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Sequential sampling plans for inspection by attributes

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Plans d'échantillonnage progressif pour le contrôle par attributs

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Sequential sampling plans for inspection by attributes

Plans d'échantillonnage progressifs pour le contrôle par attributs

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Contents

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 8422 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Sub-Committee SC 5, *Acceptance sampling*.

Annexes A, B and C form an integral part of this International Standard. Annex D is for information only.

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Sequential sampling plans for inspection by attributes

Section 1: General

1.1 Scope

1.1.1 This International Standard specifies sequential sampling plans and procedures for inspection by attributes of discrete items.

The plans in the main body of the standard are indexed in terms of the producer's risk point and the consumer's risk point.

Annex A specifies sequential sampling plans and procedures indexed in terms of the acceptable quality level (AQL) to supplement the system of sampling plans in ISO 2859-1.

The purpose of this International Standard is to provide procedures for sequential assessment of inspection results that may be used to induce the supplier through the economic and psychological pressure of non-acceptance of lots of inferior quality to supply lots of a quality having a high probability of acceptance. At the same time, the consumer is protected by a prescribed upper limit to the probability of accepting lots of poor quality.

1.1.2 The sampling plans designated in this International Standard are applicable, but not limited, to inspection in different fields, such as

— end items;

— components and raw materials;

— operations;

— materials in process;

— supplies in storage;

— maintenance operations;

— data or records;

— administrative procedures.

It contains sampling plans for inspection by attributes of discrete items. The sampling plans may be used when the extent of nonconformity is expressed either in terms of proportion (or percent) nonconforming items or in terms of nonconformities per item (per 100 items).

The sampling plans are based on the assumption that nonconformities occur randomly and with statistical independence. There may be good reasons to suspect that one nonconformity in an item could be caused by a condition also likely to cause others. If so, it may be better to consider the items just as conforming or not, and ignore multiple nonconformities.

The plans in annex A are primarily intended to be used for inspection of a continuing series of lots from the same production run. The plans in the main body of this International Standard may also be used for inspection of lots in isolation.

1.2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 2859-1:1989, *Sampling procedures for inspection by attributes — Part 1: Sampling plans indexed by acceptable quality level (AQL) for lot-by-lot inspection.*

ISO 3534-1:—[1], *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.*

ISO 3534-2:—[1], *Statistics — Vocabulary and symbols — Part 2: Statistical quality control.*

1.3 Definitions and symbols

1.3.1 Definitions

For the purposes of this International Standard, the definitions given in ISO 3534-1, ISO 3534-2 and ISO 2859-1, together with the following definitions, apply.

1.3.1.1 cumulative count (D): When sampling inspection of items from a lot is performed sequentially, the total number of nonconforming items (nonconformities) found during inspection, counting from the start of inspection up to, and including, the item last inspected.

1.3.1.2 cumulative sample size (n_{cum}): When sampling inspection of items from a lot is performed sequentially, the total of inspected items, counting from the start of the inspection up to, and including, the item last inspected.

1.3.1.3 acceptance value for sequential sampling (A): A value derived from the specified parameters of the sampling plan and the cumulative sample size. Whether the lot may yet be accepted is determined by comparing the cumulative count with the acceptance number.

NOTE 1 In sampling by attributes, acceptance values are integers and are referred to as acceptance numbers. This latter term is used throughout this International Standard.

1.3.1.4 rejection value for sequential sampling (R): A value derived from the specified parameters of the sampling plan and the cumulative sample size. Whether the lot shall yet be considered not acceptable is determined by comparing the cumulative count with the rejection number.

NOTE 2 In sampling by attributes, rejection values are integers and are referred to as rejection numbers. This latter term is used throughout this International Standard.

1.3.2 Symbols

The symbols used in this International Standard are as follows:

A_0 Acceptance number for a corresponding single sampling plan.

A Acceptance number for sequential sampling.

A_t Acceptance number corresponding to the curtailed value of the cumulative sample size.

CRQ Consumer's risk quality level (in percent nonconforming).

D Cumulative count.

g Multiplier of the cumulative sample size that is used to determine the acceptance and the rejection numbers (slope of the acceptance and rejection lines).

h_A Constant that is used to determine the acceptance numbers (intercept of the acceptance line).

h_R Constant that is used to determine the rejection numbers (intercept of the rejection line).

n_0 Sample size for a corresponding single sampling plan.

n_{av} Average sample size.

n_{cum} Cumulative sample size.

n_t Curtailment value of the cumulative sample size.

p Lot or process quality level (in proportion nonconforming or nonconformities per item).

NOTE 3 To convert p to percent nonconforming or nonconformities per 100 items, multiply by 100.

p_A Producer's risk quality level. $P_a = 1 - \alpha$ when $p = p_A$.

p_R Consumer's risk quality level. $P_a = \beta$ when $p = p_R$.

P_a The probability of acceptance.

PRQ Producer's risk quality level (in percent nonconforming).

R Rejection number for sequential sampling.

R_t Rejection number corresponding to the curtailed value of the cumulative sample size.

1) To be published.

α The producer's risk.[2)]

β The consumer's risk.[2)]

λ Index parameter that is used to determine approximations to the OC curve at general quality levels. (See C.2.2.)

1.4 Principle of a sequential sampling plan by attributes

Under a sequential sampling plan by attributes, items are selected at random and subjected to inspection one by one, and a cumulative count is kept of the number of nonconforming items (or of the number of nonconformities). After the inspection of each item, the cumulative count is used to assess whether there is sufficient information to sentence the lot at that stage of the inspection.

If, at a given stage, the cumulative count is such that the risk of accepting a lot of an unsatisfactory quality level (the consumer's risk) is sufficiently low, the lot is considered acceptable and sampling of that lot is terminated.

If, on the other hand, the cumulative count is such that the risk of non-acceptance for a lot of a satisfactory quality level (the producer's risk) is sufficiently low, the lot shall be considered not acceptable, and sampling of that lot is terminated.

If the cumulative count does not allow either of the above decisions to be taken, then an additional item is inspected. The process is continued until sufficient sample information has been accumulated to warrant a decision that the lot is acceptable or not acceptable.

2) α and β may be considered to be the type I and type II risks, respectively, when testing the null hypothesis

$$H_0: p = p_A$$

against the alternative hypothesis

$$H_1: p = p_R$$

Section 2: Choice of sampling plan

2.1 Choice between sequential, single, double and multiple sampling plans

2.1.1 Advantages and disadvantages of sequential plans

The *average* sample size is the average of the various sample sizes which may occur under a sampling plan for a given lot or process quality level. Like double and multiple sampling plans, the use of sequential sampling plans leads to a smaller average sample size than single sampling plans having the same operating characteristic. However, the average savings are even greater when using a sequential sampling plan than when a double or multiple sampling plan is used.

For good quality lots, the savings may reach, or even exceed, 50 % compared with a maximum saving of 37 % with double sampling. Annex C gives a method for determining approximate values of the average sample size.

On the other hand, the actual number of items inspected for a particular lot when using a double, multiple or sequential sampling plan may exceed that of the corresponding single sampling plan. For double and multiple sampling plans there is an upper limit to the actual number of items to be inspected.

For sequential sampling plans, there is generally no such limit, and the number of inspected items may considerably exceed the sample size of the corresponding single sampling plan and even exceed the lot size. For the sequential sampling plans in this International Standard, a curtailment rule (see 2.1.3) has been introduced in order to limit the potential number of inspected items.

As the ultimate sample size from a particular lot is not known in advance, the selection of the sample may present organizational difficulties when sequential sampling plans are used. Moreover, the sheduling of inspection operations may present difficulties when using a double, multiple or sequential sampling plan. A further disadvantage is that the execution of a sequential sampling plan is more easily misunderstood by the inspectors than the simpler rules for single sampling.

The balance between the advantages of a smaller average sample size and the organizational disadvantages associated with a fluctuating inspection load results in sequential sampling being suitable only when inspection of individual items is costly in comparison with inspection overheads.

2.1.2 Caution

The choice between single, double, multiple and sequential sampling plans shall be made before the inspection of a lot is started. During the inspection of a lot, it is not permitted to switch from one type of sampling plan to another as the operating characteristic of the plan may be drastically changed if the actual inspection results influence the choice of acceptance criterion.

2.1.3 Curtailment of the sample size

Although a sequential sampling plan is on average much more economical than the equivalent single sampling plan, it may occur, during the inspection of a particular lot, that acceptance or non-acceptance comes at a very late stage because the cumulative count remains between the acceptance number and the rejection number for a long time. With the graphical method this corresponds to the random progress of the step curve remaining in the indecision zone. Such a situation is most likely to occur when the quality level of the lot (in proportion nonconforming or in nonconformities per item) is close to g, where g is the slope of the acceptance and rejection lines.

In order to alleviate this disadvantage, a maximum cumulative sample size n_t is set before sampling begins, and inspection is stopped if the cumulative sample size reaches the curtailment value, n_t, without a decision having been made. The acceptance or non-acceptance of the lot is then determined in accordance with a rule which is also agreed in advance of sampling. The curtailment rules of this International Standard have been determined in such a way that the producer's and consumer's risks are hardly affected by this deviation from the principles underlying the statistical theory of sequential sampling inspection. The curtailment rules to be used are given in 2.4.2.

2.2 Particular reservations on the inspection of small lots

The statistical theory underlying the sequential sampling plans in this International Standard is based on the assumption that the samples taken from the lot are "with replacement", i.e. each sampled item is replaced before the next item in the sample is selected. When, as is usual, sampling is without replacement, the theory remains valid for all practical purposes if the cumulative sample size does not exceed one-tenth of N, where N denotes the lot size; the theory remains approximately valid even for cumulative sample sizes up to one-seventh of N. Unfortunately, in contrast to the situation for single sampling plans, the actual cumulative sample size that is necessary in a sequential sampling plan will not be known in advance.

In the case of a small lot it is therefore advisable to ensure that the size of the lot is sufficiently large to allow a curtailed sequential sampling plan to operate under sampling without replacement, in accordance with the specified producer's and consumer's risks. For the general sequential sampling plans described in 2.3.2 and 2.4.1, it is therefore recommended that the lot size exceed $7n_t$, where n_t is the curtailment value of the sequential sampling plan.

NOTE 4 If the lot size is not sufficiently large to satisfy the above requirement, both the consumer's and the producer's risks will generally become less than their specified values. If, however, the acceptance number of the corresponding single sampling plan is zero, then the producer's risk may slightly exceed the specified values.

2.3 Selection of a sampling plan

2.3.1 Plans matching those of ISO 2859-1

If it is required to find a sequential sampling plan matching a plan from ISO 2859-1:1989, then annex A may be used. Annex A contains sequential sampling plans indexed by acceptable quality level (AQL) and sample size code letter. The operating characteristic curves of these sequential sampling plans match, as closely as practicable, those of the corresponding plans in ISO 2859-1.

2.3.2 General plans

The general method described in 2.3.2 and in 2.4.1 is used when the requirements of the sequential sampling plan are specified in terms of two points on the operating characteristic curve of the plan. The point corresponding to the higher probability of acceptance shall be designated the *producer's risk point*; the other shall be designated the *consumer's risk point*.

The first step when designing a sequential sampling plan is to choose these two points, if they have not already been dictated by circumstances. For this purpose, a producer's risk of $\alpha = 0{,}05$ and a consumer's risk of $\beta = 0{,}10$ are often used. (See figure 1.)

When the desired sequential sampling plan is required to have approximately the same operating characteristic as an existing single, double or multiple sampling plan, the producer's risk point and the consumer's risk point may be read off from a graph or a table of the operating characteristic of that plan. When no such plan exists, the producer's and consumer's risk points have to be determined from direct considerations of the conditions under which the sampling plan will operate.

2.4 Pre-operation preparations

2.4.1 Obtaining the parameters h_A, h_R and g

The criteria for acceptance or non-acceptance of a lot that are invoked at each stage of the inspection are determined from the parameters h_A, h_R and g.

The values of these parameters corresponding to a producer's risk of $\alpha = 0{,}05$, a consumer's risk of $\beta = 0{,}10$ and preferred values of the producer's and consumer's risk quality levels are given in tables 1-A and 1-B.

Annex B gives general procedures for determining h_A, h_R and g for any combination of producer's and consumer's risk points.

EXAMPLE

The specification for a type of electrical insulator is given as the nominal withstand value 1 000 kV. An inspection agency is using a single sampling plan with sample size 65 and acceptance number 6 for determination of the acceptability of production lots of this insulator type. Since the build-up of the test voltage is consuming in both time and energy, it has been decided that a sequential sampling plan should be used in future to determine the acceptability of production lots of the insulator. The sequential sampling plan is to have an operating characteristic similar to that of the single sampling plan being used.

The single sampling plan has the following properties:

— if 5 % of the insulators from the production fail at the nominal voltage then the probability of accepting a lot is 0,95;

— if 16 % of the insulators from the production fail at the nominal voltage then the probability of accepting the lot is 0.10.

These requirements correspond to fixing

a) the producer's risk quality (PRQ) at 5 % with 95 % of lots expected to be accepted; i.e. such that the producer's risk is 5 %, or $\alpha = 0{,}05$.

b) the consumer's risk quality (CRQ) at 16 % nonconforming with 10 % of lots expected to be accepted; i.e. such that the consumer's risk is 10 %, or $\beta = 0{,}10$.

The requirements are indicated on the graph of the operating characteristic curve in figure 1.

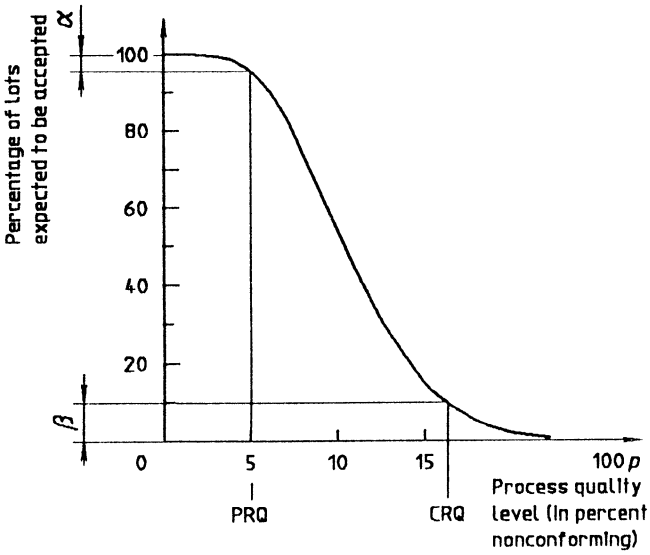

Figure 1 — Operating characteristic curve for a sampling plan with producer's risk $\alpha = 0,05$ and consumer's risk $\beta = 0,10$

From table 1-A it is found that the parameters of the sequential sampling plan that satisfies these requirements are

$h_A = 1,750$

$h_R = 2,247$

$g = 0,0957$

The same values could have been found by calculation using the procedure given in annex B.

2.4.2 Determining the curtailment value of the sample size

2.4.2.1 Standard procedures

a) If the sample size n_0 of the single sampling plan that is equivalent to the sequential sampling plan under consideration is known, the curtailment value for the cumulative sample size is determined as $n_t = 1,5n_0$, rounded up to the nearest integer.

b) If the sample size of the equivalent single sampling plan is not known, the curtailment value under inspection for percent nonconforming is determined as

$$n_t = \frac{2h_A h_R}{g(1-g)}$$

rounded up to the nearest integer and, in the case of inspection for nonconformities per 100 items, n_t is determined as

$$n_t = \frac{2h_A h_R}{g}$$

rounded up to the nearest integer.

2.4.2.2 Truncation for small lots

If the resulting value of n_t exceeds the lot size, then the sequential sampling plan shall be used with the curtailment value n_t of the sample size equal to the lot size.

2.4.2.3 Example

Consider the sequential sampling plan for inspection for percent nonconforming with parameters $h_A = 1,750$, $h_R = 2,247$ and $g = 0,095\,7$ that were determined in the example given in 2.4.1. The plan was chosen to match the single sampling plan with $n_0 = 65$ and $A_0 = 6$.

The curtailment value for the cumulative sample size is therefore $n_t = 98$.

Had the corresponding single sampling plan not been known, the curtailment value would have been determined by 2.4.2.1 b). Substituting the values of h_A, h_R and g in the formula in 2.4.2.1 b) leads to the curtailed sample size $n_t = 91$.

2.4.3 Choosing the form of the sampling plan

This International Standard gives two methods of operating a sequential sampling plan: a numerical method and a graphical method.

The numerical method has the advantage of being accurate, thereby avoiding disputes about acceptance or non-acceptance.

The graphical method is well suited to the inspection of series of lots, as the chart needs only to be drawn once, but the method is less accurate due to the inaccuracy inherent in plotting points and in drawing straight lines. On the other hand, the method does have the advantage of displaying the increase in the information on the quality of the lot as additional items are inspected, information being represented by the progress of a broken line within the indecision zone until the line reaches, or crosses, one of the boundaries of that zone.

The numerical method is the standard method, so far as acceptance or non-acceptance of a lot is concerned. See the caution in 3.4.2.

2.4.3.1 Numerical method

For each value, n_{cum}, of the cumulative sample size that is less than the curtailment value of the sample size, the acceptance number A is found by rounding the quantity

$$gn_{cum} - h_A \quad \ldots (2.1)$$

down to the nearest integer. The rejection number R is found by rounding the quantity

$$gn_{cum} + h_R \quad \ldots (2.2)$$

up to the nearest integer.

The acceptance number, A_t, corresponding to the curtailed sample size is determined as

$$A_t = gn_t$$

rounded down to the nearest integer.

The corresponding rejection number is calculated as

$$R_t = A_t + 1$$

Whenever the value of equation (2.1) is negative, the cumulative sample size is too small to allow acceptance of the lot. Conversely, whenever the value of equation (2.2) is larger than the cumulative sample size, the cumulative sample size is too small to permit non-acceptance of the lot under inspection for proportion nonconforming.

The quantities given by equations (2.1) and (2.2) shall be determined to three decimal places before rounding.

The smallest cumulative sample size permitting acceptance of the lot is obtained by rounding h_A/g up to the nearest integer.

The smallest cumulative sample size permitting non-acceptance of the lot under inspection for proportion nonconforming is obtained by rounding $h_R/(1-g)$ up to the nearest integer.

EXAMPLE

For the sequential sampling plan with parameters $h_A = 1{,}750$, $h_R = 2{,}247$ and $g = 0{,}095\,7$ that were determined in the example given in 2.4.1, the curtailment value of the sample size was determined in the example given in 2.4.2.3 to be $n_t = 98$. The corresponding acceptance number is found by rounding $gn_t = 9{,}38$ down to the nearest integer, hence the acceptance number A_t is 9 and the rejection number R_t is 10.

The formula for the acceptance number A becomes

$$0{,}095\,7n_{cum} - 1{,}750$$

rounded down to the nearest integer, and the formula for the rejection number R becomes

$$0{,}095\,7n_{cum} + 2{,}247$$

rounded up to the nearest integer.

The acceptance and rejection numbers corresponding to the cumulative sample sizes $n_{cum} = 1, 2, ..., 97$ are determined by successively inserting the values of n_{cum} in these formulae and rounding the result as described above. The result is shown in figure 2.

Cumulative sample size n_{cum}	$gn_{cum} - h_A$ [equation (2.1)]	Acceptance number A	$gn_{cum} + h_R$ [equation (2.2)]	Rejection number R
1	− 1,654	*	2,343	**
2	− 1,559	*	2,438	**
3	− 1,463	*	2,534	3
4	− 1,367	*	2,630	3
5	− 1,272	*	2,726	3
6	− 1,176	*	2,821	3
7	− 1,080	*	2,917	3
8	− 0,985	*	3,013	4
9	− 0,889	*	3,108	4
10	− 0,793	*	3,204	4
11	− 0,697	*	3,300	4
12	− 0,602	*	3,395	4
13	− 0,506	*	3,491	4
14	− 0,410	*	3,587	4
15	− 0,315	*	3,683	4
16	− 0,219	*	3,778	4
17	− 0,123	*	3,874	4
18	− 0,028	*	3,970	4
19	0,068	0	4,065	5
20	0,164	0	4,161	5
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
97	7,533	7	11,530	12
98	—	9	—	10

* indicates that the cumulative sample size is too small to permit acceptance.

** indicates that the cumulative sample size is too small to permit non-acceptance.

Figure 2 — Inspection record sheet for the sequential sampling plan considered in the example given in 2.4.3.1

2.4.3.2 Graphical method

Prepare a graph as shown in figure 3, with the cumulative sample size as the horizontal axis, the cumulative count as the vertical axis, and with the quantities given by equations (2.1) and (2.2) represented by two straight lines with the same slope, g. The lower line, with intercept $-h_A$, is designated the *acceptance line*, and the upper line, with intercept h_R, is designated the *rejection line*.

Add a vertical line, the *curtailment line*, at a cumulative sample size n_t.

The lines define three zones on the chart.

— The *acceptance zone* is the zone below (and including) the acceptance line, together with that part of the curtailment line that is below (and includes) the point (n_t; A_t).

— The *rejection zone* is the zone above (and including) the rejection line together with that part of the curtailment line that is above (and includes) the point (n_t; R_t).

— The *indecision zone* is the strip between the acceptance and rejection lines that is to the left of the curtailment line.