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Guide for the statistical analysis of electrical insulation breakdown data

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INTERNATIONAL ELECTROTECHNICAL COMMISSION

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GUIDE FOR THE STATISTICAL ANALYSIS OF ELECTRICAL INSULATION BREAKDOWN DATA

FOREWORD

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IEEE Guide for the Statistical Analysis of Electrical Insulation Breakdown Data

Sponsor

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IEEE Dielectrics and Electrical Insulation Society

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Abstract: This guide describes, with examples, statistical methods to analyze times to break down and breakdown voltage data obtained from electrical testing of solid insulating materials, for purposes including characterization of the system, comparison with another insulator system, and prediction of the probability of breakdown at given times or voltages.
Keywords: breakdown voltage and time, Gumbel, Lognormal distributions, statistical methods, statistical confidence limits, Weibull

IEEE Introduction

This introduction in not part of IEEE Std 930-2004, IEEE Guide for the Statistical Analysis of Electrical Insulation Breakdown Data.

Endurance and strength of insulation systems and materials subjected to electrical stress may be tested using constant stress tests in which times to breakdown are measured for a number of test specimens, and progressive stress tests in which breakdown voltages may be measured. In either case it will be found that a different result is obtained for each specimen and that, for given test conditions, the data obtained may be represented by a statistical distribution.

Failure of solid insulation can be mostly described by extreme-value statistics, such as the Weibull and Gumbel distributions, but, historically, also the lognormal function has been used. Methods for determining whether data fit to either of these distributions, graphical and computer-based techniques for estimating the most likely parameters of the distributions, computer-based techniques for estimating statistical confidence intervals, and techniques for comparing data sets and some case studies are addressed in this guide.

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GUIDE FOR THE STATISTICAL ANALYSIS OF ELECTRICAL INSULATION BREAKDOWN DATA

1. Scope

Electrical insulation systems and materials may be tested using constant stress tests in which times to breakdown are measured for a number of test specimens, and progressive stress tests in which breakdown voltages may be measured. In either case, it will be found that a different result is obtained for each specimen and that, for given test conditions, the data obtained may be represented by a statistical distribution. This guide describes, with examples, statistical methods to analyze such data.

The purpose of this guide is to define statistical methods to analyze times to breakdown and breakdown voltage data obtained from electrical testing of solid insulating materials, for purposes including characterization of the system, comparison with another insulator system, and prediction of the probability of breakdown at given times or voltages.

Methods are given for analyzing complete data sets and also censored data sets in which not all the specimens broke down. The guide includes methods, with examples, for determining whether the data is a good fit to the distribution, graphical and computer-based techniques for estimating the most likely parameters of the distribution, computer-based techniques for estimating statistical confidence intervals, and techniques for comparing data sets and some case studies. The methods of analysis are fully described for the Weibull distribution. Some methods are also presented for the Gumbel and lognormal distributions. All the examples of computer-based techniques used in this guide may be downloaded from the following web site “http://grouper.ieee.org/groups/930/IEEEGuide.xls.” Methods to ascertain the short time withstand voltage or operating voltage of an insulation system are not presented in this guide. Mathematical techniques contained in this guide may not apply directly to the estimation of equipment life.

2. References

The following publications may be used when applicable in conjunction with this guide. When the following standards are superseded by an approved revision, the revision shall apply.

ASTM D149-97a(2004) Standard Test Method for Dielectric Breakdown Voltage and Dielectric Strength of Solid Electrical Insulating Materials at Commercial Power Frequencies.[1]

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[1] ASTM publications are available from the American Society for Testing and Materials, 100 Barr Harbor Drive, West Conshohocken, PA 19428-2959, USA (http://www.astm.org/).

BS 2918-2, Methods of test for electric strength of solid insulating materials.[2]

IEC 60243 series, Electrical strength of insulating materials—Test Methods—Part 1: Tests at power frequencies.[3]

3. Steps required for analysis of breakdown data

3.1 Data acquisition

3.1.1 Commonly used testing techniques

There are two commonly used breakdown tests for electrical insulation: *constant stress* tests and p*rogressive stress* tests. In these tests a number of identical specimens are subjected to identical test regimes intended to cause electrical breakdown. In constant stress tests the same voltage is applied to each specimen (they are often tested in parallel) and the times to breakdown are measured. The times to breakdown may be widely distributed with the longest time often being more than two orders of magnitude that of the shortest. In progressive stress tests an increasing voltage is applied to each specimen, usually breakdown voltages are measured. The voltage may be increased continuously with time or in small steps. Other protocols, for example impulse testing, may also be used. Breakdown voltages may be much less widely distributed with the highest voltage sometimes only being 2% more than the lowest voltage.

Various international standards, e.g. BS 2918-2 and IEC 60243 series, give appropriate experimental procedures for constant and progressive stress tests. This guide is intended to provide a more rigorous treatment for the breakdown data obtained in this way.

3.1.2 Other data

Breakdown data may also be available from other sources; for example, times to breakdown of the insulation in service may be available. Such data is generally much more difficult to analyze since the history of each failed insulator may not be the same (see 3.1.4), particularly as units that failed will have been replaced. It may also be unclear how many such insulation systems are in service and hence what proportion of them have failed. The techniques described in this guide are, nevertheless, appropriate for such data provided sufficient care is exercised in their application.

3.1.3 Data requirements

The number of data points required depends upon the number of parameters that describes the distribution and the confidence demanded in the results. If possible, failure data on at least ten specimens should be obtained; serious errors may result with less than five specimens (see also 3.2.2).

If all the specimens break down, the data is referred to as complete. In some cases, not all the specimens break down, the data is then referred to as censored. Censored data may be encountered in constant stress tests where the data are analyzed or the test is terminated before all the specimens break down. Censored data can also occur with progressive stress tests where the power supply has insufficient voltage capability to break down all the samples. In these cases, the data associated with a single group of specimens, those with the highest strength, are not known and the data set is said to be singly censored.[4] Data may also be progressively censored. In this case, specimens may be withdrawn (or their data discounted) at any time or

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[2] Bristish Standards are available from IHS Engineering/IHS International, 15 Iverness Way East, Englewood, CO 80112, USA.

[3] IEC publications are available from the Sales Department of the International Electrotechnical Commission, Case Postale 131, 3, rue de Varembé, CH-1211, Genève 20, Switzerland/Suisse (http://www.iec.ch/). IEC publications are also available in the United States from the Sales Department, American National Standards Institute, 25 West 43rd Street, 4th Floor, New York, NY 10036, USA (http://www.ansi.org/).

voltage; such data are often referred to as “suspended.” This may be the case where specimen breakdown is due to a spurious mechanism such as termination failure or flashover or where the specimen is deliberately withdrawn for alternative analysis. Censoring can occur by plan or by accident in many insulation tests and it is essential that this is taken into account in the data analysis. Less confidence can be placed in the analysis of a censored data set than in a complete set of data with the same number of specimens. If possible censored data sets should include at least ten (non-censored) data points and at least 30% of the specimens should have broken down.

3.1.4 Practical precautions in data capture

Specimens should, as far as possible, be identical, have the same history prior to testing, and be tested under the same conditions. In measuring the breakdown characteristics of materials it should be noted that the breakdown field (kilovolt per millimeter)[5] is usually dependent upon the rate of voltage rise, specimen thickness, electrode material, configuration and method of attachment, temperature, area, and frequency if an alternating voltage is applied. Other factors such as humidity and specimen age may also be important. With insulating systems such as cables and bushings, surface and interfacial partial discharges must be minimized and stress enhancements due to protrusions, contaminants and voids are likely to reduce breakdown strengths considerably.

The scope of this guide is limited to ac voltage testing, but the techniques may be applied to other failure tests (such as impulse or dc testing) with care. Knowledge of the failure mechanism may be required in order to establish the appropriate parameters to be measured. In pulse energized dc systems, for example, it may be more appropriate to measure the number of pulses to breakdown than the dc time to failure. Precautions in data capture are described more fully elsewhere, e.g., Abernethy [B1].[6]

3.2 Characterizing data using a probability function

3.2.1 Types of failure distribution

Failure data, such as that described in breakdown of electrical insulation, may be represented in a histogram form as numbers of specimens failed in consecutive periods. For example, the times to breakdown of polymer coated wires subject to constant ac stress are shown in Figure A.1 as a histogram. The mean and standard deviation of this data set is easily found using a scientific calculator and the corresponding Normal probability density function can be superimposed on the histogram. Whilst the Normal is probably the best known and its parameters (the mean and standard deviation) are easily calculated; it is not usually appropriate to electrical breakdown data. For example, it can be seen in Figure A.1 that its shape is rather different to the histogram. In particular the Normal distribution has a finite probability of failure at (physically impossible) negative times. An important step in analyzing breakdown data is the selection of an appropriate distribution.

Distributions for electrical breakdown include the Weibull, Gumbel, and lognormal. The most common for solid insulation is the Weibull and is the main distribution described in this guide. It is found to have wide applicability and is a type of extreme value distribution in which the system fails when the weakest link fails. The Gumbel distribution, another extreme value distribution, may have applicability in breakdown involving percolation, in liquids and in cases where fault sites such as voids are exponentially distributed. The effect of the size of test specimens (thickness, area, volume) on life or breakdown voltage can be modeled using extreme value distributions. The lognormal distribution may be useful where specimens break

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[4] This is also known as “right” censored data since specimens beyond a certain time or voltage are not tested. It is possible to have “left” censored data but this does not usually occur in electrical breakdown testing. In this guide, “singly” censored data always refers to “right” censoring.

[5] To convert this unit value from kV/mm to kV/inch multiply the value in kV/mm value by 25.4.

[6] The numbers in brackets correspond to those of the bibliography in Annex B.

down due to unrelated causes or mechanisms. The lognormal distribution may be closely approximated by the Weibull distribution.

The previous distributions may be described in terms of two parameters (as the normal distribution is described in terms of the mean and standard deviation). To give more generality, however, a third parameter may be included corresponding to a time before, or a voltage below, which a specimen will not break down.

In some cases two or more mechanisms may be operative, this may necessitate combining two or more distributions functions.

Mathematical descriptions of these distributions are given in Clause 4.

3.2.2 Testing the adequacy of a distribution

Having chosen a distribution to represent a set of breakdown data, it is necessary to check that the distribution is adequate for this purpose. It was seen in 3.2.1 that, although the parameters of a Normal distribution could be found for a given set of data, this did not imply that the distribution was an adequate representation (e.g., Figure A.1). The most common technique to test the adequacy of the distribution is to plot data points on special probability paper associated with the distribution in question. Such paper is available for all the distributions thus far mentioned. A good fit to a distribution will result in a straight line plot (5.1 and 5.2). Statistical techniques are also available for assessing the adequacy of a distribution; a simple technique is given in 5.4.

3.2.3 Estimating parameters and confidence limits

Probability plots can also be used for graphical estimation of the parameters of the distribution (Clause 6) but this is not recommended; more accurate computation techniques are readily available (Clause 7).

The parameters obtained from all such techniques are only estimates because the measured data points are randomly distributed according to a given failure mechanism. For example, if 100 experiments were performed each with ten specimens, the analysis of each of the 100 experiments would give 100 estimates for the parameters of the probability distribution each of which are slightly different. In such a case, it may be possible to state with (for example) 90% confidence that the true value of the given parameter lies between the fifth largest and fifth smallest value obtained. It is common to calculate (9.1), for each parameter estimate, a statistical confidence interval that encloses the true parameter with high probability. In general, the more specimens tested, the narrower the confidence interval. Enough specimens should be tested so as to obtain sufficiently narrow confidence intervals for practical purposes. If the confidence intervals are calculated to be adequate before all the specimens have failed, the test could be aborted.

If an experiment is poorly performed, for example, if the applied voltage is not held constant in a constant stress test, the statistical confidence intervals are inaccurate. Statistical confidence intervals are valid therefore only for identically tested specimens. If the variation in testing conditions is known it may be possible to estimate confidence intervals, but this is beyond the scope of this guide.

3.3 Hypothesis testing

The estimation of the parameters (and confidence intervals) of the distribution describing an insulating specimen or system may be required for a number of reasons, including:

- Reporting the characteristics of the insulating system following a manufacturing development.
- Testing of a batch of insulating systems and comparing them to another batch for quality control or for development.

- Estimating whether early failures in the system are due to a mechanism likely to cause failure in the remaining parts of the system.
- Estimating equipment life.
- Establishing operating conditions.

Examples of some of these processes are given as case studies in this guide (Clause 11).

4. Probability distributions for failure data

A brief introduction to these distributions has been given in this clause.

4.1 The Weibull distribution

The expression for the cumulative density function for the two-parameter Weibull distribution is shown in Equation (1):

$$F(t;\alpha,\beta) = 1 - \exp\left\{-\left(\frac{t}{\alpha}\right)^{\beta}\right\} \qquad (1)$$

where:

t is the measured variable, usually time to break down or the breakdown voltage,
F(t) is the probability of failure at a voltage or time less than or equal to t. For tests with large numbers of specimens, this is approximately the proportion of specimens broken down by time or voltage, *t*.
α is the scale parameter and is positive, and
β is the shape parameter and is positive.

The probability of failure *F(t)* is zero at $t = 0$. The probability of failure rises continuously as t increases. As the time or voltage increases, the probability of failure approaches certainty, that is, $F(\infty) = 1$.

The scale parameter α represents the time (or voltage) for which the failure probability is 0.632 (that is $1 - 1/e$ where e is the exponential constant). It is analogous to the mean of the Normal distribution (e.g., Cochran and Snedecor [B2]). The units of α are the same as *t*, that is, voltage, electric stress, time, number of cycles to failure etc.

The shape parameter β is a measure of the range of the failure times or voltages. The larger β is, the smaller is the range of breakdown voltages or times. It is analogous to the inverse of the standard deviation of the Normal distribution, Cochran and Snedecor [B2].

The two-parameter Weibull distribution of Equation (1) is a special case of the three-parameter Weibull distribution that has the cumulative distribution function shown in Equation (2).

$$F(t) = 1 - \exp\left\{-\left(\frac{t-\gamma}{\alpha}\right)^{\beta}\right\}; t \ge \gamma \qquad (2)$$

$$0; t < \gamma$$

The additional term γ is called the location parameter. *F(t)* = 0 for $t = \gamma$, that is the probability of failure for $t < \gamma$ is zero.

4.2 The Gumbel distribution

A cumulative Gumbel distribution function is given by Equation (3).

$$F_G(t) = 1 - \exp\left[-\exp\left\{\frac{t-u}{b}\right\}\right]; -\infty \le t + \infty \tag{3}$$

where:

u is the location parameter and may have any value, and

b is the scale parameter and is positive.

The Gumbel distribution is asymmetrical and can have a physically impossible finite probability of breakdown for $t < 0$. This distribution is also called the smallest extreme-value (that is, weakest link) distribution. If t is voltage, then the units of u and b are also voltage.

The Gumbel distribution is closely related to the Weibull distribution. That is, if t has a Weibull distribution then $y = \ln(t)$ has a Gumbel distribution where: $u = \ln(\alpha)$ and $b = 1/\beta$. Estimation techniques for one distribution (Gumbel or Weibull) apply to the other if this transformation is utilized.

4.3 The lognormal distribution

The lognormal distribution has sometimes been used to represent failure data from insulation systems, but it has not been used nearly as often as the extreme-value distributions in 4.1 and 4.2. However, since this probability distribution is a simple logarithmic transformation of the well-known Normal distribution, methods for data analysis are available in all standard statistical references. The probability density function of the lognormal distribution is shown in Equation (4).

$$f_{\ln}(z) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{(z-\mu)^2}{2\sigma^2}\right\} \tag{4}$$

where:

$z = \log(t)$,

μ = logarithmic mean, and

σ = logarithmic standard deviation.

The cumulative density function is the integral of the above. There is no closed-form equation for the integral. Values of the distribution are in Cochran and Snedecor [B2] and Natrella [B12] or can be obtained from statistical calculators or computer programs.

4.4 Mixed distributions

It is not uncommon to find that more than one breakdown mechanism is operative in a given specimen. The probability that such a specimen survives to a given value or voltage or time t is $1 - F(t)$. If the probability of failure due to mechanism 1 is $F_1(t)$ and due to mechanism 2 is $F_2(t)$, then the probability of survival is

$$1 - F(t) = [1 - F_1(t)] \times [1 - F_2(t)] \tag{5}$$

If both may be described by the two-parameter Weibull distribution, then we have