the target value than the RPL will have a risk of acceptance smaller than β.

The process levels lying between the APL and RPL would yield a product of borderline quality. That is, process levels falling between the APL and RPL would represent quality which is not so good that it would be a waste of time, or represent over-control, if the process were adjusted, and not so bad that the product could not be used if no shift in level were made. This region is often called the "indifference zone". The width of this zone is a function of the requirements for a particular process and the risks one is willing to take in connection with it. The narrower the zone, i.e. the closer the APL and RPL are to each other, the larger will the sample size have to be. This approach will permit a realistic appraisal of the effectiveness of any acceptance control system, and will provide a descriptive method for showing just what any given control system is intended to do.

As with any acceptance sampling system, four elements are required for the definition of an acceptance control chart. They are the following:

a) an acceptable process level (APL) associated with a one-sided α-risk;

b) a rejectable process level (RPL) associated with a one-sided β-risk;

c) an action criterion or acceptance control limit (ACL);

d) the sample size (n).

NOTE 1 Generally, the defined risks are one-sided in this International Standard. In the case of two-sided specifications, the risks are a 5 % risk to go above an upper limit or a 5 % risk to go below a lower limit. This results in a 5 % (not 10 %) total risk.

Simplicity of operation is of critical importance to the use of a procedure such as an acceptance control chart. Only the acceptance control limits and the sampling instructions such as sample size, frequency, or method of selection need be known to the operator who uses the chart, although training him to understand the derivation is not difficult and can be helpful. It is thus no more complicated to use than the Shewhart chart. The supervisor, quality expert, or trained operator will derive these limits without much effort from the above considerations and will obtain a more meaningful insight into the process acceptance procedure, and a better understanding of the control implications.

6 Acceptance control of a process

6.1 Plotting the chart

The sample average value of the quality characteristic is plotted on acceptance control charts in the following way. A point is plotted on the chart for each sample with an identification number (numerical order, time order, etc.) on the horizontal scale, and the corresponding sample average on the vertical scale.

6.2 Interpreting the chart

When the plotted point falls above the upper acceptance control limit ACL_U or below the lower acceptance control limit ACL_L, the process shall be considered non-acceptable.

If a plotted point is close to the control line, the numerical values shall be used to make the decision.

7 Specifications

The specification of the values of any two of the defining elements APL (with risk α), RPL (with risk β), acceptance control limit (ACL) or sample size (n) of an acceptance control chart system determines the remaining two values. In addition, the within-rational subgroup value of σ must be known or have been estimated by the usual control chart techniques such as using $\hat{\sigma} = \overline{R}/d_2$ or $\overline{s}/c_4$,

$$\hat{\sigma}_p = \sqrt{p(1-p)/n}$$

or $\hat{\sigma}_c = \sqrt{c}$ (see ISO 8258). It is essential that the inherent random variability be in a state of statistical control in order for the risk computations to be meaningful. This can be monitored through the use of a Shewhart-type control chart for ranges or standard deviations. (See ISO 8258.)

Several selections of pairs of defining elements may be chosen.

a) Definition of the APL and RPL along with their respective α- and β-risks, and determination of the sample size (n) and the acceptance control limit (ACL).

Often, $\alpha = 0{,}05$ is chosen in acceptance control chart applications since there are few instances where a process continuously runs at the APL. This means that the risk of rejection on each side of the target value, T, should always be smaller than α.

This option is generally used when

1) acceptable processes are defined either for economic or other practical reasons in terms of process capabilities that include allowance for small discrete shifts in process level in addition to inherent random variation, or in terms of an acceptable quality level described by the percentage of items exceeding specification limits, and

2) when rejectable processes are defined either for practical reasons in terms of unnecessarily large shifts in process level, or in terms of a process level yielding an unsatisfactory percentage of items exceeding specification limits.

b) Definition of the APL (with α) and the sample size n, and determination of the RPL for a given β-risk and the ACL.

This option is used when acceptable processes are defined as in 1) above, and when there is a restriction determining the allowable sample size.

c) Definition of the RPL (with β) and n, and determination of the APL for a given α-risk and the ACL.

This option is used when rejectable processes are defined as in 2) above, and when there is a restriction determining the allowable sample size.

d) Definition of the ACL and n, and determination of the APL for a given α-risk and the RPL for a given β-risk.

This option is used primarily to interpret the meaning of a given control chart system by revealing its effective acceptable and rejectable process levels.

The remaining combinations of defining elements (APL and ACL or RPL and RCL) can be developed by similar approaches, but are less frequently encountered. The examples in this International Standard deal with variables data and are described in terms of two-sided specifications with limits and levels defined both above and below the target value. However, the method is equally valid for one-sided specification limits. In addition, there is no requirement that the values selected above and below the target value be symmetrical should more latitude be desired on either side. If different values are selected above and below the target, the sample size for the more stringent situation (i.e. smaller distance between the APL and RPL) shall be used (see 8.1.1).

8 Calculation procedures

8.1 Selection of pairs of elements

8.1.1 Defining elements APL and RPL

In the case of variables $\bar{X}$, the APL may be selected in several ways. If the specification limits are known, as well as the underlying distribution of the individual population items, the APL may be defined in terms of an acceptable proportion (or percentage) p_0 of nonconforming items which would occur when the process is centred at the APL. See figure 2. If the underlying distribution is normal (Gaussian), a one-tailed table of normal distribution values can be used. Correction factors to adjust for the two-tail probabilities required for APLs located very near to, or at, the target value are given in table 1 and illustrated in example 5 (in 9.5).

For samples of four or more, the assumption of a normal distribution for control purposes is generally valid for $\bar{X}$ charting. However, the interpretation of the proportion (percentage) of nonconforming items associated with the APL and RPL levels is dependent on the underlying distribution. Thus, for other distributions, appropriate tables should be followed and the standard normal deviate values z_{p_i} replaced accordingly. (In some references, the symbols U or t_∞ are used instead of z.) The choice of z is to emphasize that the distance represented is the absolute difference between the distribution centre and the tail area, whereas U generally represents the difference between $-\infty$ and the tail area. The advantage of the z approach in this application is that the limits and defining elements fall above and below the centre, so that it is convenient to have identical α and β values on both sides of the target rather than having to deal with α and $1-\alpha$ or β and $1-\beta$, depending on which side of the centre is involved. This also aids in a geometric interpretation such as

$$z_\alpha \sigma_{\bar{X}} + z_\beta \sigma_{\bar{X}} = (\text{RPL} - \text{APL})$$

$$\text{Upper APL } (\text{APL}_\text{U}) = \text{USL} - z_{p_0}\sigma_\text{w}$$

$$\text{Lower APL } (\text{APL}_\text{L}) = \text{LSL} + z_{p_0}\sigma_\text{w}$$

where

USL is the upper specification limit;

LSL is the lower specification limit;

z_{p_0} is the cut-off point in the normal distribution table for a proportion p_0;

σ_w is the within-rational subgroup standard deviation.

See example 1 in 9.1 where $\overline{X}$ charts with the APL and RPL are defined in terms of the percentage of nonconforming items.

In some cases, the selection of an APL value may not be directly related to the specification limits, but may be chosen on an arbitrary basis. Experience may show that the "uneconomic" or "not readily adjustable" causes for shifts in process level correspond to a narrow band. The edge of this band may be arbitrarily designated as the APL (see example 2 in 9.2). In this case, the normal distribution assumption is not invoked since the APL is not directly related to the specification limits.

In a similar fashion, the RPL may be selected in several ways. It can be related to the specification limits by defining an unacceptable proportion (percentage) p_1 of nonconforming items which would occur when the process is centred at the RPL.

$$\text{Upper RPL } (\text{RPL}_\text{U}) = \text{USL} - z_{p_1}\sigma_\text{W}$$

$$\text{Lower RPL } (\text{RPL}_\text{L}) = \text{LSL} + z_{p_1}\sigma_\text{W}$$

where

USL is the upper specification limit;

LSL is the lower specification limit;

z_{p_1} is the cut-off point in the normal distribution table for a proportion p_1.

Table 1 — Acceptance control limit factors

α = 0,05				α = 0,01			
Difference between APL and target	z	**Difference between ACL and target**	P_a	**Difference between APL and target**	z	**Difference between ACL and target**	P_a
1	2	3 = 1 + 2	4	5	6	7 = 5 + 6	8
⩾ 0,85	1,65	⩾ 2,50	0,950	⩾ 0,67	2,33	⩾ 3,00	0,990
0,80	1,65	2,45	0,951	0,60	2,33	2,93	0,990
0,70	1,66	2,36	0,952	0,50	2,33	2,83	0,990
0,60	1,67	2,27	0,953	0,40	2,37	2,77	0,991
0,50	1,68	2,18	0,954	0,30	2,37	2,67	0,991
0,40	1,71	2,11	0,956	0,20	2,41	2,61	0,992
0,30	1,75	2,05	0,960	0,10	2,52	2,62	0,994
0,20	1,80	2,00	0,964	0,00	2,58	2,58	0,995
0,10	1,87	1,97	0,969				
0,00	1,96	1,96	0,975				

NOTES

1 For applications, see example 5 in 9.5.

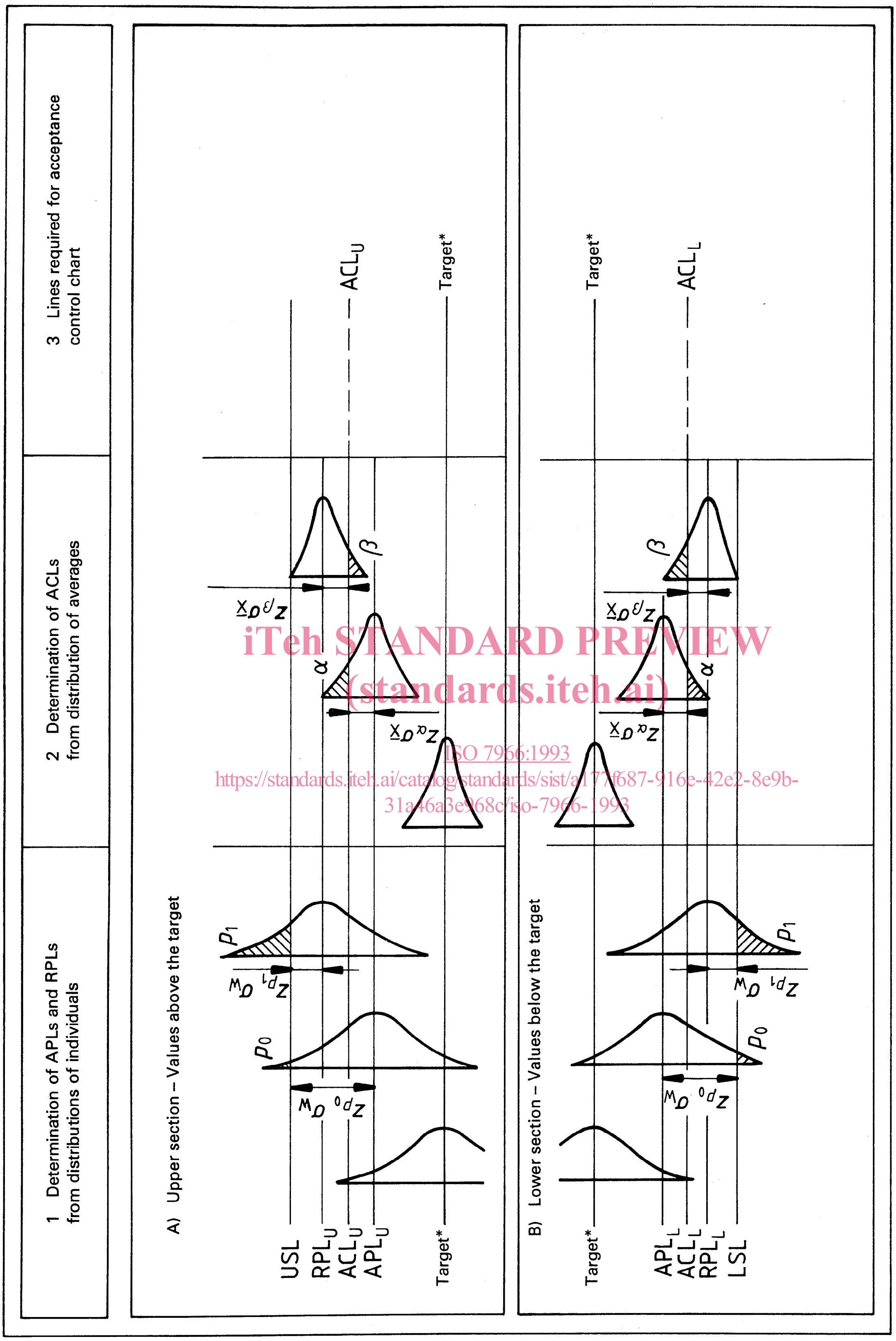
2 The control limit factors given in table 1 are for use in locating acceptance and control limit lines:

APL = target value ± (factor[1]) $(\sigma_\text{W}/\sqrt{n})$

ACL = target value ± (factor[2]) $(\sigma_\text{W}/\sqrt{n})$

1) Use appropriate factor from column 1 or 5.

2) Use appropriate factor from column 3 or 7.



* The two target lines should coincide. They have been separated to avoid overlap of distributions.

Figure 2 — Limits and defining elements of acceptance control charts

Alternatively, the selection may be arbitrary, such as a feeling that there is no reason for the process to exceed a certain distance from the target value.

Once the APL and α, and RPL and β, values are defined, the upper acceptance control limit (ACL_U) is located at

$$ACL_U = APL_U + \left(\frac{z_\alpha}{z_\alpha + z_\beta} \right)(RPL_U - APL_U)$$

where z_α and z_β are the cut-off points for a proportion α and β respectively.

The lower limit is located at

$$ACL_L = APL_L - \left(\frac{z_\alpha}{z_\alpha + z_\beta} \right)(APL_L - RPL_L)$$

When the α- and β-risks are selected to be equal, the acceptance control limit lies halfway between the APL and RPL.

The sample size can be calculated as

$$n = \left[\frac{(z_\alpha + z_\beta)\sigma_W}{(RPL - APL)} \right]^2$$

For asymmetrical limits, as at the end of clause 7:

$$n = \max\left\{ \left[\frac{(z_{\alpha,U} + z_{\beta,U})\sigma_W}{RPL_U - APL_U} \right]^2 \text{ or } \left[\frac{(z_{\alpha,L} + z_{\beta,L})\sigma_W}{APL_L - RPL_L} \right]^2 \right\}$$

A nomograph, which also provides an OC (operating characteristic) curve, can be used instead of these calculations. Both the calculation and nomograph methods are easy to use (see annex A).

8.1.2 Defining elements APL, α, β and n

The APL may be selected as specified in 8.1.1. The sample size may be specified as a matter of convenience in the operation, or it may be entered as a trial proposal to discover what kind of RPL and β values will result. If these are unsatisfactory, the process can be iterated or one of the other combinations used so that n is calculated. Given the APL, α and n values:

$$ACL_U = APL_U + z_\alpha \sigma_W / \sqrt{n}$$

$$ACL_L = APL_L - z_\alpha \sigma_W / \sqrt{n}$$

$$RPL_U = ACL_U + z_\beta \sigma_W / \sqrt{n}$$

$$RPL_L = ACL_L - z_\beta \sigma_W / \sqrt{n}$$

See example 2 in 9.2.

8.1.3 Defining elements RPL, α β and n

The RPL may be selected as specified in 8.1.1. As with the combination specified in 8.1.2, the sample size may be a convenient number or a value derived through iteration of the process. Given the RPL, β and n values:

$$ACL_U = RPL_U - z_\beta \sigma_W / \sqrt{n}$$

$$ACL_L = RPL_L + z_\beta \sigma_W / \sqrt{n}$$

$$APL_U = ACL_U - z_\alpha \sigma_W / \sqrt{n}$$

$$APL_L = ACL_L + z_\alpha \sigma_W / \sqrt{n}$$

See example 3 in 9.3.

8.1.4 Defining elements ACL, α, β and n

The ACL and n values may be selected from an existing Shewhart control system in order to calculate the APL (α) and RPL (β) values.

Given the ACL and n values:

$$APL_U = ACL_U - z_\alpha \sigma_W / \sqrt{n}$$

$$APL_L = ACL_L + z_\alpha \sigma_W / \sqrt{n}$$

$$RPL_U = ACL_U + z_\beta \sigma_W / \sqrt{n}$$

$$RPL_L = ACL_L - z_\beta \sigma_W / \sqrt{n}$$

See example 4 in 9.4.

8.2 Frequency of sampling

The relationship between sample size and the α- and β-risks has been discussed above. The determination of frequency of sampling will not be treated in this International Standard. If the history of a process is one of well-behaved inherent variability and of level shifts usually within the zone of acceptable processes, the sampling frequency may be relatively low when compared to that for processes exhibiting less stability. The costs of erroneous decisions are to some extent considered in the selection of the α and β values, but are clearly related to the frequency of sampling as well.

8.3 Other cases

For the attributes cases, such as the proportion (percentage) of nonconforming items, p, or the count of nonconformities, c, the same type of considerations hold. For p charts, the APL is defined directly as p_0 and the RPL as p_1. If some lesser category of imperfection than a nonconformity is selected, a value of p_0 and p_1 not related to the percentage of items exceeding specification limits may be selected. For c charts, c_0 and c_1 will usually not be related to the number of items exceeding limits. The regular Shewhart p and c