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Measurement of liquid flow in open channels — Computing stream flow using an unsteady flow model

Mesure de débit de liquides dans les canaux découverts — Calcul de l'écoulement dans un cours d'eau à l'aide d'un modèle d'écoulement non permanent

ISO/TR 11627:1998(E)

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare International Standards, but in exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

— type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;

— type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;

— type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 11627, which is a Technical Report of type 2, was prepared by Technical Committee ISO/TC 113, *Hydrometric determinations,* Subcommittee SC 1, *Velocity area methods*.

This document is being issued in the Technical Report (type 2) series of publications (according to subclause G.3.2.2 of part 1 of the ISO/IEC Directives) as a “prospective standard for provisional application” in the field of hydrometric determinations because there is an urgent need for

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guidance on how standards in this field should be used to meet an identified need.

This document is not to be regarded as an “International Standard”. It is proposed for provisional application so that information and experience of its use in practice may be gathered. Comments on the content of this document should be sent to the ISO Central Secretariat.

A review of this Technical Report (type 2) will be carried out not later than three years after its publication with the options of: extension for another three years; conversion into an International Standard; or withdrawal.

Measurement of liquid flow in open channels — Computing stream flow using an unsteady flow model

1 SCOPE

This Technical Report describes a method for computing continuous records of stream flow in an open channel through the numerical solution of the one-dimensional unsteady flow equations. Such an approach is typically identified as an unsteady flow model and generally involves the use of computers for solution of the flow equations.

Unsteady flow models are appropriate for computing stream flow records at locations where (1) a single-valued stage-discharge relation does not exist, (2) backwater affects the discharge under selected or all conditions, (3) flows are affected by tides, or (4) it is not possible to gage the flow using velocity-area methods. Unsteady flow models also are appropriate for evaluating the effects of changes in a managed flow regime on downstream conditions prior to the implementation of any changes.

This Technical Report is applicable to steady and unsteady flows, to nominee flows, and to tidal flows in which there are no significant longitudinal and vertical density gradients. The method is considered equivalent to, or better than, the commonly used stage-fall-discharge technique (ISO 1100-1:1996, 7.2 and Annex C) because the method uses information on the physical characteristics of the channel, including the cross-sectional geometry, channel rugosity and channel slope, and the method is based on a mathematical description of the physics of fluid flow.

This Technical Report describes the theoretical basis and fundamental assumptions of the technique, and provides a summary of selected numerical methods used to solve the unsteady flow equations. Also provided are details on the application of an unsteady flow model, including data requirements, procedures for model calibration, testing, and applications, and identification of uncertainties associated with the method. This Technical Report does not provide sufficient information for the development of a computer program for solving the unsteady flow equations, but rather is based on the assumption that an adequately documented computer program is available

2 REFERENCES

ISO 748:1997, *Measurement of liquid flow in open channels — Velocity-area methods.*

ISO 772:1996, *Hydrometric determinations — Vocabulary and symbols.*

ISO 1070:1992, *Liquid flow measurement in open channels — Slope-area method.*

ISO 1100-1:1996, *Measurement of liquid flow in open channels — Part 1: Establishment and operation of a gauging station*.

ISO 1100-2:—[1)], *Measurement of liquid flow in open channels — Part 2: Determination of the stage-discharge relation*.

1) To be published. (Revision of ISO 1100-2:1982)

ISO 2425:—[2], *Methods for hydrometric measurements under tidal conditions.*

ISO 2537:1988, *Liquid flow measurement in open channels — Rotating element current-meters.*

ISO 3454:1983, *Liquid flow measurement in open channels — Direct depth sounding and suspension equipment.*

ISO 4373:1995, *Measurement of liquid flow in open channels — Water-level measuring devices.*

ISO 6416:1992, *Measurement of liquid flow in open channels — Measurement of discharge by the ultrasonic (acoustic) method.*

ISO 9555-1:1994, *Measurement of liquid flow in open channels — Tracer dilution methods for the measurement of steady flow — Part 1: General.*

ISO 9555-2:1992, *Measurement of liquid flow in open channels — Tracer dilution methods for the measurement of steady flow — Part 2: Radioactive tracers.*

ISO 9555-3:1992, *Measurement of liquid flow in open channels — Tracer dilution methods for the measurement of steady flow — Part 3: Chemical tracers.*

ISO 9555-4:1992, *Measurement of liquid flow in open channels — Tracer dilution methods for the measurement of steady flow — Part 4: Fluorescent tracers.*

3 DEFINITIONS

For the purposes of this Technical Report, the definitions given in ISO 772 and the following definitions apply.

3.1 Boundary condition: A boundary condition is a condition that a dependent variable of a differential equation must satisfy along the boundary of the model domain. Boundary conditions for the dependent variables must be specified at the physical extremities of the modeled region for the duration of model application.

3.2 Courant condition: The usual condition for the numerical stability of the explicit formulation of a numerical scheme which requires that the ratio of the propagation speed of a physical disturbance to that of a numerical signal should not exceed unity.

3.3 Explicit finite-difference numerical scheme: Explicit numerical schemes convert either the characteristic equations or the governing equations to a system of linear algebraic equations from which the unknowns may be solved directly (explicitly) without iterative computations. Dependent variables on the advanced time level are determined one point at a time from known values and conditions at the present or previous time levels. Explicit schemes are only conditionally stable, meaning that errors may grow as the solution progresses, and the errors are a function of the time and distance finite-difference step sizes. Explicit schemes are generally stable when the courant conditions is met, which results in limitations on the distance step and maximum time which can be used.

3.4 Gradually-varied, unsteady flow: Generally nonuniform flow in which there are no abrupt changes in depth along the longitudinal axis of the channel, and in which depth (and velocity and discharge) change with time.

2) To be published. (Revision of ISO 2425:1974)

3.5 Hydrograph: A relation in graphical, equational, or tabular form between time and flow variables such as discharge, depth, velocity, and stage. Stage and dischar hydrographs are typically used for open channel flows.

3.6 Implicit finite-difference numerical scheme: Implicit numerical schemes convert either the characteristic equations or the governing equations to a system on nonlinear algebraic equations from which the unknowns must be solved iteratively. All of the unknowns within the model domain are determined simultaneously, rather than point-by-point as with explicit methods. Implicit methods are generally stable, and are more computationally efficient than explicit schemes, but implicit schemes require more complex computer algorithms than do explicit schemes.

3.7 Initial conditions: A description of the dynamic conditions (typically, discharge and depth of flow for unsteady flow models) in the model domain at some specified time, usually the beginning of the simulation period. For all subsequent times, the governing equations and the boundary conditions describe the state of the system.

3.8 Method of characteristics: The method of characteristics is a mathematical approach for solving boundary-value problems by transforming the original partial differential equations representing the physical system into corresponding characteristic equations. The characteristic equations are ordinary differential equations and generally are more amenable to numerical solution than are the original partial differential equations.

3.9 Momentum coefficient: The momentum coefficient, also known as the Boussinesq coefficient, quantifies the deviation of the velocity at any point in a cross section from uniform velocity distribution in the same cross section. A value of unity indicates that a uniform velocity distribution is present in the cross section. The momentum coefficient generally varies between about 1.01 and 1.12 for fairly straight, prismatic channels; coefficients are typically smaller for large, deep channels than for small channels.

4 UNITS OF MEASUREMENT

The units of measurement used in this International Standard are SI units.

5 PRINCIPLES OF UNSTEADY FLOW MODELS

5.1 Governing equations

The foundations for the fundamental derivation of the governing one-dimensional unsteady flow equations were laid by the 19th century hydraulicians Coriolis, Boussinesq, and Saint Venant. The governing equations are the one-dimensional, cross-sectionally averaged expressions for (1) the conservation of mass (or equation of continuity),

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \qquad (1)$$

and, (2) conservation of linear momentum

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\beta \frac{Q^2}{A}\right) + gA\frac{\partial z}{\partial x} + gA\,(S_f - S_o) = qu' \qquad (2)$$

where:

A is the cross-sectional area of the channel, and varies with x, t, and z;
t is time;
Q is the discharge, and varies with x and t;
u' is longitudinal component of the lateral inflow velocity, and varies with x and t;
x is the longitudinal position along the channel axis;
z is the depth of flow, and varies with x and t;
g is the acceleration of gravity;
β is the momentum coefficient, and varies with x, z, and t;
q is the lateral inflow per unit length of channel, and varies with x and t;
S_o is the bed slope, and varies with x; and
S_f is the friction slope, and varies with x, t, and z.

The momentum coefficient may be computed as

$$\beta = \int \frac{u^2\,dA}{U^2 A} \qquad (3)$$

where:

u is the velocity in some elemental area dA, and
U is the mean velocity in the same cross section having a total area A.

The friction slope, S_f, accounts for the resistance due to external boundary stresses. The friction slope is generally written as

$$S_f = \frac{Q|Q|n^2}{AR^{4/3}} \qquad (4)$$

where

R is the hydraulic radius, and
n is the Manning coefficient.

Both R and n can vary as a function of x, z, and t. Equation 4 is based on the assumption that the Manning equation for steady, uniform flow provides a reasonable approximation for S_f in unsteady, nonuniform flow.

Equation 2 can be modified to include a term accounting for the momentum imparted to the water by a temporally and spatially varying wind. Equations 1 and 2 also can be written with (1) depth and velocity, (2) stage and velocity, or (3) stage and discharge as the dependent variables.

Equations 1 and 2 apply to the unsteady, spatially-varied, turbulent free-surface flow of an incompressible, viscous fluid in an open channel of arbitrary cross section and alignment. The equations are solved simultaneously for the unknowns z (depth of flow) and Q (discharge) as a function of time (t) and longitudinal position (x).

5.2 Assumptions upon which governing equations are based

Equations 1 and 2 are derived from first principles, and may be obtained directly from the three-dimensional equation of mass continuity and the Navier-Stokes equations, which are general, three-dimensional statements of the conservation of momentum for any fluid flow. A number of assumptions are required to derive equations 1 and 2. An unsteady flow model which is based on equations 1 and 2 should generally be applied to those conditions in which none of the major assumptions are severely violated. The assumptions are as follows.

- The flow is approximately one-dimensional, meaning that the predominant spatial variation in dynamic conditions (discharge, velocity, and stage) is in the longitudinal direction.
- The fluid density is homogeneous throughout the modeled reach.
- Vertical accelerations are negligible (the hydrostatic pressure distribution is applicable).
- Velocity is uniformly distributed in a given cross section. Inclusion of the momentum coefficient in equation 2 allows this assumption to be violated somewhat, but there should be no flow separation, and streamline should not be highly curvilinear.
- Neither aggradation nor degradation of the flow channel occurs.
- Turbulence and energy dissipation can be described by resistance laws formulated for steady, uniform flow (required for equation 4).
- There are no abrupt changes in channel shape or alignment.
- The velocity is zero at the channel boundary.
- There is no superelevation of the water level at any cross section.
- Surface tension and the density of air at the free surface are negligible.

5.3 Simplified models

A number of techniques have been used to simplify equations 1 and 2 to provide approximate unsteady flow models. These simplified models generally provide results with less computational effort and fewer data than is required for solution of the full equations. However, the models have limited applicability, and it is more appropriate to use a general unsteady flow model based on equations 1 and 2 to obtain reliable records of discharge under a wide range of conditions. A brief summary of simplified models follows.

5.3.1 Empirical models

Empirical models are based on observations of past flood events. These models are limited to applications in which sufficient observations of inflows and outflows of a river section are available to calibrate essential empirical relations or routing coefficients. These models are typically applied to slowly fluctuating rivers with negligible lateral inflows and backwater effects.

5.3.2 Hydrologic models

Hydrologic models are based on the continuity equation written as

$$I - O = dS/dt \qquad (5)$$

where

I is the inflow to the modeled river section;
O is the outflow from the section; and
dS is the change in storage within the section during the time interval dt.

The storage is generally assumed to be related to the inflow or outflow by some empirically-determined storage constant. Hydrologic models are limited to applications in which the stage-discharge relation is single-valued, and are not applicable to flows having backwater effects, significant lateral inflows, or looped stage-discharge relations. Difficulties in solving equation 5 are often encountered when flows are changing rapidly with time.

5.3.3 Linearized models

Linearized models are derived from equations 1 and 2 by ignoring or linearizing nonlinear terms in the equations. The linearized equations can then be analytically integrated with less computational effort than is required for numerical integration of equations 1 and 2. The most common simplifying assumptions for these models are:

- the acceleration term (second term) in the momentum equation (equation 2) is negligible;
- the cross-sectional area (A) and channel bottom slope (S_o) are constant;
- the friction slope (S_f) is linearized with respect to discharge and depth;
- there is no lateral inflow; and
- the routed flood wave has a simple shape described by an analytical expression.

These assumptions severely limit the applicability of linearized models.

5.3.4 Kinematic wave model

The kinematic wave model is derived by assuming that all terms in the momentum equation are negligible relative to the friction slope (S_f) and the bed slope (S_o), and that there is no lateral inflow,

so that

$$S_f = S_o \tag{6}$$

As a consequence of equation 6, the discharge for a kinematic flow is equal to the normal discharge. This means that the momentum of the unsteady flow is described by an expression, such as the Manning or Chezy equations, in which flow is a single-valued function of depth of flow. Moreover, kinematic waves travel without attenuation of the peak flow, but the shape of the flood wave is modified as the wave is translated downstream. The kinematic wave model allows only the downstream propagation of flow disturbances, so that backwater and tidal effects cannot be modeled. Numerous analytical solutions exist for applications of the kinematic wave model to specific flow geometries, and these models are most widely used in the routing of overland flow of precipitation runoff.

5.3.5 Diffusion analogy model

The diffusion analogy model is obtained by assuming that the channel is prismatic, that the local and convective acceleration terms in the momentum equation are negligible, and that there is no lateral inflow. The continuity and momentum equations may then be combined to form a single parabolic partial-differential equation, which is in the form of the so-called convective-diffusion equation with the single unknown of discharge. The local and convective acceleration terms, the first two terms in equation 2, are often small in steep streams.

The diffusion analogy model can be used to compute flows affected by backwater conditions. However, the diffusion model is limited to applications in which flows change relatively slowly, and in which the channel has a rather uniform geometry throughout the modeled reach

5.4 Numerical techniques for solution of governing equations

No known analytical solutions exist for equations 1 and 2. Consequently, numerical techniques are used to convert equations 1 and 2 into algebraic equations that may be solved for z and Q at finite, incremental values of x and t. This solution depends on the proper description of the cross-sectional area as a function of x and t, and on the availability of accurate boundary condition data.

A variety of numerical techniques have been proposed and used to solve the unsteady flow equations. Although finite element methods may be used to solve the equations, finite-difference techniques generally are more appropriate for the solution of the one-dimensional partial-differential equations describing unsteady open-channel flow. The three broad categories of numerical techniques are (1) method of characteristics, (2) explicit finite-difference methods, and