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First edition
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Quantities and units —

Part 2: Mathematical signs and symbols to be used in the natural sciences and technology

Grandeurs et unités —

Partie 2: Signes et symboles mathématiques à employer dans les sciences de la nature et dans la technique

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 80000-2 was prepared by Technical Committee ISO/TC 12, *Quantities and units*, in collaboration with IEC/TC 25, *Quantities and units*.

This first edition cancels and replaces ISO 31-11:1992, which has been technically revised. The major technical changes from the previous standard are the following:

— Four clauses have been added, i.e. “Standard number sets and intervals”, “Elementary geometry”, “Combinatorics” and “Transforms”.

ISO 80000 consists of the following parts, under the general title *Quantities and units*:

— *Part 1: General*

— *Part 2: Mathematical signs and symbols to be used in the natural sciences and technology*[1)]

— *Part 3: Space and time*

— *Part 4: Mechanics*

— *Part 5: Thermodynamics*

— *Part 7: Light*

— *Part 8: Acoustics*

— *Part 9: Physical chemistry and molecular physics*

— *Part 10: Atomic and nuclear physics*

— *Part 11: Characteristic numbers*

— *Part 12: Solid state physics*

1) Title to be shortened to read “Mathematics” in the second edition of ISO 80000-2.

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IEC 80000 consists of the following parts, under the general title *Quantities and units*:

— *Part 6: Electromagnetism*

— *Part 13: Information science and technology*

— *Part 14: Telebiometrics related to human physiology*

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Introduction

Arrangement of the tables

The first column “Item No.” of the tables contains the number of the item, followed by either the number of the corresponding item in ISO 31-11 in parentheses, or a dash when the item in question did not appear in ISO 31-11.

The second column “Sign, symbol, expression” gives the sign or symbol under consideration, usually in the context of a typical expression. If more than one sign, symbol or expression is given for the same item, they are on an equal footing. In some cases, e.g. for exponentiation, there is only a typical expression and no symbol.

The third column “Meaning, verbal equivalent” gives a hint on the meaning or how the expression may be read. This is for the identification of the concept and is not intended to be a complete mathematical definition.

The fourth column “Remarks and examples” gives further information. Definitions are given if they are short enough to fit into the column. Definitions need not be mathematically complete.

The arrangement of the table in Clause 16 “Coordinate systems” is somewhat different.

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Quantities and units —

Part 2:
Mathematical signs and symbols to be used in the natural sciences and technology

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1 Scope

ISO 80000-2 gives general information about mathematical signs and symbols, their meanings, verbal equivalents and applications.

The recommendations in ISO 80000-2 are intended mainly for use in the natural sciences and technology, but also apply to other areas where mathematics is used.

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2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

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ISO 80000-1:—[2], *Quantities and units — Part 1: General*

3 Variables, functions and operators

Variables such as x, y, etc., and running numbers, such as i in $\Sigma_i\, x_i$ are printed in italic (sloping) type. Parameters, such as a, b, etc., which may be considered as constant in a particular context, are printed in italic (sloping) type. The same applies to functions in general, e.g. f, g.

An explicitly defined function not depending on the context is, however, printed in Roman (upright) type, e.g. sin, exp, ln, Γ. Mathematical constants, the values of which never change, are printed in Roman (upright) type, e.g. e = 2,718 218 8…; π = 3,141 592…; $\mathrm{i}^2 = -1$. Well-defined operators are also printed in Roman (upright) style, e.g. div, δ in δx and each d in $\mathrm{d}f/\mathrm{d}x$.

Numbers expressed in the form of digits are always printed in Roman (upright) style, e.g. 351 204; 1,32; 7/8.

The argument of a function is written in parentheses after the symbol for the function, without a space between the symbol for the function and the first parenthesis, e.g. $f(x)$, $\cos(\omega t + \varphi)$. If the symbol for the function consists of two or more letters and the argument contains no operation symbol, such as +, –, ×, · or / , the parentheses around the argument may be omitted. In these cases, there should be a thin space between the symbol for the function and the argument, e.g. int 2,4; sin $n\pi$; arcosh $2A$; Ei x.

If there is any risk of confusion, parentheses should always be inserted. For example, write $\cos(x) + y$; do not write $\cos x + y$, which could be mistaken for $\cos(x + y)$.

2) To be published. (Revision of ISO 31-0:1992)

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A comma, semicolon or other appropriate symbol can be used as a separator between numbers or expressions. The comma is generally preferred, except when numbers with a decimal comma are used.

If an expression or equation must be split into two or more lines, one of the following methods shall be used.

a) Place the line breaks immediately after one of the symbols =, +, −, ± or ∓, or, if necessary, immediately after one of the symbols ×, ·, or /. In this case, the symbol indicates that the expression continues on the next line or next page.

b) Place the line breaks immediately before one of the symbols =, +, −, ± or ∓, or, if necessary, immediately before one of the symbols ×, ·, or /. In this case, the symbol indicates that the expression is a continuation of the previous line or page.

The symbol shall not be given twice around the line break; two minus signs could for example give rise to sign errors. Only one of these methods should be used in one document. If possible, the line break should not be inside of an expression in parentheses.

It is customary to use different sorts of letters for different sorts of entities. This makes formulas more readable and helps in setting up an appropriate context. There are no strict rules for the use of letter fonts which should, however, be explained if necessary.

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4 Mathematical logic

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-4.1 (*11-3.1*)	$p \wedge q$	conjunction of p and q, p and q	
2-4.2 (*11-3.2*)	$p \vee q$	disjunction of p and q, p or q	This "or" is inclusive, i.e. $p \vee q$ is true, if either p or q, or both are true.
2-4.3 (*11-3.3*)	$\neg p$	negation of p, not p	
2-4.4 (*11-3.4*)	$p \Rightarrow q$	p implies q, if p, then q	$q \Leftarrow p$ has the same meaning as $p \Rightarrow q$. $\Rightarrow$ is the implication symbol.
2-4.5 (*11-3.5*)	$p \Leftrightarrow q$	p is equivalent to q	$(p \Rightarrow q) \wedge (q \Rightarrow p)$ has the same meaning as $p \Leftrightarrow q$. $\Leftrightarrow$ is the equivalence symbol.
2-4.6 (*11-3.6*)	$\forall x \in A \quad p(x)$	for every x belonging to A, the proposition $p(x)$ is true	If it is clear from the context which set A is being considered, the notation $\forall x\, p(x)$ can be used. $\forall$ is the universal quantifier. For $x \in A$, see 2-5.1.
2-4.7 (*11-3.7*)	$\exists x \in A \quad p(x)$	there exists an x belonging to A for which $p(x)$ is true	If it is clear from the context which set A is being considered, the notation $\exists x\, p(x)$ can be used. $\exists$ is the existential quantifier. For $x \in A$, see 2-5.1. $\exists^1 x\, p(x)$ is used to indicate that there is exactly one element for which $p(x)$ is true. $\exists!$ is also used for $\exists^1$.

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5 Sets

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-5.1 (*11-4.1*)	$x \in A$	x belongs to A, x is an element of the set A	$A \ni x$ has the same meaning as $x \in A$.
2-5.2 (*11-4.2*)	$y \notin A$	y does not belong to A, y is not an element of the set A	$A \not\ni y$ has the same meaning as $y \notin A$. The negating stroke may also be vertical.
2-5.3 (*11-4.5*)	$\{x_1, x_2, \ldots, x_n\}$	set with elements $x_1, x_2, \ldots, x_n$	Also $\{x_i \mid i \in I\}$, where I denotes a set of subscripts.
2-5.4 (*11-4.6*)	$\{x \in A \mid p(x)\}$	set of those elements of A for which the proposition $p(x)$ is true	EXAMPLE $\{x \in \mathbf{R} \mid x \leqslant 5\}$ If it is clear from the context which set A is being considered, the notation $\{x \mid p(x)\}$ can be used (for example $\{x \mid x \leqslant 5\}$, if it is clear that x is a variable for real numbers).
2-5.5 (*11-4.7*)	card A $\lvert A \rvert$	number of elements in A, cardinality of A	The cardinality can be a transfinite number. See also 2-9.16.
2-5.6 (*11-4.8*)	$\varnothing$	the empty set	
2-5.7 (*11-4.18*)	$B \subseteq A$	B is included in A, B is a subset of A	Every element of B belongs to A. $\subset$ is also used, but see remark to 2-5.8. $A \supseteq B$ has the same meaning as $B \subseteq A$.
2-5.8 (*11-4.19*)	$B \subset A$	B is properly included in A, B is a proper subset of A	Every element of B belongs to A, but at least one element of A does not belong to B. If $\subset$ is used for 2-5.7, then $\subsetneq$ shall be used for 2-5.8. $A \supset B$ has the same meaning as $B \subset A$.
2-5.9 (*11-4.24*)	$A \cup B$	union of A and B	The set of elements which belong to A or to B or to both A and B. $A \cup B = \{x \mid x \in A \vee x \in B\}$
2-5.10 (*11-4.26*)	$A \cap B$	intersection of A and B	The set of elements which belong to both A and B. $A \cap B = \{x \mid x \in A \wedge x \in B\}$
2-5.11 (*11-4.25*)	$\bigcup_{i=1}^{n} A_i$ $A_1 \cup A_2 \cup \ldots \cup A_n$	union of the sets $A_1, A_2, \ldots, A_n$	The set of elements belonging to at least one of the sets $A_1, A_2, \ldots, A_n$ $\bigcup_{i=1}^{n}$, $\bigcup_{i \in I}$ and $\bigcup_{i \in I}$ are also used, where I denotes a set of subscripts.
2-5.12 (*11-4.27*)	$\bigcap_{i=1}^{n} A_i$ $A_1 \cap A_2 \cap \ldots \cap A_n$	intersection of the sets $A_1, \ldots, A_n$	The set of elements belonging to all sets $A_1, A_2, \ldots, A_n$ $\bigcap_{i=1}^{n}$, $\bigcap_{i \in I}$ and $\bigcap_{i \in I}$ are also used, where I denotes a set of subscripts.

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Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-5.13 (*11-4.28*)	$A \setminus B$	difference of A and B, A minus B	The set of elements which belong to A but not to B. $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ $A - B$ should not be used. $\complement_A B$ is also used. $\complement_A B$ is mainly used when B is a subset of A, and the symbol A may be omitted if it is clear from the context which set A is being considered.
2-5.14 (*11-4.30*)	(a, b)	ordered pair a, b, couple a, b	$(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. If the comma can be mistaken as the decimal sign, then the semicolon (;) or a stroke (|) may be used as separator.
2-5.15 (*11-4.31*)	$(a_1, a_2, \ldots, a_n)$	ordered n-tuple	See remark to 2-5.14.
2-5.16 (*11-4.32*)	$A \times B$	Cartesian product of the sets A and B	The set of ordered pairs (a, b) such that $a \in A$ and $b \in B$. $A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$
2-5.17 (—)	$\prod_{i=1}^{n} A_i$ $A_1 \times A_2 \times \ldots \times A_n$	Cartesian product of the sets $A_1, A_2, \ldots, A_n$	The set of ordered n-tuples $(x_1, x_2, \ldots, x_n)$ such that $x_1 \in A_1, x_2 \in A_2, \ldots, x_n \in A_n$. $A \times A \times \ldots \times A$ is denoted by A^n, where n is the number of factors in the product.
2-5.18 (*11-4.33*)	id_A	identity relation on A, diagonal of $A \times A$	id_A is the set of all pairs (x, x) where $x \in A$. If the set A is clear from the context, the subscript A can be omitted.

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6 Standard number sets and intervals

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-6.1 (*11.4.9*)	**N**	the set of natural numbers, the set of positive integers and zero	**N** = {0, 1, 2, 3, …} **N*** = {1, 2, 3, …} Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{N}_{>5} = \{n \in \mathbf{N} \mid n > 5\}$ The symbols $\mathbb{N}$ and $\mathbb{N}$ are also used.
2-6.2 (*11.4.10*)	**Z**	the set of integers	**Z** = {…, –2, –1, 0, 1, 2, …} $\mathbf{Z}^* = \{n \in \mathbf{Z} \mid n \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{Z}_{\geqslant -3} = \{n \in \mathbf{Z} \mid n \geqslant -3\}$ The symbol $\mathbb{Z}$ is also used.
2-6.3 (*11.4.11*)	**Q**	the set of rational numbers	$\mathbf{Q}^* = \{r \in \mathbf{Q} \mid r \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{Q}_{<0} = \{r \in \mathbf{Q} \mid r < 0\}$ The symbols $\mathbb{Q}$ and $\mathbb{Q}$ are also used.
2-6.4 (*11.4.12*)	**R**	the set of real numbers	$\mathbf{R}^* = \{x \in \mathbf{R} \mid x \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{R}_{\geqslant 0} = \{x \in \mathbf{R} \mid x \geqslant 0\}$ The symbols $\mathbb{R}$ and $\mathbb{R}$ are also used.
2-6.5 (*11.4.13*)	**C**	the set of complex numbers	$\mathbf{C}^* = \{z \in \mathbf{C} \mid z \neq 0\}$ The symbols $\mathbb{C}$ and $\mathbb{C}$ are also used.
2-6.6 (—)	**P**	the set of prime numbers	**P** = {2, 3, 5, 7, 11, 13, 17, …} The symbols $\mathbb{P}$ and $\mathbb{P}$ are also used.
2-6.7 (*11.4.14*)	$[a, b]$	closed interval from a included to b included	$[a, b] = \{x \in \mathbf{R} \mid a \leqslant x \leqslant b\}$
2-6.8 (*11.4.15*)	$(a, b]$	left half-open interval from a excluded to b included	$(a, b] = \{x \in \mathbf{R} \mid a < x \leqslant b\}$ The notation $]a, b]$ is also used.
2-6.9 (*11.4.16*)	$[a, b)$	right half-open interval from a included to b excluded	$[a, b) = \{x \in \mathbf{R} \mid a \leqslant x < b\}$ The notation $[a, b[$ is also used.
2-6.10 (*11.4.17*)	(a, b)	open interval from a excluded to b excluded	$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$ The notation $]a, b[$ is also used.
2-6.11 (—)	$(-\infty, b]$	closed unbounded interval up to b included	$(-\infty, b] = \{x \in \mathbf{R} \mid x \leqslant b\}$ The notation $]-\infty, b]$ is also used.

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Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-6.12 (—)	$(-\infty, b)$	open unbounded interval up to b excluded	$(-\infty, b) = \{x \in \mathbf{R} \mid x < b\}$ The notation $]-\infty, b[$ is also used.
2-6.13 (—)	$[a, +\infty)$	closed unbounded interval onward from a included	$[a, +\infty) = \{x \in \mathbf{R} \mid a \leqslant x\}$ The notations $[a, \infty[$, $[a, +\infty[$ and $[a, \infty)$ are also used.
2-6.14 (—)	$(a, +\infty)$	open unbounded interval onward from a excluded	$(a, +\infty) = \{x \in \mathbf{R} \mid a < x\}$ The notations $]a, +\infty[$, $]a, \infty[$ and (a, ∞) are also used.

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