

TECHNICAL REPORT



**Fibre optic communication system design guides –
Part 9: Guidance on polarization mode dispersion measurements and theory**

IEC TR 61282-9:2016

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FIBRE OPTIC COMMUNICATION SYSTEM DESIGN GUIDES –

Part 9: Guidance on polarization mode dispersion
measurements and theory

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IEC TR 61282-9, which is a Technical Report, has been prepared by subcommittee 86C: Fibre optic systems and active devices, of IEC technical committee 86: Fibre optics.

This second edition cancels and replaces the first edition published in 2006.

This second edition includes the following significant technical changes with respect to the previous edition:

- a) much of the theory has been condensed – focusing only on content that is needed to explain the test method;
- b) symbols have been removed, but abbreviations are retained;

- c) the material in the Clause 5 has been significantly reduced in an effort to avoid repeating what is already in the actual International Standards. Instead, the focus is on explaining the International Standards;
- d) measurement methods that are not found in International Standards have been removed;
- e) there are significant corrections to the modulation phase shift method, particularly in regard to the Mueller set technique;
- f) there are significant corrections to the polarization phase shift method;
- g) the proof of the GINTY interferometric method is presented. This proof also extends to the Fixed Analyser Cosine transfer technique;
- h) another Fixed Analyser method is suggested. This is based on the proof of the GINTY method and is called "spectral differentiation method";
- i) Clause 6 has been renamed "Limitations" and refocused on the limitations of the test methods. This Technical Report is not intended to be an engineering manual;
- j) the annexes have been removed;
- k) the bibliography has been much reduced in size;
- l) the introduction has been expanded to include some information on system impairments.

The text of this Technical Report is based on the following documents:

Enquiry draft	Report on voting
86C/1342/DTR	86C/1366/RVC

Full information on the voting for the approval of this Technical Report can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all parts in the IEC 61282 series, published under the general title *Fibre optic communication system design guides*, can be found on the IEC website.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC website under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

A bilingual version of this publication may be issued at a later date.

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INTRODUCTION

This Technical Report is complementary to the International Standards describing PMD procedures (IEC 60793-1-48, IEC 61280-4-4, IEC 61290-11-1, IEC 61290-11-2 and IEC 61300-3-32) and other design guides on PMD (IEC 61282-3 and IEC 61292-5), as well as ITU-T Recommendation G.650.2.

The system power penalty associated with PMD varies depending on transmission format and bit rate. It also varies with optical frequency and state of polarization (SOP) of the light source. At the output of a link, the signal can shift from a maximum delay to a minimum delay as a result of using different SOPs at the source. The difference in these delays is called the differential group delay (DGD), which is associated with two extremes of input SOP. At these extremes, a signal in the form of a single pulse appears shifted up or down by half the DGD, about a midpoint, at the output. At intermediate SOPs, the single pulse appears as a weighted total of two pulses at the output, one shifted up by half the DGD and one shifted down by half the DGD. This weighted total of two shifted pulses is what causes signal distortion.

The system power penalty is partly defined in terms of a maximum allowed bit error rate and a minimum received power. In the absence of distortion, there is a minimum received power that will produce the maximum allowed bit error rate. In the presence of distortion, the received power should be increased to produce the maximum bit error rate. The magnitude of the required increase of received power is the power penalty of the distortion.

The term PMD is used to describe two distinctly different ideas.

One idea is associated with the signal distortion induced by transmission media for which the output SOP varies with optical frequency. This is the fundamental source of signal distortion.

The other idea is that of a number (value) associated with the measurement of a single-mode fibre transmission link or element of that link. There are several measurement methods with different strengths and capabilities. They are all based on quantifying the magnitude of possible variation in output SOP with optical frequency. The objective of this Technical Report is to explain the commonality of the different methods.

The DGD at the source's optical frequency is what controls the maximum penalty across all possible SOPs. However, in most links, the DGD varies randomly across optical frequency and time. The PMD value associated with measurements, and which is specified, is a statistical metric that describes the DGD distribution. There are two main metrics, linear average and root-mean square (RMS), that exist in the literature and in the measurement methods. For most situations, one metric can be calculated from the other using a conversion formula. The reason for the dual metrics is an accident of history. If history could be corrected, the RMS definition would be the most suitable.

For the non-return to zero transmission format, DGD equal to 0,3 of the bit period yields approximately 1 dB maximum penalty. Because DGD varies randomly, a rule of thumb emerged in the system standardization groups: keep PMD less than 0,1 of the bit period for less than 1 dB penalty. This assumes that DGD larger than three times the PMD, and that the source output SOP produces the worst case distortion, is not very likely. For 10 Gbit/s non-return to zero, this rule yields a design rule: keep the link PMD less than 10 ps. ITU-T G.sup.39 [1]¹ has more information on the relationship of PMD and system penalties.

¹ Numbers in square brackets refer to the Bibliography.

FIBRE OPTIC COMMUNICATION SYSTEM DESIGN GUIDES –

Part 9: Guidance on polarization mode dispersion measurements and theory

1 Scope

This part of IEC 61282, which is a Technical Report, describes effects and theory of polarization mode dispersion (PMD) and provides guidance on PMD measurements.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60793-1-48, *Optical fibres – Part 1-48: Measurement methods and test procedures – Polarization mode dispersion*

IEC 61280-4-4, *Fibre optic communication subsystem test procedures – Part 4-4: Cable plants and links – Polarization mode dispersion measurement for installed links*

IEC 61290-11-1, *Optical amplifier – Test methods – Part 11-1: Polarization mode dispersion parameter – Jones matrix eigenanalysis (JME)*

IEC 61290-11-2, *Optical amplifier – Test methods – Part 11-1: Polarization mode dispersion parameter – Poincaré sphere analysis method*

IEC 61300-3-32, *Fibre optic interconnecting devices and passive components – Basic tests and measurement procedures – Part 3-32: Examinations and measurements – Polarization mode dispersion measurement for passive optical components*

3 Terms, definitions, and abbreviations

3.1 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1.1

PMD phenomenon

polarization mode dispersion phenomenon

signal of fibre-optic transmission signal induced by variation in the signal output state of polarization with optical frequency

Note 1 to entry: PMD can limit the bit rate-length product of digital systems.

3.1.2

PMD value

polarization mode dispersion value

magnitude of polarization mode dispersion phenomenon associated with a single-mode fibre, optical component and sub-system, or installed link

Note 1 to entry: The polarization mode dispersion value is usually expressed in ps.

3.2 Abbreviations

CFT	cosine Fourier transform
DGD	differential group delay
DOP	degree of polarization
EC	extrema counting
FA	fixed analyser
FT	Fourier transform
GINTY	general interferometric method
JME	Jones matrix eigenanalysis
MPS	modulation phase shift
OTDR	optical time domain reflectometer
PDL	polarization dependent loss
PDV	polarization dispersion vector
PMD	polarization mode dispersion
PPS	polarization phase shift
PSA	Poincaré sphere analysis
PSP	principal state of polarization
SMF	single-mode fibre
SOP	state of polarization
SPE	Stokes parameter evaluation
TINTY	traditional interferometric method
WSOSA	wavelength scanning OTDR and SOP analysis method

4 Theoretical framework

4.1 Limitations and outline

The theory presented in Clause 4 does not include the effects of polarization dependent loss or gain, or nonlinear effects. See 6.3 for information on polarization dependent loss.

The outline for Clause 4 is

- optical field and state of polarization;
- measurement of SOP, Stokes vector, and rotation;
- first order polarization mode dispersion;
- birefringence vector, concatenations, and mode coupling;
- the statistics of PMD and second order PMD.

4.2 Optical field and state of polarization

This subclause is intended to show the linkage between the propagation of an optical field in a single-mode fibre (SMF) and the transmission signal state of polarization (SOP). This information is fundamental to the PMD phenomena because variation in output SOP with optical frequency is the distortion inducing mechanism.

The solution of the wave equation has degenerated eigenvalues. This means that even the fundamental solution is degenerated. A SMF supports a pair of polarization modes for a monochromatic light source. In particular, the lowest order mode, namely the fundamental

mode HE_{11} (LP_{01}) can be defined to have its transverse electric field predominately along the x -direction; the orthogonal polarization is an independent mode, as shown in Figure 1.

In a lossless SMF, the electric field vector of a monochromatic electromagnetic wave propagating along the z -direction can be described by a linear superposition of these two modes in the x - y transverse plane as shown in Equation (1) and in Figure 1.

$$E = \left[j_x \exp(i\beta_x z) \right] + \left[j_y \exp(i\beta_y z) \right] \exp(-i\omega t)$$

$$= \left[j_x \exp(-\Delta\beta z / 2) + j_y \exp(i\Delta\beta z / 2) \right] \exp\left[-i\left(\omega t - \bar{\beta} z\right)\right] \quad (1)$$

where

j_x and j_y	are complex coefficients describing the amplitude and phase of the x/y initial SOPs;
$E_x(x,y)$ and $E_y(x,y)$	are the spatial variation (in the x - y transverse plane) of the E vector of the PM along the x/y -direction (see Figure 1);
β_x and β_y	are the propagation constants (also called effective index or wavenumber) of the PM along the x/y -directions with the index of refraction n_x/n_y . Using $i = x$ or y , $\beta_i = k n_i$. The index of refraction has a dependence on frequency ω , frequency ν , or wavelength λ ;
$\Delta\beta$	is the difference of β_x and β_y ;
$\bar{\beta}$	is the average of β_x and β_y ;
k	is the propagation constant with the wavelength λ in vacuum ($= 2\pi\nu/c = 2\pi/\lambda$);
ν	is the frequency in s^{-1} or Hz;
ω	is the angular frequency in rad/s (the bar indicates absolute frequency rather than deviation from some particular value);
c	is the speed of light in vacuum (299792458 m/s);
Δ'	is the birefringence coefficient (s/m), $= \Delta n/c$
z	is the distance (m) in the DUT along the optical axis (axis of propagation); $z = L$ at the output of the DUT with length L .

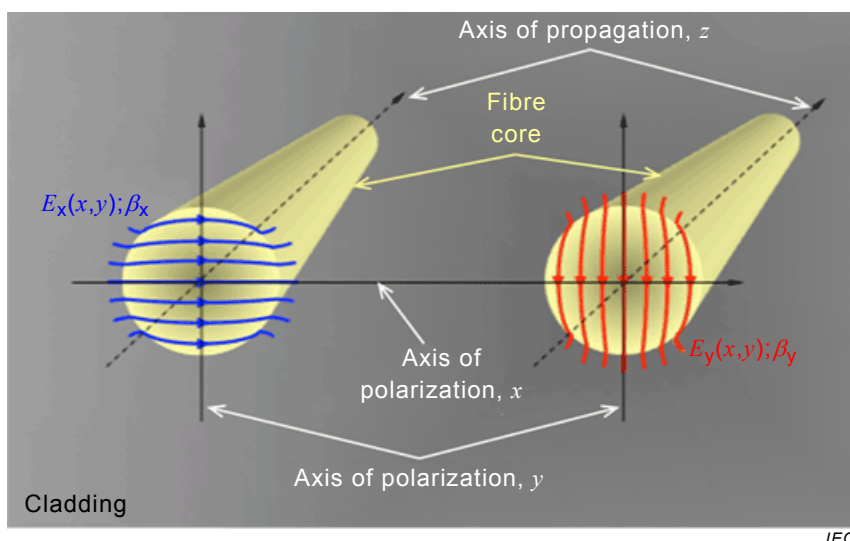


Figure 1 – Two electric field vector polarizations of the HE₁₁ mode in a SMF

The complex pair, $[j_x \exp(-i\Delta\beta z/2), j_y \exp(i\Delta\beta z/2)]$, describes the SOP defined in the x - y plane of the wave propagating along the z -direction. This pair can be considered as a vector, and is often called a Jones vector.

In the case where the transmission media is an ideal SMF with perfect circular symmetry, $\beta_y = \beta_x$,

- the two polarization modes are degenerate (when two solutions have the same eigenvalue, they are said to be degenerate);
- any wave with a defined input SOP will propagate unchanged along the z -direction throughout the output of the SMF.

However, in a practical SMF, the circular symmetry is broken by imperfections produced by the fabrication process, cabling, field installation/use or the installation environment:

- $\beta_y \neq \beta_x$, implying a phase difference, an index-of-refraction difference Δn , and a phase-velocity difference between the two PMs;
- the degeneracy of the two polarization modes is lifted;
- the SOP of an input wave will change along the z -direction throughout the output of the SMF.

The difference between β_y and β_x , namely $\Delta\beta$, is called the phase birefringence or simply the birefringence and has units of inverse length (m^{-1}). Birefringence may also be referred to as the index difference, Δn , or as the birefringence coefficient, the difference divided by length. Birefringence coefficient values typically vary between 0,25 fs/m and 2,5 fs/m in commonly available SMFs.

As the SOPs travel through the fibre, they will return to the initial state at positions that are increments of $2\pi/\Delta\beta$. At these positions, the two field components will beat. The difference between these positions is called the beat length.

Birefringence can be induced by a number of factors such as core non-circularity or asymmetric stresses that can be induced by bends, twist, and compression. These factors change over the length of the fibre and can change over time due to changes in configuration, or temperature. These factors will also vary with optical frequency. Equation (1) is also defined with an arbitrary coordinate system that will not generally correspond to the laboratory or field test equipment coordinate system. The mapping of the input SOP to the output SOP

over a particular length and optical frequency, L and ω_0 , is represented with the Jones matrix, T , and the input and output Jones vectors, \vec{j}_{IN} and \vec{j}_0 , as:

$$\vec{j}_0 = T\vec{j}_{IN} \quad (2)$$

where

$$T = V_T S_T V_T^\dagger \quad (3)$$

$$S_T = \begin{bmatrix} \exp(-i\xi_T/2) & 0 \\ 0 & \exp(i\xi_T/2) \end{bmatrix} \quad (4)$$

$$V_T = \begin{bmatrix} \cos\theta_T \exp(-i\mu_T/2) & -\sin\theta_T \exp(-i\mu_T/2) \\ \sin\theta_T \exp(i\mu_T/2) & \cos\theta_T \exp(i\mu_T/2) \end{bmatrix} = S(i\mu_T/2)R(\theta_T) \quad (5)$$

NOTE 1 The subscript T is used to distinguish the matrix parameters from similar parameters used later.

The main operation of Equation (3), corresponding to Equation (1), is found in the matrix, S_T . Pre and post-multiplying by V_T and V_T^\dagger is a change of coordinates. The notation V^\dagger indicates the transpose conjugate for matrices and vectors.

The diagonal expressions in Equation (4) are intended to show the connection to Equation (1). Equation (1), however, is only applicable locally, while Equation (3) is used to indicate change over the entire transmission media. There is another expression that uses $\gamma_T = \xi_T/2$ that is found in ITU-T Recommendation G.650.2. On the Poincaré sphere, the action of Equation (4) is a rotation of γ_T from an input SOP to an output SOP. When equations are found in both documents, the γ_T notation will be used.

Unit Jones vectors, which are one way to represent SOPs, can be specified with a (θ, μ) pair as:

$$\vec{j} = \begin{bmatrix} \cos\theta \exp(-i\mu/2) \\ \sin\theta \exp(i\mu/2) \end{bmatrix} \quad (6)$$

The Jones matrix, T , is unitary in that $T^\dagger T = I$, the identity matrix. The columns of V_T are the eigenvectors, and the diagonal elements of S_T are the eigenvalues. When the input Jones vector is equal to either of the eigenvectors, the output SOP is the same as the input SOP because the SOP is not affected by a multiplication by a constant. These states are sometimes called the eigenstates.

NOTE 2 All the parameters of T can change with optical frequency, as well as with changes caused by fibre movement over time or temperature change.

4.3 SOP measurements, Stokes vectors, and Poincaré sphere rotations

The SOP is usually measured with a polarimeter, which yields a Stokes vector. The measurement is actually done with a series of power measurement differences through various states of a polarizer/analyser.

An ideal polarizer may be defined as:

$$Pol(\theta_P, \mu_P) = V_{Pol} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} V_{Pol}^\dagger = \begin{bmatrix} \cos^2 \theta_P & \frac{1}{2} \sin 2\theta_P \exp(-i\mu_P) \\ \frac{1}{2} \sin 2\theta_P \exp(i\mu_P) & \sin^2 \theta_P \end{bmatrix} \quad (7)$$

where V_{Pol} is of the same form as Equation (5), but with different parameters.

For a given Jones vector, the power through the polarizer is:

$$P(\theta_P, \mu_P) = \vec{j}^\dagger Pol^\dagger(\theta_P, \mu_P) Pol(\theta_P, \mu_P) \vec{j} \quad (8)$$

The measured Stokes vector is given as:

$$\vec{S} = \begin{bmatrix} P(0,0) + P(\pi/2,0) \\ P(0,0) - P(\pi/2,0) \\ P(\pi/4,0) - P(-\pi/4,0) \\ P(\pi/4,\pi/2) - P(\pi/4,-\pi/2) \end{bmatrix} \text{ indexed with } j = 0, 1, 2, 3 \quad (9)$$

The normalized Stokes vector is designated with a small \vec{s} and has three elements indexed: 1, 2, and 3. The normalized Stokes vector elements are the measured Stokes vector element values with the same index, divided by S_0 . The normalized Stokes vector has a length of one.

SOPs where s_3 is zero are linear states. When s_3 is nonzero but with absolute value less than one, the polarization is elliptical. When s_3 is ± 1 , the polarization is circular.

NOTE 1 In the rest of this Technical Report, the normalized Stokes vector will be referred to simply as the Stokes vector.

The degree of polarization (DOP) is normally expressed as a per cent and is equal to $100 \cdot S_0 / P$, where P is the power without the polarizer.

A unit Jones vector can be represented either as an x/y pair or as Equation (6). The relationship of the normalized Stokes vector to the Jones vector is given as:

$$\vec{s} = \begin{bmatrix} xx^* - yy^* \\ xy^* + yx^* \\ i(xy^* - yx^*) \end{bmatrix} = \begin{bmatrix} \cos 2\theta \\ \sin 2\theta \cos \mu \\ \sin 2\theta \sin \mu \end{bmatrix} \quad (10)$$

There is an ambiguity in trying to calculate the Jones vector from the Stokes vector. One must assume something like $0 < \theta < \pi$. This is due to the fact that the Stokes vector is not affected by multiplying the Jones vector by any unit complex number (a number, c , for which $cc^* = 1$), including ± 1 . This can be called a one π ambiguity. This property is one reason to think of the Stokes vector as the primary definition of the SOP: the SOP is not changed when either of the eigenstates are used as inputs, but the output Jones vector is multiplied by $\exp(\pm i\pi/2)$.

Unit three term vectors can be represented on a sphere. In the case of Stokes vectors, the sphere is called the Poincaré sphere.

Examination of the rightmost expression of Equation (10) and the different parts of Equations (3), (4) and (5) shows that the action of T is consistent with the following right-hand-rule rotations applied to the input Stokes vector that corresponds to the input Jones vector:

- a) anti-rotation of μ_T about the (1,0,0) vector, which describes a linear SOP;
- b) anti-rotation of $2\theta_T$ about the (0,0,1) vector, which describes a circular SOP;
- c) rotation of ξ_T ($= 2\gamma_T$) about the (1,0,0) vector;
- d) rotation of $2\theta_T$ about the (0,0,1) vector;
- e) rotation of μ_T about the (1,0,0) vector.

These steps can be combined into a single rotation, from input Stokes vector to output Stokes vector, as:

$$\vec{s}_0 = R_T \vec{s}_{IN} \quad (11)$$

where

$$R_T = \vec{y}\vec{y}^T (1 - \cos \xi_T) + I \cos \xi_T + [\vec{y} \times] \sin \xi_T \quad (12)$$

where \vec{y}^T is the transpose of \vec{y} , I is the identity matrix, and $[\vec{y} \times]$ is the cross product operator,

$$\begin{bmatrix} 0 & -y_3 & y_2 \\ y_3 & 0 & -y_1 \\ -y_2 & y_1 & 0 \end{bmatrix}.$$

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This is a rotation of ξ_T about the rotation vector, \vec{y} . The rotation vector is found by converting the first column of V_T to a Stokes vector. Figure 2 illustrates a rotation on the Poincaré sphere. This is a 2π rotation about the (1,0,0) axis.

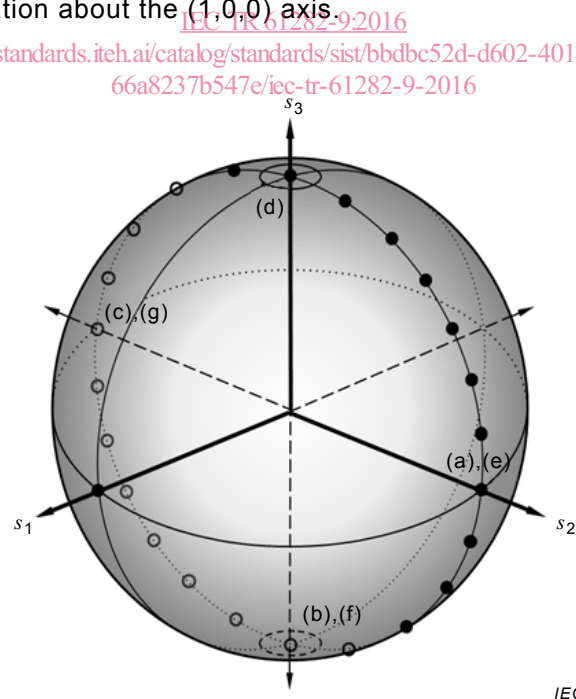


Figure 2 – A rotation on the Poincaré sphere

The T matrix can now be written in a simplified form:

$$T = \begin{bmatrix} \cos \gamma_T - iy_1 \sin \gamma_T & -(y_3 + iy_2) \sin \gamma_T \\ (y_3 - iy_2) \sin \gamma_T & \cos \gamma_T + iy_1 \sin \gamma_T \end{bmatrix}$$