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An American National Standard

Standard Test Method for Moduli of Elasticity and Fundamental Frequencies of Carbon and Graphite Materials by Sonic Resonance[1]

This standard is issued under the fixed designation C 747; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method covers the measurement of the fundamental transverse, longitudinal, and torsional frequencies of isotropic and anisotropic carbon and graphite materials. These measured resonant frequencies are used to calculate dynamic elastic moduli for any grain orientations.

1.2 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

C 215 Test Method for Fundamental Transverse, Longitudinal, and Torsional Frequencies of Concrete Specimens[2]

C 559 Test Method for Bulk Density by Physical Measurement of Manufactured Carbon and Graphite Articles[3]

C 885 Test Method for Young's Modulus of Refractory Shapes by Sonic Resonance[3]

E 111 Test Method for Young's Modulus, Tangent Modulus, and Chord Modulus[4]

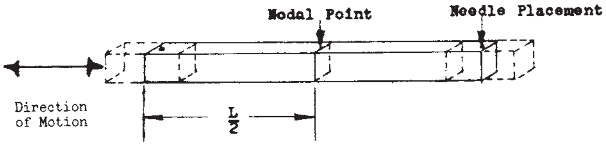
3. Terminology

3.1 *Definitions of Terms Specific to This Standard:*

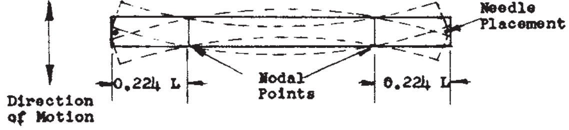
3.1.1 *elastic modulus*—the initial tangent modulus as defined in Test Method E 111.

3.1.2 *slender rod or bar*—a specimen whose ratio of length to minimum cross-sectional dimension is at least 5 but not more than 20.

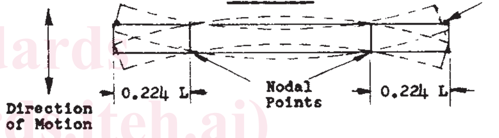
3.1.3 *longitudinal vibrations*—when the oscillations in a slender rod or bar are in a plane parallel to the length dimension, the vibrations are said to be in the longitudinal mode (Fig. 1(*a*)).



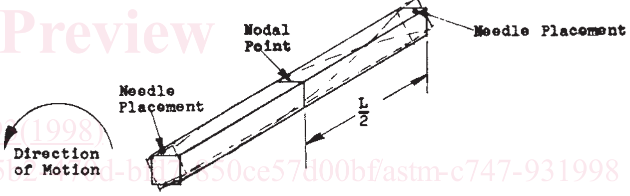
(*a*) Longitudinal Vibration Mode



(*b*) Transverse Vibration Mode



(*c*) Flexural Vibration Mode



(*d*) Torsional Vibration Mode

FIG. 1 Resonance Modes

3.1.4 *transverse vibrations*—when the oscillations in a slender rod or bar are in a horizontal plane normal to the length dimension, the vibrations are said to be in the transverse mode (Fig. 1(*b*)). This mode is also commonly referred to as the flexural mode when the oscillations are in a vertical plane (Fig. 1(*c*)). Either the transverse or flexural mode of specimen vibration will yield the correct fundamental frequency, subject to the geometric considerations given in 9.1.

3.1.5 *torsional vibrations*—when the oscillations in each cross-sectional plane of a slender rod or bar are such that the plane twists around the length dimension axis, the vibrations are said to be in the torsional mode (Fig. 1(*d*)).

3.1.6 *resonance*—a slender rod or bar driven into one of the above modes of vibration is said to be in resonance when the imposed frequency is such that resultant displacements for a

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[1] This test method is under the jurisdiction of ASTM Committee D02 on Petroleum Products and Lubricants and is the direct responsibility of Subcommittee D02.F on Manufactured Carbon and Graphite Products.

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[2] *Annual Book of ASTM Standards*, Vol 04.02.

[3] *Annual Book of ASTM Standards*, Vol 05.05.

[4] *Annual Book of ASTM Standards*, Vol 03.01.

given amount of driving force (voltage) are at a maximum. The resonant frequency is a natural vibration frequency which is determined by the elastic moduli, density, and dimensions of the test specimen.

3.1.7 *nodal points*—a slender rod or bar in resonance contains one or more points having zero displacement, called nodal points. In the longitudinal and torsional fundamental resonances of a uniform rod or bar, the mid-length point is the nodal point (Fig. 1(*a*) and Fig. 1(*d*)). For the fundamental transverse or flexural resonance, the nodal points are located at 0.224 *L* from each end, where *L* is the length of the specimen (Fig. 1(*b*) and Fig. 1(*c*)).

4. Summary of Test Method

4.1 The dynamic methods of determining the elastic moduli are based on the measurement of the fundamental resonant frequencies of a slender rod of circular or rectangular cross section. The resonant frequencies are related to the specimen dimensions and material properties as follows:

4.1.1 *Transverse or Flexural Mode*—The equation for the fundamental resonant frequency of the transverse or flexural mode of vibration is as follows:

$$E = CMf^2 \tag{1}$$

where:

E = elastic modulus, Pa,

C = a dimensional constant that depends upon the shape and size of the specimen, and Poisson's ratio. The units of C are to be consistent with those of E, M, and f,

M = mass of the specimen, kg, and

f = frequency of fundamental transverse or flexural mode of vibration, Hz.

4.1.2 *Longitudinal Mode*—The equation for the fundamental resonant frequency of the longitudinal mode of variation is as follows:

$$E = Df^{2}L^{2}\rho \tag{2}$$

where:

E = elastic modulus, Pa,

D = a constant consistent with the units of E, f, and L,

f = frequency of fundamental longitudinal mode of vibration, Hz,

L = length of the specimen, m, and

ρ = density of the specimen as determined by Test Method C 559, kg/m^3.

4.1.3 *Torsional Mode*—The equation for the fundamental resonant frequency of the torsional mode of vibration is as follows:

$$G = RBf^{2}L^{2}\rho \tag{3}$$

where:

G = modulus of rigidity, Pa,

R = ratio of the polar moment of inertia to the shape factor for torsional rigidity,

B = a constant consistent with the units of G, R, f, L, and ρ,

f = frequency of fundamental torsional mode of vibration, Hz,

L = length of the specimen, m, and

ρ = density of the specimen as determined by Test Method C 559, kg/m^3.

5. Significance and Use

5.1 This test method is primarily concerned with the room temperature determination of the dynamic moduli of elasticity and rigidity of slender rods or bars composed of homogeneously distributed carbon or graphite particles.

5.2 This test method can be adapted for other materials that are elastic in their initial stress-strain behavior, as defined in Test Method E 111.

5.3 This basic test method can be modified to determine elastic moduli behavior at temperatures from −75°C to +2500°C. Thin graphite rods may be used to project the specimen extremities into ambient temperature conditions to provide resonant frequency detection by the use of transducers as described in 6.1.

6. Apparatus

6.1 The fundamental resonant frequencies for the different modes of vibration of a test specimen can be determined by several established testing procedures. The apparatus described herein uses phonograph record pickup cartridges as a convenient method of generating and detecting these frequencies. A typical testing apparatus is shown schematically in Fig. 2.

6.1.1 *Driving Circuit*—The driving circuit consists of a variable-frequency oscillator and a record pickup cartridge assembly. It is recommended that a variable-frequency oscillator be used in conjunction with a digital-frequency counter. The oscillator shall have sufficient power output to induce detectable vibrations in the test specimen at frequencies above and below the fundamental frequency under consideration. Means for controlling the output of the oscillator shall be

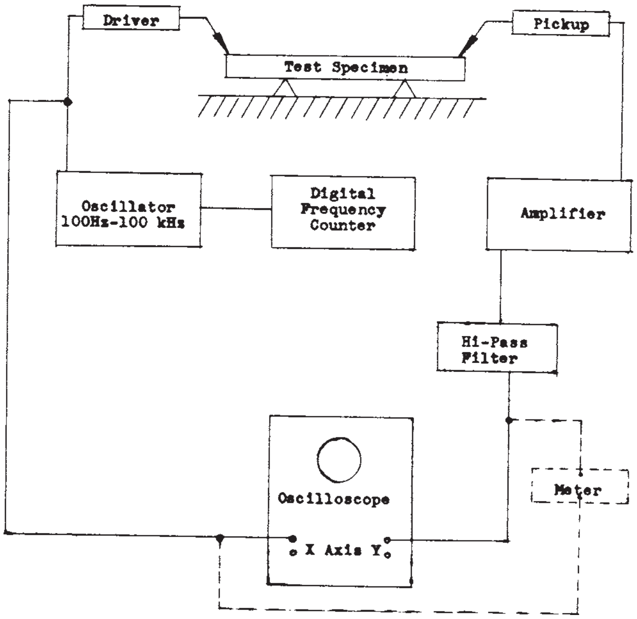


FIG. 2 Schematic Diagram of Typical Dynamic Elastic Modulus Detection Apparatus

provided. The vibrating needle of the driving unit shall be small in mass as compared to the test specimen, and a means shall be provided to maintain a minimal contact pressure on the specimen. Either a piezoelectric or magnetic driving unit meeting these requirements may be used.

6.1.2 *Pickup Circuit*—The pickup circuit consists of a record pickup cartridge, amplifier, optional high-pass filter, and an indicating meter or cathode-ray oscilloscope. The pickup unit shall generate a voltage proportional to the amplitude, velocity, or acceleration of the test specimen. Either a piezoelectric or magnetic pickup unit meeting these conditions may be used. The amplifier shall have a controllable output of sufficient magnitude to sharply peak out the resonant frequencies on the indicating meter or the cathode-ray oscilloscope display tube. It may be necessary to use a high-pass filter in order to reduce room noise and spurious vibrations. The indicating meter may be a voltmeter, microammeter or oscilloscope. An oscilloscope is recommended because it enables the operator to postively identify resonances, including higher order harmonics, by Lissajous figure analysis.

6.1.3 *Specimen Supports*—The supports shall permit the specimen to oscillate without significant restriction in the desired mode. This is accomplished for all modes by supporting the specimen at its transverse fundamental nodal points (0.224 *L* from each end). The supports should have minimal area in contact with the specimen and shall be of cork, rubber, or similar material. In order to properly identify resonant frequencies, the receiver record pickup cartridge must be movable along the total specimen length. Provisions shall be made to adjust contact pressures of both record pickup cartridges in order to accommodate specimen size variations. The entire specimen support structure shall be mounted on a massive base plate resting on vibration isolators.

7. Test Specimens

7.1 *Selection and Preparation of Specimens*—In the selection and preparation of test specimens, take special care to obtain representative specimens that are straight, uniform in cross section, and free of extraneous liquids.

7.2 *Measurement of Weight and Dimensions*—Determine the weight and the average length of the specimens within ±0.5 %. Determine average specimen cross-sectional dimension within ±1 %.

7.3 *Limitations on Dimensional Ratio*— Specimens having either very small or very large ratios of length to thickness may be difficult to excite in the fundamental modes of vibration. For this method, the ratio must be between 5 and 20 (slender rod limitations).

8. Procedure

8.1 Switch on all electrical equipment and allow to stabilize in accordance with the manufacturers' recommendations. (Use of a metal bar as a calibration standard is recommended to check equipment response and accuracy. Dimensional measurements and weight shall meet the requirements of 7.2.)

8.2 *Transverse Fundamental Resonance Frequency*:

8.2.1 Place the specimen on the supports, which are located at the fundamental transverse nodal points (0.224 *L* from each end). Place the driving and pickup-unit vibrating needles on the specimen center line at its extreme opposite ends with a minimal contact pressure consistent with good response. The vibrating direction of the driving and pickup needles must be perpendicular to the length of the specimen (Fig. 1(*b*)).

8.2.2 Force the test specimen to vibrate at various frequencies and simultaneously observe the amplified output on an indicating meter or oscilloscope. Record the frequency of vibration of the specimen that results in a maximum displacement, having a well-defined peak on the indicator, where nodal point tracking indicates fundamental transverse resonance.

8.2.3 A basic understanding of Lissajous patterns as displayed on an oscilloscope cathode ray tube (CRT), will aid in the proper identification of the modes of vibration and harmonic frequencies observed. As the oscillator frequency level is increased from a point well below expected resonance, a single closed loop Lissajous pattern tilted from the horizontal reference plane, will eventually be displayed on the CRT. This pattern denotes a resonance mode. The nodal points dynamic modulus tracking guide template (Fig. 3) may be used to identify any resonant mode.

8.2.4 Move the pickup cartridge needle slowly toward the specimen center and observe the Lissajous pattern loop. Fundamental transverse resonance is indicated when the following conditions prevail:

8.2.4.1 The loop pattern flattens to a horizontal line with the pickup needle over the specimen support.

8.2.4.2 The CRT pattern opens up to a full loop in a direction normal to its original direction, with the pickup needle over the specimen center.

8.2.5 Return the pickup needle to its original position at the specimen end.

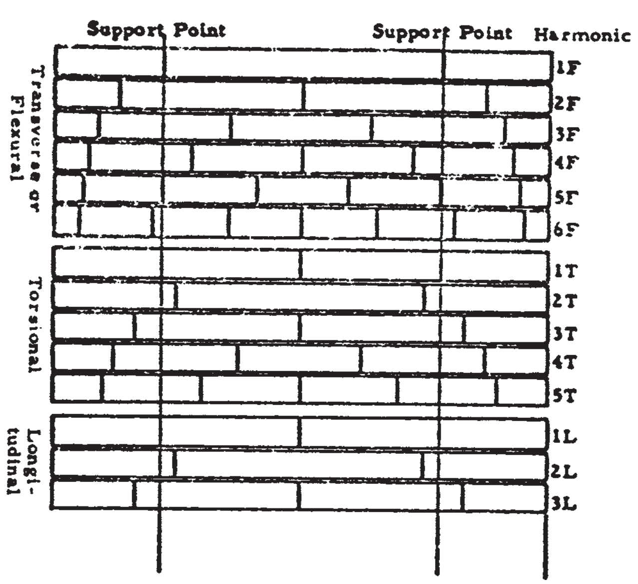
8.2.6 Spurious resonating frequency modes may mask or attenuate the fundamental transverse frequency indication. Investigation of higher order harmonic resonating frequencies by use of the tracking guide template (Fig. 3) will help to identify the correct fundamental frequency mode. A plot of the ratio of harmonic to fundamental frequency for transverse mode of vibration (Fig. 4) may then be used to calculate the fundamental transverse resonant frequency mode.

8.3 *Longitudinal Fundamental Resonance Frequency*:

8.3.1 Leave the specimen supported at the fundamental transverse mode nodal points as in 8.2.1. Rotate the driving unit and pickup cartridge needles so as to induce vibrations parallel to the specimen length (Fig. 1(*a*)).

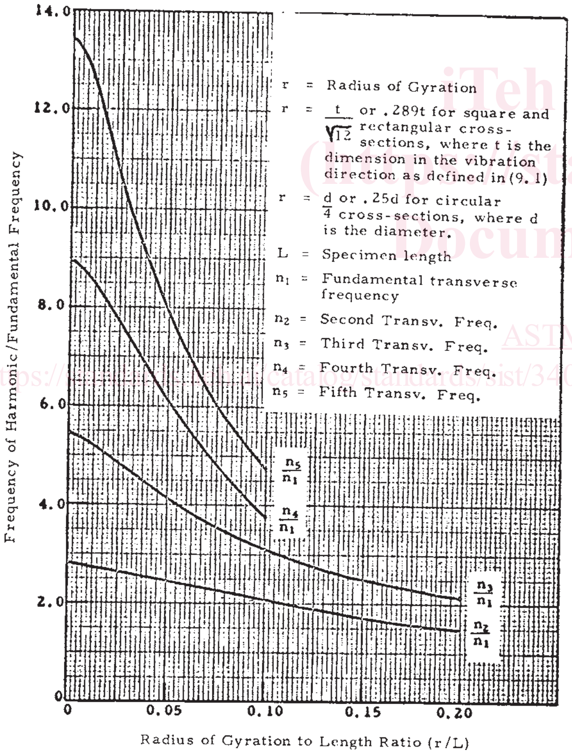
8.3.2 Force the test specimen to vibrate as in 8.2.2. Record the frequency of vibration of the test specimen, where nodal point tracking indicates fundamental longitudinal resonance. The second harmonic longitudinal resonant frequency is twice the fundamental longitudinal resonant frequency.

8.4 *Torsional Fundamental Resonance Frequency*:



Harmonic	Distance from end for each nodal point in fraction of total length
1F	0.224
2F	0.132, 0.500
3F	0.0944, 0.3558
4F	0.0733, 0.277, 0.500
5F	0.0600, 0.266, 0.409
6F	0.0508, 0.192, 0.346, 0.500
1T	0.500
2T	0.250
3T	0.167, 0.500
4T	0.125, 0.375
5T	0.100, 0.300, 0.500
1L	0.500
2L	0.250
3L	0.167, 0.500

FIG. 3 Nodal Points Dynamic Modulus Tracking Guide Template



NOTE—Taken from Pickett, Gerald, "Equations for Computing Elastic Constants from Flexural and Torsional Resonant Frequencies of Vibration of Prisms and Cylinders,'' *Proceedings*, ASTM, Vol 45, 1945.

FIG. 4 Ratio of Harmonic to Fundamental Frequency for Transverse Mode of Vibration

8.4.1 Leave the specimen supported as in 8.2.1. Rotate the driving unit and pickup cartridge needles so as to induce vibrations perpendicular to the length of the sample (Fig. 1 (*d*)).

8.4.2 Force the specimen to vibrate as in 8.2.2. Record the frequency of vibration of the test specimen, where nodal point tracking indicates fundamental torsional resonance. The second harmonic torsional resonant frequency is twice the fundamental torsional resonant frequency.

9. Calculation

9.1 Calculate the dynamic modulus of elasticity for the transverse or flexural mode of vibration from the fundamental transverse frequency, weight, and dimensions of the test specimen as follows:

$$\text{Dynamic } E = CMf^{2} \qquad (4)$$

where units are as defined in 3.1.1. The evaluation of the constant *C*, because of the complexity of its determination, is in tabular form. Eq 4 may be rewritten in the forms:

$$\text{Dynamic } E\ (\text{pascals}) = A_c M f^{2}/d \text{ for rods with circular cross sections} \qquad (5)$$

where *d* is the diameter of the rod in metres, and

$$\text{Dynamic } E\ (\text{pascals}) = A_R M f^{2}/w \text{ for bars with square or rectangular cross sections} \qquad (6)$$

where *w* is the width dimension of the bar in metres.

9.1.1 Values of A_c and A_R are shown in Annex A1 under Table A1.1 and Table A1.2. The value of A_c is given as a function of the diameter-to-length ratio of the sample. The value of A_R is given as a function of the ratio of the dimension in the direction of vibration, *t*, to the length. The dimension, *w*, is perpendicular to the vibration direction, as shown in Fig. 5. Table A1.1 and Table A1.2 have been calculated for three values of Poisson's ratio (μ). The value of (μ) = ⅙ is normally used for carbon-graphite materials.

9.2 The dynamic modulus of elasticity in pascals may also be calculated from the fundamental longitudinal frequency, weight, and dimensions of the test specimen as follows:

$$\text{Dynamic } E = 4.000\ f^{2} L^{2} \rho \text{ for rods and bars} \qquad (7)$$