IEC/TR 61156-1-2

Edition 1.0 2009-05

TECHNICAL REPORT

**Multicore and symmetrical pair/quad cables for digital communications –
Part 1-2: Electrical transmission characteristics and test methods of symmetrical pair/quad cables**

IEC/TR 61156-1-2:2009(E)

About IEC publications

The technical content of IEC publications is kept under constant review by the IEC. Please make sure that you have the latest edition, a corrigenda or an amendment might have been published.

- Catalogue of IEC publications: www.iec.ch/searchpub

The IEC on-line Catalogue enables you to search by a variety of criteria (reference number, text, technical committee,…). It also gives information on projects, withdrawn and replaced publications.

- IEC Just Published: www.iec.ch/online_news/justpub

Stay up to date on all new IEC publications. Just Published details twice a month all new publications released. Available on-line and also by email.

- Electropedia: www.electropedia.org

The world's leading online dictionary of electronic and electrical terms containing more than 20 000 terms and definitions in English and French, with equivalent terms in additional languages. Also known as the International Electrotechnical Vocabulary online.

- Customer Service Centre: www.iec.ch/webstore/custserv

If you wish to give us your feedback on this publication or need further assistance, please visit the Customer Service Centre FAQ or contact us:

Email: csc@iec.ch
Tel.: +41 22 919 02 11
Fax: +41 22 919 03 00

IEC/TR 61156-1-2

Edition 1.0 2009-05

TECHNICAL REPORT

Multicore and symmetrical pair/quad cables for digital communications – Part 1-2: Electrical transmission characteristics and test methods of symmetrical pair/quad cables

INTERNATIONAL
ELECTROTECHNICAL
COMMISSION

PRICE CODE X

ICS 33.120.20

ISBN 978-2-88910-418-5

CONTENTS

FOREWORD ... 4
1 Scope ... 6
2 Normative references ... 6
3 Terms, definitions, symbols, units and abbreviated terms ... 6
3.1 Terms and definitions ... 6
3.2 Symbols, units and abbreviated terms ... 6
4 Basic transmission line equations ... 9
4.1 Introduction ... 9
4.2 Characteristic impedance and propagation coefficient equations ... 10
4.2.1 General ... 10
4.2.2 Propagation coefficient ... 10
4.2.3 Characteristic impedance ... 11
4.2.4 Phase and group velocity ... 12
4.3 High frequency representation of secondary parameters ... 13
4.4 Frequency dependence of the primary and secondary parameters ... 14
4.4.1 Resistance ... 14
4.4.2 Inductance ... 14
4.4.3 Characteristic impedance ... 15
4.4.4 Attenuation coefficient ... 15
4.4.5 Phase delay and group delay ... 16
5 Measurement of characteristic impedance ... 17
5.1 General ... 17
5.2 Open/short circuit single-ended impedance measurement made with a balun (reference method) ... 18
5.2.1 Principle ... 18
5.2.2 Test equipment ... 18
5.2.3 Procedure ... 19
5.2.4 Expression of results ... 19
5.3 Function fitting the impedance magnitude and angle ... 20
5.3.1 General ... 20
5.3.2 Impedance magnitude ... 20
5.3.3 Function fitting the angle of the characteristic impedance ... 22
5.4 Characteristic impedance determined from measured phase coefficient and capacitance ... 22
5.4.1 General ... 22
5.4.2 Equations for all frequencies case and for high frequencies ... 22
5.4.3 Procedure for the measurement of the phase coefficient ... 23
5.4.4 Phase delay ... 25
5.4.5 Phase velocity ... 25
5.4.6 Procedure for the measurement of the capacitance ... 25
5.5 Determination of characteristic impedance using the terminated measurement method ... 25
5.6 Extended open/short circuit method using a balun but excluding the balun performance ... 26
5.6.1 Test equipment and cable-end preparation ... 26
5.6.2 Basic equations ... 26
5.6.3 Measurement principle ... 26

5.7 Extended open/short circuit method without using a balun....28
5.7.1 Basic equations and circuit diagrams....28
5.7.2 Measurement principle....30
5.8 Open/short impedance measurements at low frequencies with a balun....31
5.9 Characteristic impedance and propagation coefficient obtained from modal decomposition technique....32
5.9.1 General....32
5.9.2 Procedure....33
5.9.3 Measurement principle....33
5.9.4 Scattering matrix to impedance matrix....35
5.9.5 Expression of results....37
6 Measurement of return loss and structural return loss....37
6.1 General....37
6.2 Principle....37
7 Propagation coefficient effects due to periodic structural variation related to the effects appearing in the structural return loss....38
7.1 General....38
7.2 Equation for the forward echoes caused by periodic structural inhomogeneities....38
8 Unbalance attenuation....40
8.1 General....40
8.2 Unbalance attenuation near end and far end....40
8.3 Theoretical background....42
Bibliography....46

Figure 1 – Secondary parameters extending from 1 kHz to 1 GHz....16
Figure 2 – Diagram of cable pair measurement circuit....19
Figure 3 – Determining the multiplier of 2π radians to add to the phase measurement....24
Figure 4 – Measurement configurations....27
Figure 5 – Measurement principle with four terminal network theory....27
Figure 6 – Admittance measurement configurations....30
Figure 7 – Admittance measurement principle....30
Figure 8 – Transmission line system....34
Figure 9 – Differential-mode transmission in a symmetric pair....40
Figure 10 – Common-mode transmission in a symmetric pair....40
Figure 11 – Circuit of an infinitesimal element of a symmetric pair....43
Figure 12 – Calculated coupling transfer function for a capacitive coupling of 0,4 pF/m and random ±0,4 pF/m (ℓ = 100 m; $\varepsilon_{r1} = \varepsilon_{r2}$ = 2,3)....45
Figure 13 – Measured coupling transfer function of 100 m Twinax 105 Ω....45

Table 1 – Unbalance attenuation at near end....41
Table 2 – Unbalance attenuation at far end....41
Table 3 – Measurement set-up....42

INTERNATIONAL ELECTROTECHNICAL COMMISSION

MULTICORE AND SYMMETRICAL PAIR/QUAD CABLES FOR DIGITAL COMMUNICATIONS –
PART 1-2: ELECTRICAL TRANSMISSION CHARACTERISTICS AND TEST METHODS OF SYMMETRICAL PAIR/QUAD CABLES

FOREWORD

1) The International Electrotechnical Commission (IEC) is a worldwide organization for standardization comprising all national electrotechnical committees (IEC National Committees). The object of IEC is to promote international co-operation on all questions concerning standardization in the electrical and electronic fields. To this end and in addition to other activities, IEC publishes International Standards, Technical Specifications, Technical Reports, Publicly Available Specifications (PAS) and Guides (hereafter referred to as "IEC Publication(s)"). Their preparation is entrusted to technical committees; any IEC National Committee interested in the subject dealt with may participate in this preparatory work. International, governmental and non-governmental organizations liaising with the IEC also participate in this preparation. IEC collaborates closely with the International Organization for Standardization (ISO) in accordance with conditions determined by agreement between the two organizations.
2) The formal decisions or agreements of IEC on technical matters express, as nearly as possible, an international consensus of opinion on the relevant subjects since each technical committee has representation from all interested IEC National Committees.
3) IEC Publications have the form of recommendations for international use and are accepted by IEC National Committees in that sense. While all reasonable efforts are made to ensure that the technical content of IEC Publications is accurate, IEC cannot be held responsible for the way in which they are used or for any misinterpretation by any end user.
4) In order to promote international uniformity, IEC National Committees undertake to apply IEC Publications transparently to the maximum extent possible in their national and regional publications. Any divergence between any IEC Publication and the corresponding national or regional publication shall be clearly indicated in the latter.
5) IEC provides no marking procedure to indicate its approval and cannot be rendered responsible for any equipment declared to be in conformity with an IEC Publication.
6) All users should ensure that they have the latest edition of this publication.
7) No liability shall attach to IEC or its directors, employees, servants or agents including individual experts and members of its technical committees and IEC National Committees for any personal injury, property damage or other damage of any nature whatsoever, whether direct or indirect, or for costs (including legal fees) and expenses arising out of the publication, use of, or reliance upon, this IEC Publication or any other IEC Publications.
8) Attention is drawn to the Normative references cited in this publication. Use of the referenced publications is indispensable for the correct application of this publication.
9) Attention is drawn to the possibility that some of the elements of this IEC Publication may be the subject of patent rights. IEC shall not be held responsible for identifying any or all such patent rights.

The main task of IEC technical committees is to prepare International Standards. However, a technical committee may propose the publication of a technical report when it has collected data of a different kind from that which is normally published as an International Standard, for example "state of the art".

IEC 61156-1-2, which is a technical report, has been prepared by subcommittee 46C: Wires and symmetric cables, of IEC technical committee 46: Cables, wires, waveguides, R.F. connectors, R.F. and microwave passive components and accessories.

The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
46C/853/DTR	46C/889/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all parts of the IEC 61156 series, under the general title: *Multicore and symmetrical pair/quad cables for digital communications*, can be found on the IEC website.

The committee has decided that the contents of this publication will remain unchanged until the maintenance result date indicated on the IEC web site under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

A bilingual version of this publication may be issued at a later date.

MULTICORE AND SYMMETRICAL PAIR/QUAD CABLES FOR DIGITAL COMMUNICATIONS – PART 1-2: ELECTRICAL TRANSMISSION CHARACTERISTICS AND TEST METHODS OF SYMMETRICAL PAIR/QUAD CABLES

1 Scope

This technical report is a revision of the symmetrical pair/quad electrical transmission characteristics present in IEC 61156-1:2002 (Edition 2) and not carried into IEC 61156-1:2007 (Edition 3).

This technical report includes the following topics from IEC 61156-1:2002:

– the characteristic impedance test methods and function fitting procedures of 3.3.6;
– Annex A covering basic transmission line equations and test methods;
– Annex B covering the open/short-circuit method;
– Annex C covering unbalance attenuation.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-726, *International Electrotechnical Vocabulary – Part 726: Transmission lines and waveguides*

IEC 61156-1:2007, *Multicore and symmetrical pair/quad cables for digital communications – Part 1: Generic specification*

IEC/TR 62152, *Background of terms and definitions of cascaded two-ports*

3 Terms, definitions, symbols, units and abbreviated terms

3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC 60050-726 and IEC/TR 62152 apply.

3.2 Symbols, units and abbreviated terms

For the purposes of this document, the following symbols, units and abbreviated terms apply.

Transmission line equation electrical symbols and related terms and symbols:

R	pair resistance (Ω/m)
L	pair inductance (H/m)
G	pair conductance (S/m)
C	pair capacitance (F/m)
α	attenuation coefficient (Np/m)
β	phase coefficient (rad/m)

γ	propagation coefficient (Np/m, rad/m)
v_P	phase velocity of cable (m/s)
v_G	group velocity of cable (m/s)
τ_P	phase delay time (s/m)
τ_G	group delay time (s/m)
Z_C	complex characteristic impedance, or mean characteristic impedance if the pair is homogeneous or free of structure (also used to represent a function fitted result) (Ω)
$\angle Z_C$	angle of the characteristic impedance in radians
Z_∞	high frequency asymptotic value of the characteristic impedance (Ω)
l	length (m)
j	imaginary denominator
Re	real part operator for a complex variable
Im	imaginary part operator for a complex variable
ω	radian frequency (rad/s)
f	frequency (Hz)
R'	first derivative of R with respect to ω
C'	first derivative of C with respect to ω
L'	first derivative of L with respect to ω
R_0	d.c. resistance of a round solid wire with radius r (Ω/m)
R_C	constant with frequency component of resistance which is about 1/4 of the d.c. resistance (Ω/m)
R_S	square-root of frequency component of resistance (Ω/m)
L_E	external (free space) inductance (H/m)
L_I	internal inductance whose reactance equals the surface resistance at high frequencies (H/m)
σ	specific conductivity of the wire material (S/m)
ρ	resistivity of the wire material (Ω/m^2)
μ	permeability of the wire material (H/m)
r	radius of the wire (m)
δ	skin depth (not to be confused with the dissipation factor tan δ) (m) $\delta = \frac{1}{\sqrt{\pi f \mu\sigma}}$
tan δ	dissipation factor tan $\delta = G/(\omega C)$
q	forward echo coefficient at the far end of the cable at a resonant frequency
p	reflection coefficient measured from the near end of the cable at a resonant frequency, $p = 10^{-PSRL/20} = \left\|\frac{Z_{CM} - Z_C}{Z_{CM} + Z_C}\right\|$

A_Q	forward echo attenuation at a resonant frequency (dB) $A_Q = -20 \log \lvert q \rvert$
$PSRL$	structural return loss at a resonant frequency (dB) $PSRL = -20 \log \lvert p \rvert$
K	$= 2\alpha l - 1$ when $2\alpha l >> 1$ (Np)
A_Q	$= 2 \times PSRL - 20 \log(2\alpha l - 1)$ (dB) where $2\alpha l$ is in Np
Z_{OC}	complex measured open circuit impedance (Ω)
Z_{SC}	complex measured short circuit impedance (Ω)
Z_{CM}	characteristic impedance as measured (with structure) (Ω) $Z_{CM} = \sqrt{Z_{SC}\, Z_{OC}}$
Z_{MEAS}	complex measured impedance (open or short) (Ω)
Z_{IN}	input impedance of the cable when it is terminated by Z_L (Ω)
Z_{OUT}	output impedance of the cable when the input of the cable is terminated by Z_G (Ω)
Z_{CN}	nominal characteristic impedance of a cable and is the specified Z_C value at a given frequency with tolerance and the structural return loss SRL limits in dB in a frequency range (Ω)
Z_N	nominal (reference) impedance of the link and/or terminals (the system) between which the cable is operating (Ω)
Z_R	(nominal) reference impedance that is used in measurement. Normally (for actual return loss results), $Z_R = Z_N$. When using a return loss measurement to approximate SRL, it is practical to choose Z_R to give the best balance in the given frequency range (Ω)
Z_T	terminated impedance measurement made with the opposite end of the cable pair terminated in the reference impedance Z_R (Ω)
ς	reflection coefficient measured in the terminated measurement method $\varsigma = \frac{Z_R - Z_C}{Z_R + Z_C}$
Z_G	termination at the cable input when defining the output impedance of the cable Z_{OUT} (Ω)
Z_L	termination at the cable output when defining the input impedance of the cable Z_{IN} (Ω)
L_0, L_1, L_2, L_3	least squares fit coefficients for angle of the characteristic impedance
K_0, K_1, K_2, K_3	least squares fit coefficients of the characteristic impedance
$\lvert Z_C \rvert$	fitted magnitude of the characteristic impedance (Ω)
$\lvert Z_{CM} \rvert$	measured magnitude of the characteristic impedance (Ω)
$\angle(V_{1N})$	input angle relative to a reference angle in radians
$\angle(V_{1F})$	output angle relative to the same reference angle in radians
k	multiple of 2π radians
S_{11}	reflection coefficient measured with an S parameter test set

RL	return loss (dB)
SRL	structural return loss (dB)

Attenuation unbalance electrical symbols:

TA	transverse asymmetry
LA	longitudinal asymmetry
R_1, R_2	resistance of one conductor per unit length (Ω)
L_1, L_2	inductance of one conductor per unit length (H)
C_1, C_2	capacitance of one conductor to earth (F)
G_1, G_2	conductance of one conductor to earth (S)
α_u	unbalance attenuation (dB)
T_u	unbalance coupling transfer function
Z_{com}	characteristic impedance of the common-mode circuit (Ω)
Z_{diff}	characteristic impedance of the differential-mode circuit (Ω)
Z_{unbal}	unbalance impedance (Ω)
ℓ	length of transmission line (m)
x	length coordinate (m)
γ_{com}	propagation factor of the common-mode circuit (Np/m, rad/m)
γ_{diff}	propagation factor of the differential-mode circuit (Np/m, rad/m)
α_{diff}	operational differential-mode attenuation of the cable (dB)
α_{com}	operational common-mode attenuation of the cable (dB)
ΔR	resistance unbalance of the sample length (Ω)
ΔL	inductance unbalance of the sample length (H)
ΔC	capacitance unbalance to earth (F)
ΔG	conductance unbalance to earth (S)
S	summing function
U_{diff}	voltage in the differential-mode circuit (V)
U_{com}	voltage in the common-mode circuit (V)
n, f	index to designate the near end and far end, respectively

4 Basic transmission line equations

4.1 Introduction

A review of the relationships between the propagation coefficient and characteristic impedance and the primary parameters R, L, G and C is useful here. Characteristic impedance is commonly thought of as being a magnitude quantity. While this concept may suffice for high frequency applications, this quantity is actually a complex one consisting of real and imaginary components or magnitude and angle. The associated propagation coefficient is readily viewed as being complex, consisting of the real attenuation and imaginary phase coefficient components. The four secondary components are readily related to the primary components. Frequency dependence of these parameters is also developed.

The cable pair parameters are represented as frequency domain dependent quantities. The measurement methods are based on frequency domain techniques. Measurement methods based on time domain techniques and combinations of time and frequency while useful in many cases are not covered here. The present-day availability of excellent frequency domain equipment such as the network analysers and impedance meters supports the frequency domain approach.

4.2 Characteristic impedance and propagation coefficient equations

4.2.1 General

The frequency domain of the complex characteristic impedance Z_C relates to the primary parameters as:

$$Z_C = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \tag{1}$$

The propagation coefficient, γ, relates to the primary parameters as:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{2}$$

4.2.2 Propagation coefficient

4.2.2.1 Attenuation and phase coefficients

Equation (2) is separated into its real and imaginary parts, the attenuation coefficient α and the phase coefficient β:

$$\alpha = \sqrt{-\frac{1}{2}(\omega^2 LC - RG) + \frac{1}{2}\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}} \tag{3}$$

$$\beta = \sqrt{\frac{1}{2}(\omega^2 LC - RG) + \frac{1}{2}\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}} \tag{4}$$

Further, by factoring out $\omega\sqrt{LC}$ we obtain:

$$\beta = \omega\sqrt{LC}\sqrt{\frac{1}{2}\left(1 - \frac{R}{\omega L}\frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}} \tag{5}$$

It can be shown that:

$$\alpha\beta = \omega\sqrt{LC}\left(\frac{R}{2}\sqrt{\frac{C}{L}}\right) \tag{6}$$

4.2.2.2 Equations useful at high frequencies

From Equations (5) and (6) we can solve for α and thus obtain for α and β the following expressions, valid within the entire frequency range:

$$\alpha = \frac{\frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}}{\sqrt{\frac{1}{2}\left(1 - \frac{R}{\omega L}\frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}}} \qquad (7)$$

$$\beta = \omega\sqrt{LC}\sqrt{\frac{1}{2}\left(1 - \frac{R}{\omega L}\frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}} \qquad (8)$$

Equations (7) and (8) are well suited for evaluation of high frequencies.

4.2.2.3 Equations useful at low frequencies

For low frequency evaluations, the expressions given by Equations (9) and (10) are suitable.

$$\alpha = \sqrt{\frac{\omega RC}{2}}\sqrt{\left(\frac{G}{\omega C} - \frac{\omega L}{R}\right) + \sqrt{\left(1 + \frac{\omega^2 L^2}{R^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}} \qquad (9)$$

$$\beta = \sqrt{\frac{\omega RC}{2}}\sqrt{\left(\frac{\omega L}{R} - \frac{G}{\omega C}\right) + \sqrt{\left(1 + \frac{\omega^2 L^2}{R^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}} \qquad (10)$$

4.2.3 Characteristic impedance

4.2.3.1 Real and imaginary parts

The characteristic impedance Z_C can also be separated into its real and imaginary parts as developed in Equations (11) and (12).

$$Z_C = Re\ Z_C + j\ Im\ Z_C = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \frac{\alpha + j\beta}{G + j\omega C} \qquad (11)$$

$$Z_C = \frac{\frac{1}{\omega C}\left[\left(\beta + \frac{G}{\omega C}\alpha\right) - j\left(\alpha - \frac{G}{\omega C}\beta\right)\right]}{1 + \frac{G^2}{\omega^2 C^2}} \qquad (12)$$

4.2.3.2 Equations useful at high frequencies

After substituting Equations (7) and (8) into Equation (12), the real and imaginary parts of the characteristic impedance are obtained as given in Equations (13) and (14) respectively. These are well suited for simplification (see 4.3) at high frequencies:

$$Re\ Z_C = \frac{\sqrt{\frac{L}{C}}\left[\frac{1}{2}\left(1 - \frac{R}{\omega L}\frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}\right]}{\left(1 + \frac{G^2}{\omega^2 C^2}\right)\sqrt{\frac{1}{2}\left(1 - \frac{R}{\omega L}\frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}}} \qquad (13)$$

$$-\,Im\ Z_{\mathrm{C}}=\frac{\dfrac{R}{2\omega\sqrt{LC}}+\dfrac{G}{2\omega C}\sqrt{\dfrac{L}{C}}-\dfrac{G}{\omega C}\sqrt{\dfrac{L}{C}}\left[\dfrac{1}{2}\left(1-\dfrac{R}{\omega L}\dfrac{G}{\omega C}\right)+\dfrac{1}{2}\sqrt{\left(1+\dfrac{R^2}{\omega^2 L^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}\right]}{\left(1+\dfrac{G^2}{\omega^2 C^2}\right)\sqrt{\dfrac{1}{2}\left(1-\dfrac{R}{\omega L}\dfrac{G}{\omega C}\right)+\dfrac{1}{2}\sqrt{\left(1+\dfrac{R^2}{\omega^2 L^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}}} \tag{14}$$

4.2.3.3 Equations useful at low frequencies

On the other hand, by substituting Equations (9) and (10) into Equation (12), the real and imaginary parts given in Equations (15) and (16) respectively are obtained. These are useful for simplification in the low frequency range:

$$Re\ Z_{\mathrm{C}}=\frac{\sqrt{\dfrac{R}{2\omega C}}\left[\sqrt{\dfrac{\omega L}{R}-\dfrac{G}{\omega C}+\sqrt{\left(1+\dfrac{\omega^2 L^2}{R^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}}+\dfrac{G}{\omega C}\sqrt{\dfrac{G}{\omega C}-\dfrac{\omega L}{R}+\sqrt{\left(1+\dfrac{\omega^2 L^2}{R^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}}\right]}{\left(1+\dfrac{G^2}{\omega^2 C^2}\right)} \tag{15}$$

$$-\,Im\ Z_{\mathrm{C}}=\frac{\sqrt{\dfrac{R}{2\omega C}}\left[\sqrt{\dfrac{G}{\omega C}-\dfrac{\omega L}{R}+\sqrt{\left(1+\dfrac{\omega^2 L^2}{R^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}}-\dfrac{G}{\omega C}\sqrt{\dfrac{\omega L}{R}-\dfrac{G}{\omega C}+\sqrt{\left(1+\dfrac{\omega^2 L^2}{R^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}}\right]}{\left(1+\dfrac{G^2}{\omega^2 C^2}\right)} \tag{16}$$

4.2.4 Phase and group velocity

The phase propagation time (per unit length) is:

$$\tau_{\mathrm{P}}=\frac{\beta}{\omega} \tag{17}$$

By introducing β from Equations (8) and (10), we obtain:

$$\tau_{\mathrm{P}}=\sqrt{LC}\sqrt{\frac{1}{2}\left(1-\frac{R}{\omega L}\frac{G}{\omega C}\right)+\frac{1}{2}\sqrt{\left(1+\frac{R^2}{\omega^2 L^2}\right)\left(1+\frac{G^2}{\omega^2 C^2}\right)}} \tag{18}$$

and

$$\tau_{\mathrm{P}}=\sqrt{\frac{RC}{2\,\omega}}\sqrt{\left(\frac{\omega L}{R}-\frac{G}{\omega C}\right)+\sqrt{\left(1+\frac{\omega^2 L^2}{R^2}\right)\left(1+\frac{G^2}{\omega^2 C^2}\right)}} \tag{19}$$

The group propagation time (per unit length) is:

$$\tau_{\mathrm{G}}=\frac{d\beta}{d\omega} \tag{20}$$

$$\tau_{\mathrm{G}}=\frac{\beta}{\omega}+\frac{1}{2}\left(\frac{L'}{L}+\frac{C'}{C}\right)\beta+\frac{\omega^2 LC}{4\beta}\left[\left(-\frac{G}{\omega C}+\frac{\dfrac{R}{\omega L}\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}{\sqrt{\left(1+\dfrac{R^2}{\omega^2 L^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}}\right)\frac{d\left(\dfrac{R}{\omega L}\right)}{d\omega}\right]\left[+\left(-\frac{R}{\omega L}+\frac{\dfrac{G}{\omega C}\left(1+\dfrac{R^2}{\omega^2 L^2}\right)}{\sqrt{\left(1+\dfrac{R^2}{\omega^2 L^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}}\right)\frac{d\left(\dfrac{G}{\omega C}\right)}{d\omega}\right] \tag{21}$$

The phase and group velocities are, respectively,

$$v_P = \frac{1}{\tau_P} \tag{22}$$

$$v_G = \frac{1}{\tau_G} \tag{23}$$

The above expressions are accurate and valid within the whole frequency range. If C and $G/(\omega C)$ can be regarded as frequency independent coefficients, then we obtain:

$$\tau_G = \frac{\beta}{\omega} + \frac{\beta}{2}\frac{L'}{L} + \frac{C}{4\beta}\left[-\frac{G}{\omega C} + \frac{\frac{R}{\omega L}\left(1+\frac{G^2}{\omega^2 C^2}\right)}{\sqrt{\left(1+\frac{R^2}{\omega^2 L^2}\right)\left(1+\frac{G^2}{\omega^2 C^2}\right)}}\right]\left(-R+R'\omega-\frac{L'R}{L}\omega\right) \tag{24}$$

The above expressions, which are valid within the entire frequency range, can be simplified into approximate expressions, which are valid at high or low frequencies only.

4.3 High frequency representation of secondary parameters

The high frequency representations of the formulas are useful over a broad range of frequencies extending from voice frequency on up because of the range of values for the dissipation factor. $G/(\omega C) = \tan \delta < 0{,}03$ (< 3 %) even for PVC insulated cables up to 1,5 MHz and for the polyethylene (PE), insulation is very small at about 0,000 1 (0,01 %). This results in approximations, which in practice are valid for the whole frequency range as follows:

$$Re\, Z_C \approx \sqrt{\frac{L}{C}}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{R^2}{\omega^2 L^2}}} \tag{25}$$

$$-Im\, Z_C \approx \frac{R}{2\omega C\ Re\, Z_C} - \frac{G}{\omega C} Re\, Z_C + \frac{\frac{G}{2\omega C}\frac{L}{C}}{Re\, Z_C} \tag{26}$$

$$\alpha \approx \frac{R}{2\ Re\, Z_C} + \frac{G\left(\sqrt{\frac{L}{C}}\right)^2}{2\ Re\, Z_C} \tag{27}$$

$$\beta \approx \omega C\ Re\, Z_C \tag{28}$$

$$\tau_P \approx \sqrt{LC} \tag{29}$$

$$\tau_G \approx \frac{\beta}{\omega} + \frac{\beta}{2}\frac{L'}{L} + \frac{C}{4\beta}\left(-\frac{G}{\omega C} + \frac{\frac{R}{\omega L}}{\sqrt{1+\frac{R^2}{\omega^2 L^2}}}\right)\left(-R+R'\omega-\frac{L'R}{L}\omega\right) \tag{30}$$