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Sequential sampling plans for inspection by variables for percent nonconforming (known standard deviation)

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Plans d'échantillonnage progressif pour le contrôle par mesures des pourcentages de non conformes (écart-type connu)

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Sequential sampling plans for inspection by variables for percent nonconforming (known standard deviation)

Plans d'échantillonnage progressif pour le contrôle par mesures des pourcentages de non conformes (écart-type connu)

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Contents

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 8423 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Sub-Committee SC 5, *Acceptance sampling*.

Annexes A, B and C form an integral part of this International Standard. Annex D is for information only.

Sequential sampling plans for inspection by variables for percent nonconforming (known standard deviation)

Section 1: General

1.1 Scope

1.1.1 This International Standard specifies sequential sampling plans and procedures for inspection by variables of discrete items. It is complementary to ISO 8422. The plans in the main body of the standard are indexed in terms of the producer's risk point and the consumer's risk point.

Annex A specifies sequential sampling plans and procedures indexed in terms of the acceptable quality level (AQL) to supplement the system of sampling plans in ISO 3951.

The purpose of this International Standard is to provide procedures based on a sequential assessment of inspection results, that may be used to induce the supplier through the economic and psychological pressure of non-acceptance of lots of inferior quality to supply lots of a quality having a high probability of acceptance. At the same time the consumer is protected by a prescribed upper limit to the probability of accepting lots of poor quality.

1.1.2 The sampling plans in this International Standard are primarily designed for use when all of the following conditions are satisfied:

a) where the inspection procedure is to be applied to a *continuing series of lots* of discrete items all supplied by one producer using one production process. If there are different producers, this International Standard shall be applied to each one separately;

b) where only a *single quality characteristic* x of these items is taken into consideration, which must be *measurable on a continuous scale*. If several such characteristics are of importance, this International Standard does not apply;

c) where production is stable (under statistical control) and the quality characteristic x has a known standard deviation and is distributed according to a normal distribution or a close approximation to the normal distribution;

d) where a contract or standard defines an *upper specification limit* U, a *lower specification limit* L, or both; an item is qualified as nonconforming when its measured quality characteristic x satisfies one of the following inequalities:

$$x > U \quad \ldots (1.1)$$

$$x < L \quad \ldots (1.2)$$

either

$$x > U$$

or

$$x < L \quad \ldots (1.3)$$

Inequalities (1.1) and (1.2) are called cases with a *single specification limit*, and (1.3) a case with *double specification limits*. In this last situation a further distinction is made between *separate* or *combined double specification limits* according to whether the risks are considered for each limit separately or for both limits combined (see 2.3.3).

1.2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of ap-

plying the most recent editions of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 2854:1976, *Statistical interpretation of data — Techniques of estimation and tests relating to means and variances.*

ISO 2859-1:1989, *Sampling procedures for inspection by attributes — Part 1: Sampling plans indexed by acceptable quality level (AQL) for lot-by-lot inspection.*

ISO 3534-1:—[1)], *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.*

ISO 3534-2:—[1)], *Statistics — Vocabulary and symbols — Part 2: Statistical quality control.*

ISO 3951:1989, *Sampling procedures and charts for inspection by variables for percent nonconforming.*

ISO 8422:1991, *Sequential sampling plans for inspection by attributes.*

1.3 Definitions and symbols

1.3.1 Definitions

For the purposes of this International Standard, the definitions given in ISO 3534-1, ISO 3534-2 and ISO 3951, together with the following definitions, apply.

1.3.1.1 separate double specification limits: The term used when both upper and lower limits are specified and separate AQLs [or separate producer's and consumer's risks] apply to each limit. (See 2.3.3.)

NOTE 1 The words in brackets have been added to the definition given in ISO 3951 as a broader definition is required in this International Standard.

1.3.1.2 combined double specification limits: The term used when both upper and lower limits are specified and an AQL is given [or the producer's and consumer's risks] which applies to the combined percent nonconforming at the two limits. (See 2.3.3.)

1.3.1.3 limiting process standard deviation for combined double specification limits [LPSD (com.)]: The upper limit to those values of the process standard deviation for which sequential sampling is applicable in the case of combined double specification limits. (See 2.4.3.1.)

1.3.1.4 maximum process standard deviation for separate double specification limits [MPSD (sep.)]: The upper limit to those values of the process standard deviation for which acceptance sampling is applicable in the case of separate double specification limits. (See 2.4.3.2.)

1.3.1.5 cumulative sample size (n_{cum}): When sampling inspection of items from a lot is performed sequentially, the total number of inspected items, counting from the start of the inspection up to, and including, the item last inspected.

1.3.1.6 least assessable quality level (LAQ): For a given sequential sampling plan, the process quality level which entails the largest average sample size.

1.3.1.7 leeway (y): A quantity derived from the measured value on an item. In the case of a single lower specification limit and in the case of double specification limits, the leeway is obtained by subtracting the numerical value of the lower specification limit from the measured value. In the case of an upper specification limit, the leeway is obtained by subtracting the measured value from the numerical value of the upper specification limit.

1.3.1.8 cumulative leeway (Y): When sampling inspection of items from a lot is performed sequentially, the value calculated by summing the leeways obtained from the start of the inspection up to, and including, that of the item last inspected.

1.3.1.9 acceptance value for sequential sampling (A): A value derived from the specified parameters of the sampling plan and the cumulative sample size. Whether the lot may yet be accepted is determined by comparing the cumulative leeway with the acceptance value.

1.3.1.10 rejection value for sequential sampling (R): A value derived from the specified parameters of the sampling plan and the cumulative sample size. Whether the lot may yet be considered unacceptable is determined by comparing the cumulative leeway with the rejection value.

1.3.2 Symbols

The symbols used in this International Standard are as follows:

A	Acceptance value for sequential sampling.
A_t	Acceptance value corresponding to the curtailed value of the cumulative sample size.

1) To be published.

CRQ — Consumer's risk quality level (in percent nonconforming).

f — In the case of separate double specification limits, a coefficient, which in conjunction with σ and $(U - L)$ determines the applicability of acceptance sampling. (See 2.4.3.2.)

F — Standardized normal distribution function.

g — Multiplier of the cumulative sample size that is used to determine the acceptance values and the rejection values (slope of the acceptance and rejection lines).

h_A — Constant that is used to determine the acceptance values (intercept of the acceptance line).

h_R — Constant that is used to determine the rejection values (intercept of the rejection line).

L — Lower specification limit (as a superscript to a parameter or a variable, denotes its value for L).

LAQ — Least assessable quality level (in percent nonconforming).

LPSD (com.) — Limiting process standard deviation for combined double specification limits.

MPSD (sep.) — Maximum process standard deviation for separate double specification limits.

n_0 — Sample size for a single sampling plan corresponding to the sequential sampling plan.

n_{av} — Average sample size.

n_{cum} — Cumulative sample size.

n_t — Curtailment value of the cumulative sample size.

p — Process quality level (in proportion nonconforming).

NOTE 2 To convert p to percent nonconforming, multiply by 100.

p_A — Producer's risk quality level (in proportion nonconforming). $P_a = 1 - \alpha$ when $p = p_A$.

p_g — Least assessable quality level (in proportion nonconforming). $p_g = 1 - F(g)$.

p_R — Consumer's risk quality level (in proportion nonconforming). $P_a = \beta$ when $p = p_R$.

P_a — The probability of acceptance.

PRQ — Producer's risk quality level (in percent nonconforming).

R — Rejection value for sequential sampling.

U — Upper specification limit (as a superscript to a parameter or a variable, denotes its value for U).

x — Measured value of the characteristic on an item of product.

y — Leeway, defined as

$y = U - x$ for a single upper specification limit

$y = x - L$ for a single lower specification limit, and

$y = x - L$ for double specification limits

Y — Cumulative leeway obtained by adding the leeways up to, and including, the item last inspected.

z_p — The P-fractile of the standardized normal distribution:

$$z_{0,05} = -1,644\ 9$$

since

$$F(-1,644\ 9) = 0,05$$

$$z_{0,10} = -1,281\ 6$$

since

$$F(-1,281\ 6) = 0,10$$

α The producer's risk.[2)]

β The consumer's risk.[2)]

σ Standard deviation of x in the process. (σ^2, the square of the standard deviation, is known as the variance.)

ψ In the case of combined double specification limits, a coefficient which in conjunction with σ and $(U - L)$ determines the applicability of sequential sampling. (See 2.4.3.1.)

λ Index parameter that is used to determine approximations to the OC curve at general quality levels. (See C.2.2.)

1.4 Principle of sequential sampling by variables

Under a sequential sampling plan by variables, items are selected at random and subjected to inspection one by one. After the inspection of each individual item, the cumulative leeway is calculated and used to assess whether there is sufficient information to sentence the lot at that stage of the inspection.

If, at a given stage the cumulative leeway is such that the risk of accepting a lot of unsatisfactory quality (the consumer's risk) is sufficiently low, the lot is considered acceptable and sampling of that lot is terminated.

If, on the other hand, the cumulative leeway is such that the risk of non-acceptance for a lot of satisfactory quality (the producer's risk) is sufficiently low, the lot shall be considered not acceptable, and sampling of that lot is terminated.

If the cumulative leeway does not allow either of the above decisions to be taken, then an additional item is inspected. The process is continued until sufficient sample information has been accumulated to warrant a decision that the lot is acceptable or not acceptable.

NOTE 3 In the case of separate double specification limits, the assessment of the cumulative leeway is performed for each limit separately. If, at a given stage, the risk of judging a lot of satisfactory quality to be unacceptable is sufficiently low for one or other of the limits, inspection terminates and the lot is not accepted. Alternatively, if at a given stage the risk of judging a lot of unsatisfactory quality to be acceptable is sufficiently low for one of the limits, then the lot is considered to be acceptable with respect to that limit and inspection for that limit is terminated; inspection is continued until

a) satisfactory results are obtained also for the other limit, in which case the lot is considered to be acceptable, or

b) inspection for the other limit leads to non-acceptance of the lot.

2) α and β may be considered to be the type I and type II risks, respectively, when testing the null hypothesis

H_0: $p = p_A$

against the alternative hypothesis

H_1: $p = p_R$

Section 2: Choice of sampling plan

2.1 Factors determining the choice of a sequential sampling plan by variables

2.1.1 Choice between variables and attributes

2.1.1.1 The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account.

a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items of product by an attributes plan with the generally more elaborate procedure required by a variables plan, which is usually more expensive in time and money per item.

b) A variables plan can be less readily understood than an attributes plan; for example, it may at first be difficult to accept that, when inspecting by variables, a decision not to accept a lot may be based on measurements taken on a sample that does not contain any nonconforming items.

c) A comparison of the size of the samples required for sampling plans for inspection by attributes and equivalent sampling plans for inspection by variables shows that a variables sampling plan requires a smaller sample size to give the same producer's and consumer's risks than an attributes sampling plan. A variables sampling plan therefore has a substantial advantage when the inspection process is expensive, for example in the case of destructive testing.

d) The sampling plans in this International Standard only apply to the case of a single quality characteristic. If conformance to specification is to be assessed in terms of more than one quality characteristic, the standard shall be applied separately to each of these. In such a situation it is recommended that all the quality characteristics be treated as attributes and that attributes sampling plans from ISO 2859-1 or ISO 8422 be used.

2.1.1.2 The sequential sampling plans in this International Standard may only be used when there is reason to believe that the distribution of measurements is normal and when there is valid evidence that the standard deviation of the process can be considered constant and taken to be σ.

If inspection is carried out on a continuing series of lots, the hypothesis of normality can be verified using results previously obtained using a single sampling plan, and the stability of the standard deviation may emerge from a control chart measuring process variability. If it appears that the standard deviation is in control, the (weighted) root mean square value of observed standard deviations may be presumed to be σ, the "known" standard deviation of the process. In order to verify that the variability remains under control, the standard deviation in subsequent samples should still be calculated.

If inspection is carried out on an isolated lot, there will be no evidence about the standard deviation of the process, and therefore this International Standard does not apply to the inspection of isolated lots.

NOTES

4 Tests for departure from normality are dealt with in section two of ISO 2854:1976, which provides examples of graphical methods which can be used to verify that the distribution of the data is sufficiently normal to justify the use of sampling by variables.

5 A more comprehensive treatment of tests of normality is given in ISO 5479.

6 If k samples have given the estimates s_1, $s_2 \dots s_k$ of σ then the weighted root mean square estimate s is determined as

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \dots + (n_k - 1)s_k^2}{n_1 - 1 + n_2 - 1 + \dots + n_k - 1}}$$

where n_1, n_2, ..., n_k denote the sizes of the k samples.

2.1.2 Choice between sequential and single sampling plans

The average sample size is the average of the various sample sizes which may occur under a sampling plan for a given level of the process quality. The use of sequential sampling plans leads to a smaller average sample size than single sampling plans having the same operating characteristic. For good or very poor quality lots, the savings over the corresponding single sampling plans may reach, or even exceed, 50 %.

On the other hand, the actual number of items inspected for a particular lot when using a sequential sampling plan may considerably exceed that of the corresponding single sampling plan. For the sequential sampling plans in this International Standard, a curtailment rule (see 2.1.4) has been introduced in order to limit the potential number of inspected items. Annex C gives a method for determining approximate values of the average sample size.

As the ultimate sample size from a particular lot is not known in advance, the selection of the sample may present organizational difficulties when sequential sampling plans are used. Moreover, the scheduling of inspection operations may present difficulties when using a sequential sampling plan. A further disadvantage is that the execution of a sequential sampling plan is more easily misunderstood by the inspectors than the simpler rules for single sampling.

When the inspection procedure is applied to the case of separate double specification limits on a single quality characteristic, it may happen that inspection concerning one of the limits terminates favourably long before enough information has been accumulated to warrant a decision regarding the other limit. Therefore sampling has to continue for some time before an overall decision can be taken.

The balance between the advantages of a smaller average sample size and the organizational disadvantages associated with a fluctuating inspection load results in sequential sampling by variables being suitable when only a single quality characteristic is considered and when the inspection of individual items is costly in comparison with inspection overheads.

2.1.3 Caution

The choice between single and sequential sampling plans shall be made before the inspection of a lot is started. During the inspection of a lot, it is not permitted to switch between one type of sampling plan to another as the operating characteristic of the plan may be drastically changed if the actual inspection results influence the choice of acceptance criterion.

2.1.4 Curtailment of the sample size

Although a sequential sampling plan is on average much more economical than the equivalent single sampling plan, it may occur, during the inspection of a particular lot, that acceptance or non-acceptance comes at a very late stage because the cumulative leeway remains between the acceptance value and the rejection value for a long time. With the graphical method this corresponds to the random progress of the step curve remaining in the indecision zone. Such a situation is most likely to occur when the quality of the lot is close to LAQ.

In order to alleviate this disadvantage, a maximum cumulative sample size n_t is set before sampling begins, and inspection is stopped if the cumulative sample size reaches the curtailment value, n_t, without a decision having been made. The acceptance or non-acceptance of the lot is then determined in accordance with a rule which is also agreed in advance of sampling. The curtailment rules of this International Standard have been determined in such a way that the producer's and consumer's risks are hardly affected by this deviation from the principles underlying the statistical theory of sequential sampling inspection. The curtailment rules to be used are given in 2.4.2.

2.2 Particular reservations on the inspection of small lots

The statistical theory underlying the sequential sampling plans in this International Standard is based on the assumption that the samples are taken from an infinitely large population. When sampling is carried out without replacement from a lot, the theory remains valid for all practical purposes if the cumulative sample size does not exceed one-tenth of the lot size N; the theory remains approximately valid even for cumulative sample sizes up to one-seventh of N. Unfortunately, in contrast to the situation for single sampling plans, the actual cumulative sample size that is necessary in a sequential sampling plan will not be known in advance.

In the case of a small lot it is therefore advisable to ensure that the size of the lot is sufficiently large to allow a curtailed sequential sampling plan to operate under sampling without replacement, in accordance with the specified producer's and consumer's risks. For the general sequential sampling plans described in 2.3.2 and 2.4.1, it is therefore recommended that the lot size exceed $7n_t$, where n_t is the curtailment value of the sequential sampling plan.

NOTE 7 If the lot size is not sufficiently large to satisfy the above requirement, both the consumer's and the producer's risks will generally become less than their specified values.

2.3 Selection of a sampling plan

2.3.1 Plans matching those of ISO 3951

If it is required to find a sequential sampling plan matching a "σ" method plan from ISO 3951:1989, then annex A may be used. Annex A contains sequential sampling plans indexed by acceptable quality level (AQL) and sample size code letter. The operating characteristic curves of these plans match, as closely as practicable, those of the corresponding "σ" method plans in ISO 3951.

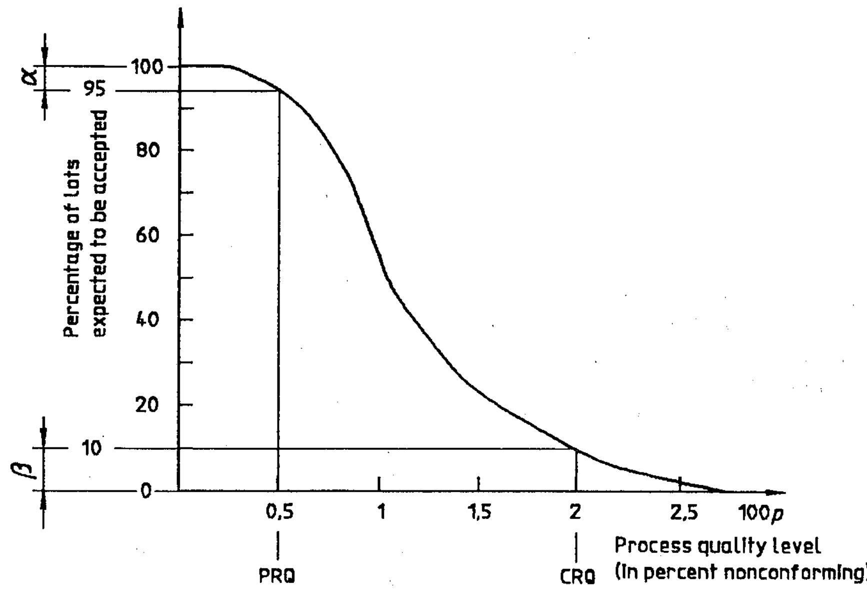

Figure 1 — Operating characteristic curve for a sampling plan with producer's risk $\alpha = 0{,}05$ and consumer's risk $\beta = 0{,}10$

2.3.2 General plans

The general method described in 2.3.2 and 2.4.1 is used when the requirements of the sequential sampling plan are specified in terms of two points on the operating characteristic curve of the plan. The point corresponding to the higher probability of acceptance shall be designated the *producer's risk point*; the other shall be designated the *consumer's risk point*.

The first step when designing a sequential sampling plan by the general method is to choose the producer's and consumer's risk points, if they have not already been dictated by circumstances. For this purpose, a producer's risk of $\alpha = 0{,}05$ and a consumer's risk of $\beta = 0{,}10$ are often used. (See figure 1.)

When the desired sequential sampling plan is required to have approximately the same operating characteristic as an existing sampling plan, the producer's risk point and the consumer's risk point may be read off from a graph or a table of the operating characteristic of that plan. When no such plan exists, the producer's and consumer's risk points have to be determined from direct considerations of the conditions under which the sampling plan will operate.

2.3.3 Specifying quality levels

When only a single specification limit is considered, the proportion nonconforming in incoming product relates to that limit.

When both upper and lower specification limits are given, it may be appropriate to consider quality levels specified separately for each limit, in which case the limits are known as "separate double specification limits". Alternatively, an overall quality level may be specified for the proportion nonconforming for the combined process at both the upper and lower limits, in which case the limits are known as "combined double specification limits".

2.4 Pre-operation preparations

2.4.1 Obtaining the parameters h_A, h_R and g

The criteria for acceptance and for non-acceptance of a lot that are invoked at each stage of the inspection are determined from the parameters h_A, h_R and g.

The values of these parameters corresponding to a producer's risk of $\alpha = 0{,}05$, a consumer's risk of $\beta = 0{,}10$ and preferred values of the producer's and consumer's risk qualities are given in table 1.

Annex B gives general procedures for determining h_A, h_R and g for any combination of producer's and consumer's risk points.

In the case of a single specification limit or combined double specification limits, only one set of parameters h_A, h_R and g shall be determined.

With separate double specification limits, two sets of parameters shall be determined: