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Measurement of liquid flow in open channels — Mixing length of a tracer

Mesure de débit des liquides dans les canaux découverts — Longueur de bon mélange d'un traceur

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare International Standards, but in exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

- type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;
- type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;
- type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 11656, which is a Technical Report of type 3, was prepared by Technical Committee ISO/TC 113, *Measurement of liquid flow in open channels*, Sub-Committee SC 4, *Dilution methods*.

Annexes A, B and C of this Technical Report are for information only.

Introduction

A variety of formulae have been developed for estimating mixing length in open channels. Some of these formulae have been developed for special flow conditions.

Most mixing-length formulae have been developed for injection of a tracer at the centre of flow. Mixing theory will also allow these formulae to be used for the injection of a tracer at one edge of the flow. However, there are times when, for a variety of reasons, a tracer is injected at a point other than the centre or edge of flow. Also, the tracer may be injected from a line source or from a multiple-point source. Thus, a mixing-length formula is needed that can estimate the mixing length for different injection situations.

Mixing-length formulae are generally developed for a condition which assumes complete mixing. However, an examination of the mixing process indicates that an infinite distance is required for theoretically complete mixing (100 %). If this theory is correct, then the existing mixing-length formulae only approximate complete mixing. Experience shows that satisfactory flow measurements can be made with less than complete mixing. Special methods can be used to minimize errors resulting from measurements at considerably less than complete mixing.

For these reasons, it is important to provide an objective means of defining the degree of mixing, and to estimate the mixing distance associated with various specified degrees of mixing.

These results may lead to the elaboration of a future International Standard.

Measurement of liquid flow in open channels — Mixing length of a tracer

1 Scope

This Technical Report investigates cross-channel mixing characteristics of solutes injected in streams. Specifically, it relates to the use of tracers for the measurement of discharge. A tracer must be well mixed, or compensating measures taken, in order to obtain a satisfactory dilution-type discharge measurement.

The purposes of this Technical Report are as follows:

a) to compare methods of defining the degree of mixing of a solute in a stream and to recommend a method;

b) to compare methods of estimating the mixing length (the downstream distance required for a solute to thoroughly mix across a stream) and to recommend a particular method;

c) to investigate the errors in dilution measurements associated with incomplete mixing;

d) to discuss methods of reducing errors in dilution-discharge measurements when mixing is incomplete.

2 Reference

The following standard contains provisions which, through reference in this text, constitute provisions of this Technical Report. At the time of publication, the edition indicated was valid. All standards are subject to revision, and parties to agreements based on this Technical Report are encouraged to investigate the possibility of applying the most recent edition of the standard indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 772:1988, *Liquid flow measurement in open channels — Vocabulary and symbols.*

3 Definitions

For the purposes of this Technical Report, the definitions given in ISO 772, except where noted, and the following definitions apply.

3.1 complete mixing: Mixing which occurs at a channel section, when the constant-injection method is used, if the steady-state tracer concentrations are equal at all points in the cross-section. Similarly, for the sudden injection method, mixing which occurs if the areas under the time/concentration curves are equal at all points sampled in a section.

3.2 degree of mixing: Measure of the extent to which mixing has been achieved in a cross-section downstream from the injection of a tracer.

The degree of mixing may vary from nearly 0 % in a cross-section immediately downstream from the injection to 100 % at a cross-section in which the tracer has been completely mixed across the entire cross-section.

3.3 mixing length: Distance, measured along the general path of flow between the injection cross-section and the downstream cross-section, at which the specified degree of mixing is obtained.

For given conditions, the mixing length is not a fixed value. It varies according to the specified degree of mixing. The higher the specified degree of mixing, the longer the mixing length.

4 Units of measurement

The units of measurements used in this Technical Report are those of the International System (SI).

5 Determination of the degree of mixing

5.1 Criteria and concepts

Determination of the degree of mixing should be readily conceptualized. It should also provide a unique value for the degree of mixing which can be rationally related to the mixing observed in the channel.

The values describing the degree of mixing range from 0 to 100 %. Where mixing has just begun, near the injection source, the degree of mixing approximates 0, and where mixing is complete, it reaches 100 %. If a tracer has been injected such that it is completely mixed in half of the flow and is not mixed at all in the other half, the degree of mixing is 50 %. The concept should hold for conditions where the tracer is fully mixed in other specified parts of the flow and is not mixed at all in the remaining flow.

For a selected downstream cross-section, the degree of mixing is determined by using the areas under N curves of concentration as a function of time for the sudden injection method, or by using N concentration values on the plateau for the constant-injection method. The areas of the concentrations must be related to some cross-sectional flow characteristic. Because of the mass balance of tracer, the appropriate characteristic is cumulative discharge, or relative cumulative discharge, measured from one edge of flow. The preferred index is the relative cumulative discharge, ranging in value from 0 to 1. Width or other cross-section characteristics vary from one cross-section to another, and are not usually adequate for accounting for the mass balance of tracer.

5.2 Formulae defining the degree of mixing

Various formulae have been proposed for defining the degree of mixing. Five such formulae are presented below.

Coefficient of variation (see [1], page 6 and [7], page 1073)

$$M_{\mathrm{CV}} = \frac{s_C}{\overline{C}} \times 100$$

$$= \frac{\left[N \sum_{i=1}^{N} C_i^2 - \left(\sum_{i=1}^{N} C_i \right)^2 \right]^{1/2}}{N\overline{C}} \times 100 \qquad \ldots (1)$$

Rimmar equation (see [1], page 6)

$$M_{\mathrm{R}} = \left[\frac{\hat{C} - \overline{C}}{\overline{C}} \right] \times 100 \qquad \ldots (2)$$

Schuster equation (see [1], page 6 and [8], page 134)

$$M_{\mathrm{S}} = \left[1 - \frac{\sum_{i=1}^{N} \left| C_i - \overline{C} \right|}{N\overline{C}} \right] \times 100 \qquad \ldots (3)$$

Cobb-Bailey equation (see [9], page C5 and [5], page 48)

$$M_{\mathrm{CB}} = \left[1 - \frac{1}{2} \int_{q/Q=0}^{1} \left| \frac{C - \overline{C}}{\overline{C}} \right| d(q/Q) \right] \times 100 \qquad \ldots (4)$$

or, in discrete form

$$M_{\mathrm{CB}} = \left\{ 1 - \frac{1}{2} \sum_{i=1}^{N} \left[\left| \frac{C_i - C}{\overline{C}} \right| \Delta(q/Q) \right] \right\} \times 100 \qquad \ldots (5)$$

Graphic (Cobb and Bailey, communication)

$$M_{\mathrm{G}} = \left(\frac{A}{A+B} \right) \times 100 \qquad \ldots (6)$$

Definition of symbols

In the above equations:

M is the degree of mixing;

s_C is the standard deviation of the concentrations observed in a section;

C is the observed tracer concentration; this is the steady-state concentration observed at a selected cross-section for the constant-injection method, or the area under the time/concentration curve for the sudden injection method;

C_i	is the discrete value of C at the ith observation point across the channel;
$\overline{C}$	is the average concentration in the channel cross-section;
$\hat{C}$	is the concentration observed in the cross-section having the greatest departure from the average concentration $\overline{C}$;
N	is the number of observation points across a section; the observations are taken at the centre of equal increments of flow;
Q	is the total stream discharge;
q	is the cumulative discharge at any point in a channel cross-section; the value of q is 0 at one bank and Q at the opposite bank;
A and B	are the areas associated with the cross-sectional distribution of tracer (see figure 1).

The characteristics of the various definitions of mixing may best be seen by looking at a number of examples. The examples A, B, C and D which follow assume that the concentration was observed at 10 points across the section. The assumed concentration distributions are shown in figure 2. The degrees of mixing computed by the various formulae are shown in table 1.

The concentration distributions shown in figure 2 are idealized distributions which can be approximated by a line injection across a part of a section. The distributions shown in figure 2 are used to demonstrate various characteristics of the formulae.

Table 1 — Degree of mixing computed by application of the various equations to the concentration distributions shown in figure 2

Values in percentage

Equation	Degree of mixing, M, for example			
	A	B	C	D
(1)	200	100	50	0
(2)	400	100	− 100	0
(3)	− 60	0	60	100
(5)	20	50	80	100
(6)	20	50	80	100

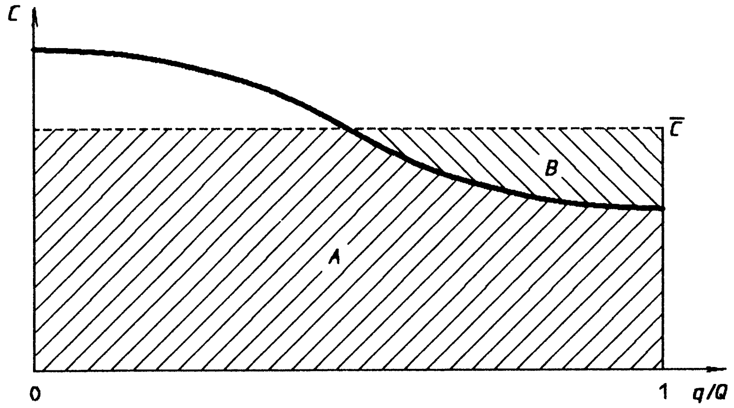
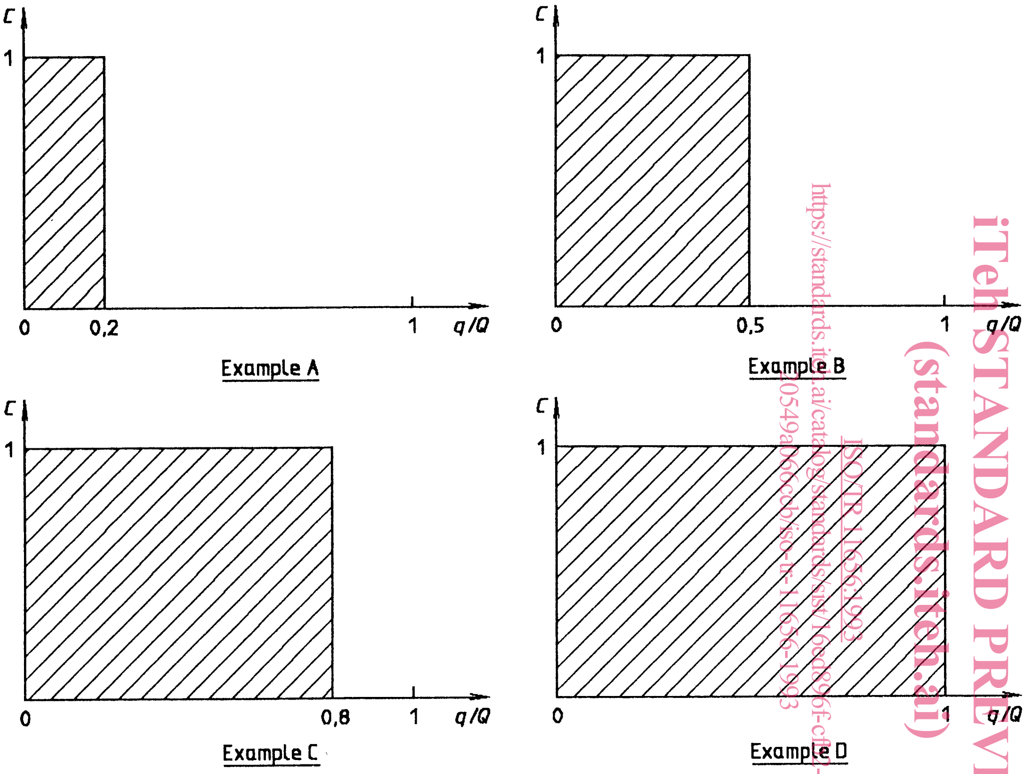


Figure 1 — Graphic description of the degree of mixing



Example	q/Q	Concentration observed, C, for an average concentration, $\overline{C}$, across a section of									
		0,05	0,15	0,25	0,35	0,45	0,55	0,65	0,75	0,85	0,95
A	0,2	1,0	1,0	0	0	0	0	0	0	0	0
B	0,5	1,0	1,0	1,0	1,0	1,0	0	0	0	0	0
C	0,8	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0	0
D	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0

Figure 2 — Hypothetical concentration distributions across a section (constant-injection method)

A number of observations can be made from this exercise. Equations (1) and (2) can give values which exceed 100 %. Equation (3) gives negative values for low degrees of mixing. Equation (2) can have negative values for some degrees of mixing.

The definition of the coefficient of variation [equation (1)] appears to be uniquely valued but shows an inverse relation with the degree of mixing. That is, the lower the computed value, the higher the degree of mixing.

The Cobb-Bailey formula [equations (4) and (5)] and the graphic formula [equation (6)] give identical numerical values. In fact, the Cobb-Bailey formula was developed to be consistent with the graphic definition. Examination of the concentration distribution curve across the injection point cross-section will reveal by the graphic method that M_G approaches zero. This can also be shown mathematically by the Cobb-Bailey formula. It is seen that only the Cobb-Bailey and the graphic formulae, [equations (4), (5) and (6)] fully meet the recommended criteria.

5.3 Recommended formula

Because of the above characteristics, as revealed in table 1, the definitions of mixing given by Rimmar and Schuster may be discarded. These equations do not define a unique degree of mixing at every condition of mixing.

The coefficient of variation [equation (1)] exhibits an inverse relation with the degree of mixing and may also give values greater than 100 %.

The graphic definition [equation (6)] provides a clear and easy-to-follow conceptual definition. Both the Cobb-Bailey and the graphic formulae [equations (4), (5) and (6)] fit the criteria established earlier and provide identical results. These two definitions thus are recommended for determining the degree of mixing.

In most cases, the discrete form, equation (5), will be necessary. The computed degree of mixing may vary slightly with the number of cross-channel observations.

In a practical situation, the problem arises of how to determine the relative flow, q/Q, across a channel without increasing the number of sub-area and velocity measurements across the channel. At times, information is available for defining the variation of the relative flow across a section. Otherwise, an approximate calculation can be made on the basis of width by substituting relative width in place of relative discharge, q/Q, in equations (4) and (5). Whether the values of q/Q are approximated or not, it is conceptually essential to have a definition that requires each concentration observation to be weighted by flow, because this is the only means of accounting for the mass balance of the tracer.

6 Mixing length

6.1 Concepts

In an open channel, mixing takes place in three directions: vertical, lateral (cross-flow) and longitudinal (parallel to the flow).

Vertical mixing takes place in most open channels relatively quickly if the tracer solution density is near that of water. Lateral mixing approaches the completely mixed condition within a finite distance depending on channel and flow characteristics. Longitudinal mixing continues to take place throughout the length of the channel.

The validity of flow measurement is dependent on adequate vertical and lateral mixing. The distance required for lateral mixing is of primary concern when making dilution-type flow measurements, since vertical mixing usually occurs rapidly.

Complete mixing as defined in 3.1 theoretically never occurs but is approached asymptotically. In a practical sense, complete mixing needs to be defined for a finite distance. For the purposes of this Technical Report, an adequate degree of mixing for most discharge measurements is considered to be 98 % as defined by the Cobb-Bailey equation.

6.2 Methods of estimating mixing length

There are three general approaches to estimating mixing lengths: direct observation, the empirical method and the theoretical method.

6.2.1 Direct observation

Direct observation involves injecting a tracer into a channel and, from measurements, determining the distance required for mixing. The results are generally valid only for that channel and for those flow conditions for which the observations were made.

6.2.2 Empirical method

A number of researchers have used the empirical approach. This involves the fitting of observed data from a limited number of streams to a set of channel or flow characteristics, either by regression analysis or by some other technique. Usually, the number of independent variables is very limited and involves measures of stream width, depth and discharge. The resulting equation is usually valid only for a limited area or type of flow condition.

The empirical formula generally takes the form:

$$X = aB^{b_1}D^{b_2}Q^{b_3}$$

where

X	is the mixing length;
B	is the channel width;
D	is the channel depth;
Q	is the flow rate;
a	is a constant;
b_1, b_2, b_3	are exponents, any one of which can be positive or negative, or in some cases equal to zero.

6.2.3 Theoretical method

The theoretical approach is usually more tedious and an attempt is made to account, in some manner, for all of the significant channel and flow characteristics which may affect the mixing process. The resulting equation for estimating mixing distance should be quasi-universal in application. The assumptions made and the lack of understanding of how some variables affect the mixing process may limit this universality, but this property qualifies the theoretical method as a satisfactory approach for an international standard.

Most theoretical equations have been developed for the injection of tracer at the centre or side of a channel. They provide an approximate value of the distance required for adequate mixing, but seldom specify the actual degree of mixing for which a formula has been developed, which can cause confusion when comparing equations.

6.3 Formulae for estimating mixing length

A review of the literature reveals that a number of relations have been developed to estimate mixing length. The following relations were considered and computational results compared with observed data for a degree of mixing, M, of 98 %.

6.3.1 Empirical relations

André formula

$$X = a_1 B Q^{1/3} \qquad \ldots (7)$$

Day formula

$$X = 25B \qquad \ldots (8)$$

Hull formula

$$X = a_2 Q^{0,33} \qquad \ldots (9)$$

6.3.2 Theoretical relations

Elder formula

$$X = 10 \frac{UD}{U_*} \qquad \ldots (10)$$

Fischer formula

$$X = K_1 \frac{UB^2}{E} \qquad \ldots (11)$$

Rimmar formula

$$X = 0{,}13B^2 \times \frac{C(0{,}7C + 2g^{1/2})}{gD} \qquad \ldots (12)$$

USSR formula

$$X = \frac{B^2_{\mathrm{m}C}}{16\alpha_y D g^{1/2} \ln (1 - \alpha_c)} \qquad \ldots (13)$$

Ward formula

$$X = K_2 \frac{B^2}{0{,}02D} \qquad \ldots (14)$$

Yotsukura formula

$$X = \frac{1}{2\alpha^2 \beta} \times \frac{UD^2}{\overline{uy}^2} \times \frac{U}{U_*} \times \frac{B^2}{D} \qquad \ldots (15)$$

6.3.3 Definition of symbols

In the formulae in 6.3.1 and 6.3.2:

X	is the mixing distance, in metres;
a_1	is a constant varying from 8 to 28; use 10 for most conditions but a higher value for very steep turbulent streams;
a_2	is a constant, equal to 150 for centre injection and 600 for side injection;
B	is the average stream surface width between injection and sampling sites, in metres;
B_m	is the distance between the point of injection and the more distant bank, in metres;
C	is the Chezy coefficient;
D	is the average stream depth between injection and sampling sites, in metres;
E	is the transverse mixing coefficient, in square metres per second;
g	is the acceleration of gravity ($g = 9{,}807$ m/s^2);
K_1	is a constant, equal to 0,1 for centre injection and 0,4 for side injection;
K_2	is a variable related to the degree of mixing, M, and the injection site (see figure 3);
Q	is the discharge, in cubic metres per second;
S	is the water-surface slope;
u	is the velocity in a stream segment, in metres per second;
U	is the mean stream velocity between the injection and sampling sites, in metres per second;
U_*	is the shear velocity [$U_* = (gDS)^{1/2}$], in metres per second;
$\frac{(UD^2)}{(\overline{uy}^2)}$	is a ratio that is usually determined as a whole and ranges in value from 0,3 to 0,9, with a value of 0,6 used for most streams;
y	is the water depth in a stream segment, in metres;

α	is a distance parameter which is a function of the degree of mixing, M, and the point of injection (see figures 4 and 5);
α_c	is a coefficient for the degree of tracer concentration variability at the site of sufficiently complete mixing, and varies from 0,15 to 0,20;
α_y	is a coefficient varying from 0,23 to 0,25;
β	is a coefficient ranging from 0,2 to 0,3 for straight channels up to 0,6 for channels with minor bends; a value of 0,2 is generally recommended for conservative estimates of the mixing length.

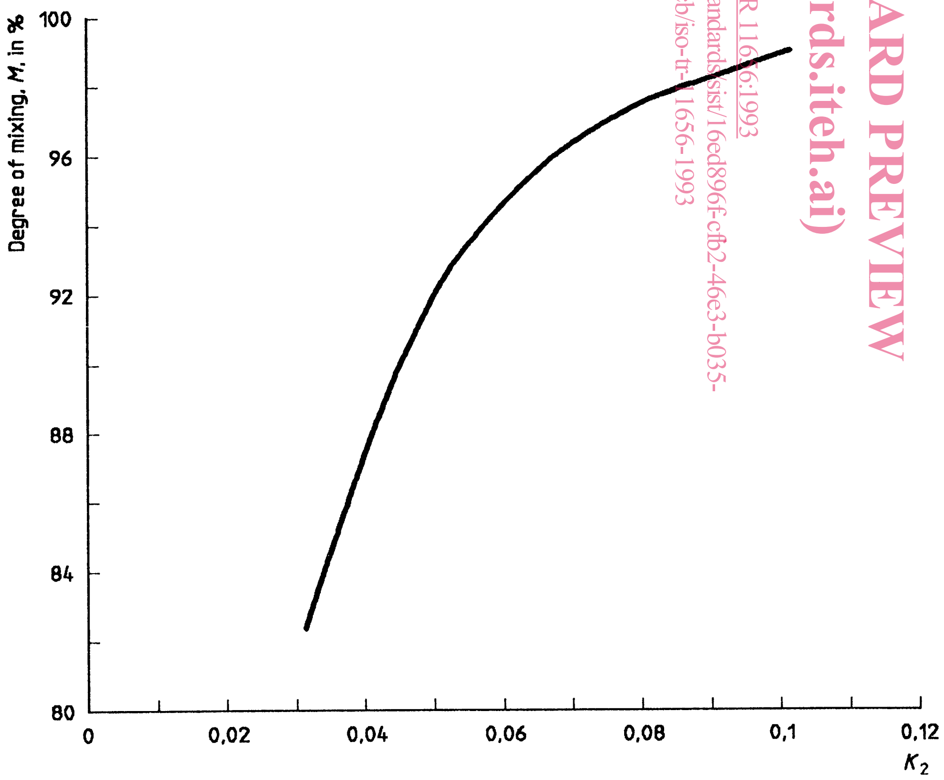


Figure 3 — Relationship between K_2 in the Ward mixing-length formula and the degree of mixing for centre injection of tracer

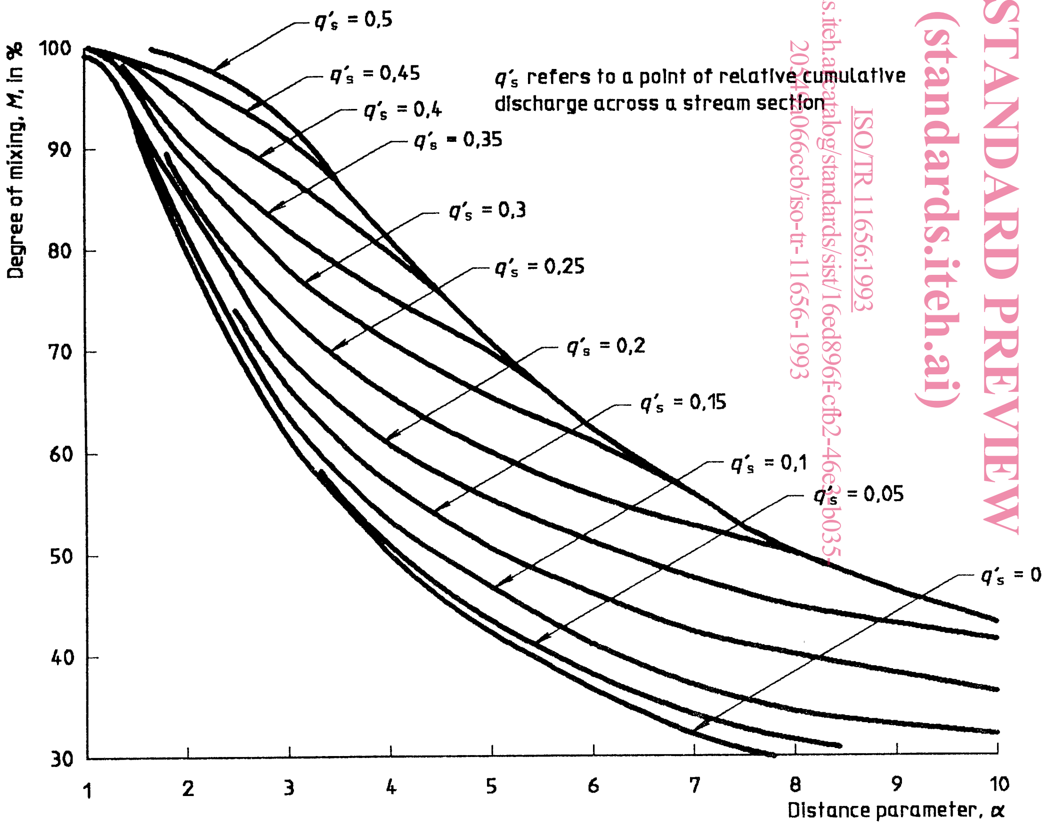


Figure 4 — Relation of the degree of mixing, M, to the distance parameter, α, and the injection site in terms of relative discharge, q'_s, for use in the Yotsukura mixing-length formula for point-source injections

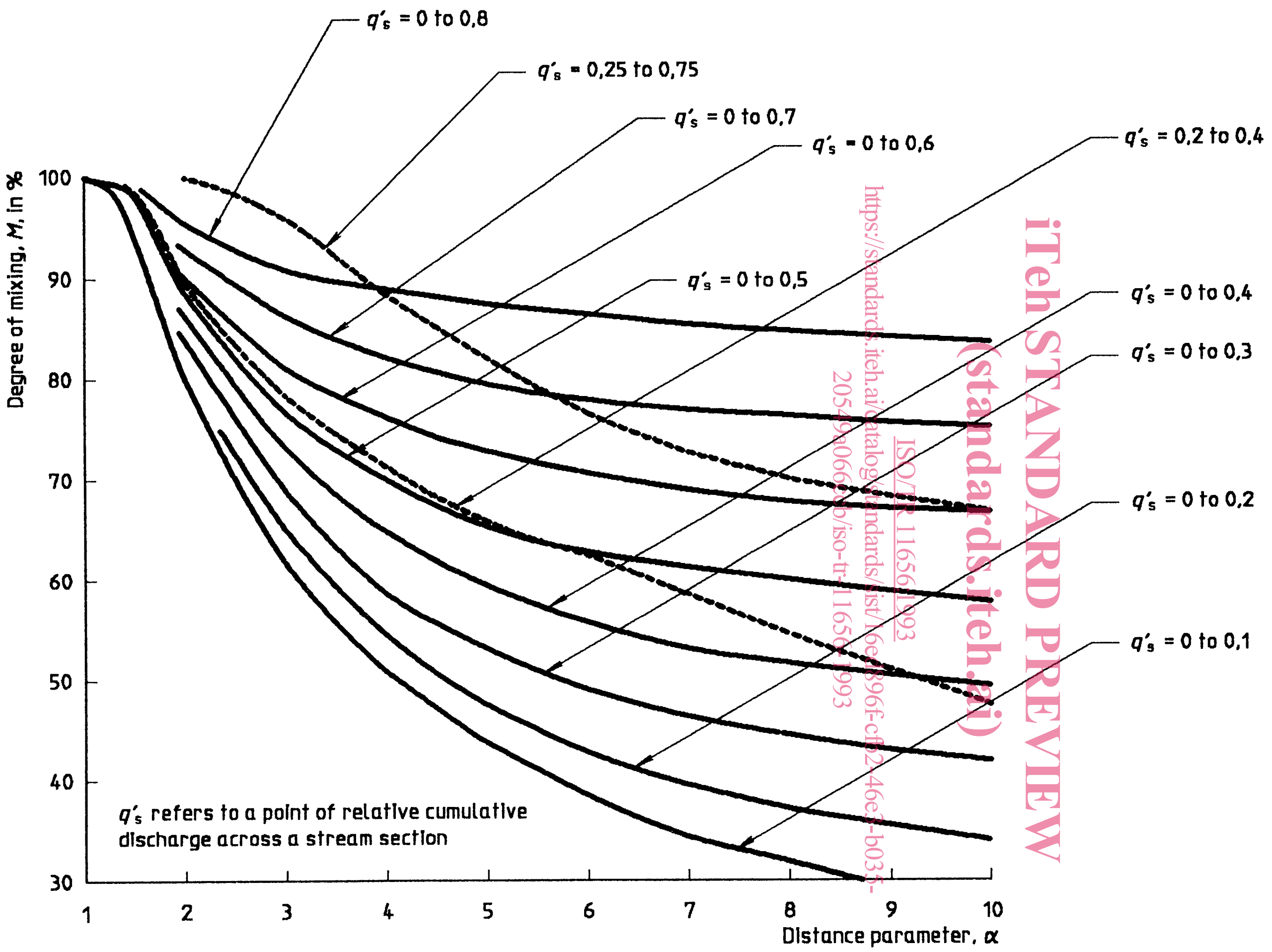


Figure 5 — Relation of the degree of mixing, M, to the distance parameter, α, and the injection site in terms of relative discharge, q'_s, for use in the Yotsukura mixing-length formula for line-source injections

6.4 Comparison of mixing-length estimates

6.4.1 Data set

Data were obtained from 22 fairly complete studies and could be used as observed data to compare with mixing-length estimates from equations (7) to (15). The data are shown in annex A. The values in parentheses are computed values and were not provided with the original data set.

When the dispersion coefficient, E, was provided but not the shear velocity, U_*, the shear velocity was determined from the following relation:

$$U_* = \frac{E}{0{,}2D} \qquad \dots (16)$$

or, conversely, E was determined from U_* using the same relation. In all cases, values for slope, S, the Manning coefficient, n, and the Chezy coefficient, C, had to be computed. Slope was determined from the following relation:

$$S = \frac{U_*^2}{gD} \qquad \dots (17)$$

The coefficient n is determined from the equation:

$$n = \frac{AR^{2/3}S^{1/2}}{Q} \qquad \dots (18)$$

where

A is the average cross-sectional area between the injection and sampling sites, in square metres ($A = DB$);

R is the average hydraulic radius in the stream reach, in metres ($R = A/WP$), where WP is the wetted perimeter estimated as $B + 2D$.

The Chezy coefficient C is determined from the equation:

$$C = \frac{1}{n} R^y \qquad \dots (19)$$