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Standard Practice for Calculating Thermal Transmission Properties Under Steady-State Conditions[1]

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1. Scope

1.1 This practice covers requirements and guidelines for the determination of thermal transmission properties based upon steady-state one dimensional heat transfer tests on a thermal insulation material or system for which values of heat flux, surface or air temperatures, and specimen geometry are reported from standard test methods.

1.2 The thermal transmission properties described include: thermal conductance, thermal resistance, apparent thermal conductivity, apparent thermal resistivity, surface conductance, surface resistance, and overall thermal resistance or transmittance.

1.3 This practice is restricted to calculation of thermal transmission properties from heat transfer data generated by standard test methods. These methods include: (1) planar geometries such as those used in Test Methods C 177, C 236, C 518, C 745, C 976, and C 1114, and (2) radial geometries such as those used in Test Methods C 335 and C 1033.

1.4 This practice includes the procedure for development of thermal conductivity as a function of temperature equation from data generated by standard test methods.

1.5 The values stated in SI units are to be regarded as the standard.

1.6 The attached appendixes provide discussions of the thermal properties of thermal insulating materials, the development of the basic relationships used in this practice, and examples of their use.

2. Referenced Documents

2.1 *ASTM Standards:*

C 168 Terminology Relating to Thermal Insulating Materials[2]
C 177 Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Guarded-Hot-Plate Apparatus[2]
C 236 Test Method for Steady-State Thermal Performance of Building Assemblies by Means of a Guarded Hot Box[2]
C 335 Test Method for Steady-State Heat Transfer Properties of Horizontal Pipe Insulations[2]
C 518 Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Heat Flow Meter Apparatus[2]
C 680 Practice for Determination of Heat Gain or Loss and the Surface Temperature of Insulated Pipe and Equipment Surfaces by the Use of a Computer Program[2]
C 745 Test Method for Heat Flux Through Evacuated Insulations Using a Guarded Flat Plate Boiloff Calorimeter,[2]
C 976 Test Method for Steady-State Thermal Performance of Building Assemblies by Means of a Calibrated Hot Box[2]
C 1033 Test Method for Steady-State Heat Transfer Properties of Pipe Insulation Installed Vertically[2]
C 1058 Practice for Selecting Temperatures for Evaluating and Reporting Properties of Thermal Insulation[2]
C 1114 Test Method for Steady-State Thermal Transmission Properties by Means of the Thin-Heater Apparatus[2]
E 122 Practice for Choice of Sample Size to Estimate the Average Quality of a Lot or Process[3]

3. Terminology

3.1 *Definitions*—The definitions and terminology of this practice are intended to be consistent with Terminology C 168. However, because exact definitions are critical to the use of this practice, the following equations are defined here for use in the calculations section of this practice.

3.2 *Symbols*—The symbols, terms and units used in this practice are the following:

A = specimen area normal to heat flux direction, m^2,
λ = thermal conductivity or apparent thermal conductivity, W/(m · K),
$\lambda(T)$ = the functional relationship between thermal conductivity and temperature, W/(m K),
λ_{exp} = the experimental thermal conductivity, W/(m K),
λ_m = mean thermal conductivity, averaged with respect to temperature from T_c to T_h, W/(m · K),
C = thermal conductance, W/(m^{2}K),
h_h = surface coefficient, hot side, W/(m^{2}K),
h_c = surface coefficient, cold side, W/(m^{2}K),
l = metering area length in the axial direction, *m*,

[1] This practice is under the jurisdiction of ASTM Committee C-16 on Thermal Insulation and is the direct responsibility of Subcommittee C16.30 on Thermal Measurements.

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[2] *Annual Book of ASTM Standards*, Vol 04.06.

[3] *Annual Book of ASTM Standards*, Vol 14.02.

q = one-dimensional heat flux (time rate of heat flow through metering area divided by the apparatus metering area A), W/m^2,
Q = time rate of one-dimensional heat flow through the metering area of the test apparatus, W,
r_i = radius of a hollow cylinder at the i th surface, m,
r' = thermal resistivity, K m/W,
R = thermal resistance, m^{2}K/W,
R_h = surface resistance, hot side, m^{2}K/W,
R_c = surface resistance, cold side, m^{2}K/W,
R_u = overall thermal resistance, m^{2}K/W,
T = temperature, K,
T_1 = area-weighted air temperature 75 mm or more from the hot side surface, K,
T_h = area-weighted temperature of specimen hot surface, K,
T_c = area-weighted temperature of the specimen cold surface, K,
T_2 = area-weighted air temperature 75 mm or more from the cold side surface, K,
T_m = specimen mean temperature, average of two opposite surface temperatures, $(T_h + T_c)/2$, K,
ΔT = temperature difference, K,
ΔT_{s-s} = temperature difference, surface to surface, $(T_h - T_c)$, K,
ΔT_{a-a} = temperature difference, air to air, $(T_1 - T_2)$, K,
U = thermal transmittance, W/(m^{2}K), and
x = linear dimension in the heat flow direction, m.

3.3 *Thermal Transmission Property Equations*:

3.3.1 *Thermal resistance*, R, is defined in Terminology C 168. It is not necessarily a unique function of temperature or material, but is rather a property determined by the specific thickness of the specimen and by the specific range of temperatures used to measure the thermal resistance.

$$R = \frac{A\,(T_h - T_c)}{Q} \tag{1}$$

3.3.2 *Thermal Conductance, C*:

$$C = \frac{Q}{A\,(T_h - T_c)} \tag{2}$$

NOTE 1—Thermal resistance, R, and the corresponding thermal conductance, C, are reciprocals, that is, their product is unity. These terms apply to specific bodies or constructions as used, either homogeneous or heterogeneous, between two specified isothermal surfaces.

NOTE 2—Subscripts h and c are used to differentiate between hot side and cold side surfaces.

3.3.3 *Apparent thermal conductivity*, λ, is defined in Terminology C 168.

Rectangular coordinates:

$$\lambda = \frac{Q\,L}{A\,(T_h - T_c)} \tag{3}$$

Cylindrical coordinates:

$$\lambda = \frac{Q \ln(r_2/r_1)}{2\,\pi\,l\,(T_h - T_c)} \tag{4}$$

where:
r_1 = inner radius,
T_h = temperature at the inner radius,
r_2 = outer radius, and
T_c = temperature at the outer radius.

3.3.4 *Apparent thermal resistivity*, r, is defined in Terminology C 168.

Rectangular coordinates:

$$r' = \frac{A\,(T_h - T_c)}{Q\,L} \tag{5}$$

Cylindrical coordinates:

$$r' = \frac{2\,\pi\,l\,(T_h - T_c)}{Q \ln(r_2/r_1)} \tag{6}$$

NOTE 3—Thermal resistivity, r', and the corresponding thermal conductivity, λ, are reciprocals, that is, their product is unity. These terms apply to specific materials tested between two specified isothermal surfaces. For this practice, materials are considered homogeneous when the value of the thermal conductivity or thermal resistivity is not significantly affected by variations in the thickness or area of the sample within the normally used range of those variables.

3.4 *Thermal Transmission Property Equations for Convective Boundary Conditions*:

3.4.1 *Surface resistance*, R_i, the quantity determined by the temperature difference at steady-state between an isothermal surface and its surrounding air that induces a unit heat flow per unit area to or from the surface. Typically, this parameter includes the combined effects of conduction, convection, and radiation. Surface resistances are calculated as follows:

$$R_h = \frac{A\,(T_1 - T_h)}{Q} \tag{7}$$

$$R_c = \frac{A\,(T_c - T_2)}{Q} \tag{8}$$

NOTE 4—Subscripts 1 and 2 are used to differentiate between the hot and cold side air, respectively.

3.4.2 *Surface coefficient*, h, is often called the film coefficient. These coefficients are calculated as follows:

$$h_h = \frac{Q}{A\,(T_1 - T_h)} \tag{9}$$

$$h_c = \frac{Q}{A\,(T_c - T_2)} \tag{10}$$

NOTE 5—The *surface coefficient, h_i*, and the *corresponding surface resistance, R_i*, are reciprocals, that is, their product is unity.

3.4.3 *Overall thermal resistance*, R_u—the quantity determined by the temperature difference at steady-state between the air temperatures on the two sides of a body or assembly that induces a unit time rate of heat flow per unit area through the body. It is the sum of the resistances of the body or assembly and of the two surface resistances and may be calculated as follows:

$$R_u = \frac{A\,(T_1 - T_2)}{Q} \tag{11}$$

$$= R_c + R + R_h$$

3.4.4 *Thermal transmittance, U* (sometimes called overall coefficient of heat transfer), is calculated as follows:

$$U = \frac{Q}{A\,(T_1 - T_2)} \tag{12}$$

The transmittance can be calculated from the thermal conductance and the surface coefficients as follows:

$$1/U = (1/h_h) + (1/C) + (1/h_c) \quad (13)$$

NOTE 6—Thermal transmittance, U, and the corresponding overall thermal resistance, R_u, are reciprocals, that is, their product is unity.

4. Significance and Use

4.1 ASTM thermal test methods are complex because of added apparatus details necessary to ensure accurate results. As a result, many users find it difficult to locate the data reduction details necessary to reduce the data obtained from these tests. This practice is designed to be referenced in the thermal test methods, thus allowing them to concentrate on experimental details rather than data reduction.

4.2 This practice is intended to provide the user with a uniform procedure for calculating the thermal transmission properties of a material or system from standard test methods used to determine heat flux and surface temperatures. This practice is intended to replace the similar calculation sections of Test Methods C 177, C 236, C 335, C 518, C 745, C 976, C 1033, and C 1114.

4.3 This practice provides the method for developing the thermal conductivity as a function of temperature for a specimen from data taken at small or large temperature differences. This relationship can be used to characterize material for comparison to material specifications and for use in calculations programs such as Practice C 680.

4.4 Two general solutions to the problem of establishing thermal transmission properties for application to end-use conditions are outlined in Practice C 1058. (Practice C 1058 should be reviewed prior to use of this practice.) One is to measure each product for each end-use condition. This solution is rather straightforward and needs no other elaboration. The second is to measure each product over the entire temperature range of application conditions and to use these data to establish the thermal-transmission property dependencies on the various end-use conditions. One advantage of the second approach is that once these dependencies have been established, they serve as the basis for estimating the performance for a given product to other conditions.

4.5 **Precaution**—The use of thermal curves developed in Section 6 must be limited to a temperature range that does not extend to less than the lowest surface temperature or higher than the highest surface temperature for the test data set used to generate the curves.

5. Determination of Thermal Transmission Properties for a Specific Temperature

5.1 Using the appropriate test method of interest, determine the steady-state heat flux and temperature data for the test.

NOTE 7—The calculation of thermal properties requires that: (1) the thermal insulation specimen is homogeneous, as defined in Terminology C 168 or, as a minimum, appears uniform across the test area; (2) the measurements are taken only after steady-state has been established; (3) the heat flows in a direction normal to the isothermal surfaces of the specimen; (4) the rate of flow of heat is known; (5) the specimen dimensions, that is, heat flow path length parallel to heat flow, and area perpendicular to heat flow, are known; and (6) both specimen surface temperatures (and equivalently, the temperature difference across the specimen) are known. In the case of a hot box systems test, both air curtain temperatures must be known.

5.2 Choose the thermal test parameter (λ or r', R or C, U or R_u) to be calculated from the test results.

5.3 Calculate the thermal property of interest using the data from the test as described in 5.1, and the appropriate equation in 3.3 or 3.4.

5.4 Using the data from the test as described in 5.1, determine the test mean temperature for the thermal property of 5.3 using the following equation:

$$T_m = (T_h + T_c)/2 \quad (14)$$

NOTE 8—The thermal transmission properties determined in 5.3 are applicable only for the conditions of the test. Further analysis is required using data from multiple tests if the relationship for the thermal property variation with mean test temperature is to be determined. If this relationship is required, the analysis to be followed is presented in Section 6.

5.5 *An Example of a Computation of Thermal Conductivity Measured in a Two-Sided Guarded Hot Plate*:

5.5.1 For a guarded hot plate apparatus in the normal, double-sided mode of operation, the heat developed in the metered area heater passes through two specimens. To reflect this fact, Eq 3 for the operational definition of the mean thermal conductivity of the pair of specimens must be modified to read:

$$\lambda_{exp} = \frac{Q}{A\,[(\Delta T/L)_1 + (\Delta T/L)_2]} \quad (15)$$

where:

$(\Delta/T_{ss}/L)_1$ = the ratio of surface to surface temperature difference to thickness for Specimen 1. A similar expression is used for Specimen 2.

5.5.2 In many experimental situations, the two temperature differences are very nearly equal (within well under 1 %), and the two thicknesses are also nearly equal (within 1 %), so that Eq 15 may be well approximated by a simpler form:

$$\lambda_{exp} = \frac{Q\,L_{average}}{2A\,\Delta T_{average}} \quad (16)$$

where:

$\Delta T_{average}$ = the arithmetic mean temperature difference $(\Delta T_1 + \Delta T_2)/2$,

$L_{average}$ = $(L_1 + L_2)/2$ is the arithmetic mean of the two specimen thicknesses, and

$2A$ = occurs because the metered power flows out through two surfaces of the metered area for this apparatus. For clarity in later discussions, use of this simpler form, Eq 16, will be assumed.

NOTE 9—The mean thermal conductivity, λ_m, is usually not the same as the thermal conductivity, λ (T_m), at the mean temperature T_m. The mean thermal conductivity, λ_m, and the thermal conductivity at the mean temperature, λ (T_m), are equal only in the special case where λ (T) is a constant or linear function of temperature **(1)**, that is, when there is no curvature (nonlinearity) in the conductivity-temperature relation. In all other cases, the conductivity, λ_{exp}, as determined by Eq 3 is not simply a function of mean temperature, but depends on the values of both T_h and T_c. This is the reason the experimental value, λ_{exp}, of thermal conductivity for a large temperature difference is not, in general, the same as that for a small difference at the same mean temperature. The discrepancy between the mean conductivity and the conductivity at the mean temperature increases as ΔT increases. Treatment of these differences is discussed in Section 6.

5.5.3 When ΔT is so large that the mean (experimental) thermal conductivity differs from the thermal conductivity at the mean specimen temperature by more than 1 %, the derived thermal conductivity (Eq 3) shall be identified as a mean value, λ_m, over the range from T_c to T_h. Reference **(1)** describes a method for establishing the actual λ versus T dependency from mean thermal conductivity measurements. Proofs of the above statements, along with some illustrative examples, are given in Appendix X3.

6. Determination of the Thermal Conductivity for a Temperature Range

6.1 Consult Practice C 1058 for the selection of appropriate test temperatures. Using the appropriate test method of interest, determine the steady-state heat flux and temperature data for each test covering the temperature range of interest.

6.2 *Small temperature differences*—The use of Eq 15 or Eq 16 is valid for determining the thermal conductivity versus mean temperature only if the temperature difference between the hot and cold surfaces is small. For the purpose of this practice, experience with most insulation materials shows that the maximum ΔT should be 25 K or 5 % of the absolute mean temperature, whichever is greater. The procedure given in 6.2.1 may be followed only when this temperature difference standard is met. The procedure of 6.3 may be used for all test data reduction.

6.2.1 The quantities on the right-hand side of Eq 15 are known for each data point; from these quantities $\lambda(T)$ may be calculated if ΔT is sufficiently small (see 6.2), for normal insulation applications. The value of $\lambda\ (T)$ so obtained is an approximation, its accuracy depends on the curvature (nonlinearity) of the thermal conductivity-temperature relationship **(1)**. It is conventional to associate the value of λ_{exp} obtained from Eq 16 with the mean temperature T_m at the given data point. For data obtained at a number of mean temperatures, a functional dependence of λ with T may be obtained, with functional coefficients to be determined from the data. For the situation where the number of data points is greater than the number of coefficients in the function, the quantities calculated from the right-hand side over determine the coefficients, and a least-squares fit to the data may be used to obtain the functional dependence of λ with T. The accuracy of the coefficients thus obtained depend not only on the experimental imprecision, but also on the extent to which the thermal conductivity-temperature relationship departs from a linear one over the temperature range defined by the isothermal boundaries of the specimen during the tests.

6.3 *Computation of thermal conductivity when temperature Differences are large*—The following sections apply to all testing results and are specifically required when the temperature difference is greater than about 5 % of the absolute mean temperature. This situation typically occurs during measurements of heat transmission in pipe insulation, Test Method C 335, but may also occur with measurements using other apparatus. Eq 17 and Eq 18 are developed in Appendix X2, but are presented here for continuity of this practice.

6.3.1 The dependence of λ on T for flat-slab geometry is:

$$\lambda_m = \frac{1}{\Delta T}\int_{T_c}^{T_h} \lambda(T)\, dT$$

or:

$$\lambda_m = QL / [2\, A\, (T_h - T_c)] \tag{17}$$

The quantities T_h, T_c, Q, and $(L/2A)$ on the right-hand side are known for each data point obtained by the user.

6.3.2 The dependence of λ on T for cylindrical geometry is:

$$\lambda_m = \frac{1}{\Delta T}\int_{T_c}^{T_h} \lambda(T)\, dT$$

or:

$$\lambda_m = \frac{Q \ln (r_2/r_1)}{2\, \pi\, l\, (T_h - T_c)} \tag{18}$$

6.4 *Thermal conductivity integral (TCI) method*—To obtain the dependence of thermal conductivity on temperature from Eq 17 or Eq 18, a specific functional dependence to represent the conductivity-temperature relation must first be chosen. After the form of the thermal conductivity equation is chosen, 6.4.1-6.4.3 are followed to determine the coefficients for that equation.

6.4.1 Integrate the selected thermal conductivity function with respect to temperature. For example, if the selected function $\lambda(T)$ were a polynomial function of the form

$$\lambda(T) = A_o + A_n T^n + A_m T^m, \tag{19}$$

then, from Eq 18, the temperature-averaged thermal conductivity would be:

$$\lambda_m = \frac{A_o(T_h - T_c)}{(T_h - T_c)} + \frac{A_n\,(T_h^{n+1} - T_c^{n+1})}{(n+1)\,(T_h - T_c)} + \frac{A_m\,(T_h^{m+1} - T_c^{m+1})}{(m+1)\,(T_h - T_c)} \tag{20}$$

6.4.2 By means of any standard least-squares fitting routine, the right-hand side of Eq 20 is fitted against the values of experimental thermal conductivity, λ_{exp}. This fit determines the coefficients in the thermal conductivity function, Eq 19 in this case.

6.4.3 Use the coefficients obtained in 6.4.2 to describe the assumed thermal conductivity function, Eq 19. Each data point is then conventionally plotted at the corresponding mean specimen temperature. When the function is plotted, it may not pass exactly through the data points. This is because each data point represents mean conductivity, λ_m, and this is not equal to the value of the thermal conductivity, $\lambda\ (T_m)$, at the mean temperature. The offset between a data point and the fitted curve depends on the size of ΔT and on the nonlinearity of the thermal conductivity function.

NOTE 10—Many equation forms other than Eq 19 can be used to represent the thermal conductivity function. If possible, the equation chosen to represent the thermal conductivity versus temperature relationship should be easily integrable with respect to temperature. However, in some instances it may be desirable to choose a form for $\lambda(T)$ that is not easily integrable. Such equations may be found to fit the data over a much wider range of temperature. Also, the user is not restricted to the use of polynomial equations to represent $\lambda(T)$, but only to equation forms that can be integrated either analytically or numerically. In cases where direct integration is not possible, one can carry out the same procedure using numerical integration.

6.5 *TCI method—A summary*—The thermal conductivity integral method of analysis is summarized in the following steps:

6.5.1 Measure several sets of λ_{exp}, T_h, and T_c over a range of temperatures.