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Calculation of load capacity of bevel gears —

Part 3:
Calculation of tooth root strength

Calcul de la capacité de charge des engrenages coniques —

Partie 3: Calcul de la résistance du pied de dent

Reference number
ISO 10300-3:2001(E)

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this part of ISO 10300 may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

International Standard ISO 10300-3 was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 2, *Gear capacity calculation*.

ISO 10300 consists of the following parts, under the general title *Calculation of load capacity of bevel gears*:

— *Part 1: Introduction and general influence factors*

— *Part 2: Calculation of surface durability (pitting)*

— *Part 3: Calculation of tooth root strength*

Annex A forms an integral part of this part of ISO 10300. Annex B is for information only.

In this corrected version of ISO 10300-3, Equation (57) has been corrected.

Introduction

Parts 1, 2 and 3 of ISO 10300, taken together with ISO 6336-5, are intended to establish general principles and procedures for the calculation of the load capacity of bevel gears. Moreover, ISO 10300 has been designed to facilitate the application of future knowledge and developments, as well as the exchange of information gained from experience. This part of ISO 10300 gives formulae for bending-strength rating in calculations for the avoidance of tooth breakage.

Failure of gear teeth by breakage can be brought about in many ways — severe instantaneous overloads, excessive pitting, case crushing and bending fatigue are some. The strength ratings determined by the formulae in this part of ISO 10300 are based on cantilever-projection theory modified to consider the following:

— compressive stress at the tooth roots caused by the radial component of the tooth load;

— non-uniform moment distribution of the load, resulting from the inclined contact lines on the teeth of spiral bevel gears;

— stress concentration at the tooth root fillet;

— load-sharing between adjacent contacting teeth;

— lack of smoothness due to a low contact ratio.

The formulae can be used for determining a load rating that will prevent tooth root fillet fracture during the design life of the gear teeth. Nevertheless, if there is insufficient material under the teeth (in the rim), a fracture can occur from the root through the rim of the gear blank or to the bore — a type of failure not covered by this part of ISO 10300. Moreover, special applications could require additional blank material to support the load.

Occasionally, surface distress (pitting or wear) may limit the strength rating, due either to stress concentration around large sharp-cornered pits, or to wear steps on the tooth surface. Neither of these effects are considered in this part of ISO 10300.

In most cases, the maximum tensile stress at the tooth root (arising from bending at the root when the load is applied to the tooth flank) can be used as the criterion for the assessment of the bending tooth root strength, as when the allowable stress number is exceeded the teeth can experience breakage. When calculating the tooth root stresses of straight bevel gears, this part of ISO 10300 starts from the assumption that the load is applied at the tooth tip of the virtual cylindrical gear. The load is subsequently converted to the outer point of single-tooth contact with the aid of the contact-ratio factor Y_ε (see clause 8). The procedure thus corresponds to method C for the tooth root stress of cylindrical gears (see ISO 6336-3).

For spiral bevel gears with a high overlap ratio ($\varepsilon_{v\beta} > 1$), the mid point in the contact zone is regarded as the critical point of load application. There is an interpolation for medium overlap ratio ($0 < \varepsilon_{v\beta} < 1$).

The breakage of a tooth generally means the end of a gear's life. It is often the case that all gear teeth are destroyed as a consequence of the breakage of a single tooth. An S_F, the safety factor against tooth breakage, higher than the safety factor against damage due to pitting, is therefore generally to be preferred (see ISO 10300-1).

Calculation of load capacity of bevel gears —

Part 3: Calculation of tooth root strength

1 Scope

This part of ISO 10300 specifies the fundamental formulae for use in the tooth-bending stress calculation of straight and helical (skew), zerol- and spiral-bevel gears with a minimum rim thickness under the root $\geqslant 3{,}5\ m_{mn}$. All load influences on tooth stress are included, insofar as they are the result of load transmitted by the gearing and able to be evaluated quantitatively. (Stresses such as those caused by the shrink-fitting of gear rims, which are superposed on stresses due to tooth loading, are to be taken into consideration in the calculation of the tooth root stress σ_F or the permissible tooth root stress σ_{FP}.)

The formulae in this part of ISO 10300 are valid for bevel gears with teeth with a transverse contact ratio of $\varepsilon_{v\alpha} < 2$, while the results are valid within the range of the applied factors given in ISO 10300-1 and ISO 6336-3.

This part of ISO 10300 does not apply to stress levels above those permitted for 10^3 cycles, as stresses in that range could exceed the elastic limit of the gear tooth.

CAUTION — The user is cautioned that when the methods are used for large spiral and pressure angles, and for large face width $b > 10\ m_{mn}$, the calculated results of ISO 10300 should be confirmed by experience.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this part of ISO 10300. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this part of ISO 10300 are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 53:1998, *Cylindrical gears for general and heavy engineering — Standard basic rack tooth profile*.

ISO 1122-1:1998, *Vocabulary of gear terms — Part 1: Definitions related to geometry*.

ISO 6336-3, *Calculation of load capacity of spur and helical gears — Part 3: Calculation of tooth bending strength*.

ISO 6336-5, *Calculation of load capacity of spur and helical gears — Part 5: Strength and quality of materials*.

ISO 10300-1:2001, *Calculation of load capacity of bevel gears — Part 1: Introduction and general influence factors*.

ISO 10300-2:2001, *Calculation of load capacity of bevel gears — Part 2: Calculation of surface durability (pitting)*.

3 Terms and definitions

For the purposes of this part of ISO 10300, the geometrical gear terms given in ISO 53 and ISO 1122-1, and the following term and definition, apply.

3.1
tooth bending strength
load capacity determined on the basis of the permissible bending stress

4 Symbols and abbreviated terms

For the purposes of this part of ISO 10300, the symbols and abbreviated terms given in Table 1 of ISO 10300-1:2001, and the following abbreviated terms, apply.

Table 1 — Abbreviated terms

Abbreviation	Description
St	steel (σ_B<800 N/mm^2)
V	through-hardened steel ($\sigma_B \geqslant$ 800 N/mm^2)
GG	grey cast iron
GGG (perl., bai., ferr.)	spheroidal cast iron (perlitic, bainitic, ferritic structure)
GTS (perl.)	black malleable cast iron (perlitic structure)
Eh	case-hardening steel, case-hardened
IF (root)	steel and GGG, flame or induction-hardened (including root fillet)
NT (nitr.)	nitriding steels, nitrided
NV (nitr.)	through-hardened and case-hardening steel, nitrided
NV (nitrocar.)	through-hardened and case-hardening steels, nitro-carburized

5 Tooth breakage and safety factors

Tooth breakage usually ends transmission service life. Sometimes the destruction of all gears in a transmission is a consequence of the breakage of one tooth, while in certain instances the transmission path between input and output shafts is broken.

Because of this, the chosen value of the safety factor, S_F, against tooth breakage should be larger than the square of the safety factor, S_H, against pitting (see ISO 10300-1 for general comments on the choice of safety factor).

The value of the minimum safety factor for bending stress, S_{Fmin}, should be $\geqslant$ 1,3 for spiral bevel gears. For straight bevel gears, or where $\beta_m \leqslant 5°$, S_{Fmin} should be $\geqslant$ 1,5. It is recommended that the manufacturer and customer agree on the value of the minimum safety factor.

6 Gear-tooth rating formulae

6.1 General

The capacity of a gear tooth to resist bending shall be determined by the comparison of the following stress values:

— **bending stress**, based on the geometry of the tooth, the accuracy of its manufacture, the rigidity of the gear blanks, bearings and housing, and the operating torque, expressed by the bending stress formula (see 6.2);

— **allowable stress number**, and the effect of the working conditions under which the gears operate, expressed by the permissible bending stress formula (see 6.3).

The determined tooth root stress, σ_F, shall be $\leqslant \sigma_{FP}$, which is the permissible tooth root stress.

NOTE In respect of the allowable stress, reference is made to a stress "number", a designation adopted because pure stress, as determined by laboratory testing, is not calculated by the formulae in this part of ISO 10300. Instead, an arbitrary value is calculated and used throughout, with accompanying changes to the allowable stress number in order to maintain consistency for design comparison.

6.2 Tooth root stress

6.2.1 General

The tooth root stress is determined separately for pinion and wheel:

$$\sigma_F = \sigma_{FO} K_A K_v K_{F\beta} K_{F\alpha} \leqslant \sigma_{FP} \quad (1)$$

where

σ_{F0} is the local tooth root stress defined as the maximum tensile stress arising at the tooth root due to the nominal torque when a perfect gear is loaded.

See ISO 10300-1 for K_A, K_v, $K_{F\beta}$, $K_{F\alpha}$

6.2.2 Local tooth root stress, $\sigma_{F0\text{-}B1}$ — Method B1

The calculation of the local tooth root stress is based on the maximum tensile stress at the tooth root (30° tangent to the tooth root fillet). The determinant position of load application is:

a) the outer limit of single tooth contact ($\varepsilon_{v\beta} = 0$);

b) the mid-point of the zone of contact ($\varepsilon_{v\beta} \geqslant 1$);

c) interpolation between a) and b) ($0 < \varepsilon_{v\beta} < 1$).

The transformation from tip to this determinant position is done by Y_ε:

$$\sigma_{F0\text{-}B1} = \frac{F_{mt}}{b m_{mn}} Y_{Fa} Y_{Sa} Y_\varepsilon Y_K Y_{LS} \quad (2)$$

where

F_{mt} is the nominal tangential force at the reference cone at mid-face width (see ISO 10300-1);

b is the face width;

Y_{Fa} is the tooth form factor (see clause 7), which accounts for the influence of the tooth form on the nominal bending stress for load application at the tooth tip;

Y_{Sa} is the stress correction factor (see clause 7), which accounts for the conversion of the nominal bending stress for load application at tooth tip to the corresponding local tooth root stress. Thus Y_{Sa} accounts for the stress-increasing effect of the notch (in the root fillet), as well as for the fact that the stress condition in the critical root section is complex, but not for the influence of the bending moment arm;

Y_{ε} is the contact-ratio factor (see clause 8), which accounts for the conversion of the local stress determined for the load application at the tooth tip to the determinant position;

Y_K is the bevel-gear factor, which accounts for smaller values for l_b' compared to total face width b and the inclined lines of contact;

Y_{LS} is the load sharing factor, which accounts for load distribution between two or more pairs of teeth.

6.2.3 Local tooth root stress, $\sigma_{F0\text{-}B2}$ — Method B2

When applying method B2, the combined geometry factor Y_P replaces the factors Y_{Fa}, Y_{Sa}, Y_{ε}, Y_{LS} and Y_K in the local tooth root stress Equation such that Equation (2) becomes:

$$\sigma_{F0\text{-}B2} = \frac{F_{mt}}{b m_{mn}} Y_P \tag{3}$$

The value of Y_P is determined by:

$$Y_P = \frac{Y_A}{Y_J} \frac{m_{mt}\, m_{mn}}{m_{et}^2} \tag{4}$$

Substitution in Equation (3):

$$\sigma_{F0\text{-}B2} = \frac{F_{mt}}{b} \frac{m_{mt}}{m_{et}^2} \frac{Y_A}{Y_J} \tag{5}$$

where

Y_A is the bevel-gear adjustment factor for method B2, for standard carburized and case-hardened bevel gears (see annex A);

Y_J is the bending strength geometry factor for method B2 (see 9.2).

The bending-strength geometry factor, Y_J, evaluates the shape of the tooth, the position at which the most damaging load is applied, the stress concentration due to the geometric shape of the root fillet, the sharing of load between adjacent pairs of teeth, the tooth-thickness balance between the wheel and mating pinion, the effective face width due to lengthwise crowning of the teeth, and the buttressing effect of an extended face width on one member of the pair. Both the tangential (bending) and radial (compressive) components of the tooth load are included.

6.3 Permissible tooth root stress

6.3.1 General

The permissible tooth root stress, σ_{FP}, is determined separately for pinion and wheel. It should be calculated on the basis of the strength determined at an actual gear, as this way the reference value for geometrical similarity, course of movement and manufacture will lie within the field of application.

$$\sigma_{FP} = \frac{\sigma_{FE}\, Y_{NT}}{S_{F\min}}\, Y_{\delta\,rel\,T}\, Y_{R\,rel\,T}\, Y_X \quad (6)$$

$$\sigma_{FP} = \frac{\sigma_{F\,lim}\, Y_{ST}\, Y_{NT}}{S_{F\,min}}\, Y_{\delta\,rel\,T}\, Y_{R\,rel\,T}\, Y_X \quad (7)$$

where

σ_{FE} is the allowable stress number (bending),

σ_{FE} = $\sigma_{F\,lim}\, Y_{ST}$, the basic bending strength of the un-notched specimen under the assumption that the material (including heat treatment) is fully elastic;

$\sigma_{F\,lim}$ is the bending stress number for the nominal stress in bending of the test gear, which accounts for material, heat treatment, and surface influence at test gear dimensions (see ISO 6336-5);

Y_{ST} is the stress-correction factor for the dimensions of the standard test gear Y_{ST} = 2,0;

$S_{F\,min}$ is the minimum safety factor (see ISO 10300-1);

$Y_{\delta\,rel\,T}$ is the relative sensitivity factor (see clause 10) for the allowable stress number, related to the conditions at the standard test gear ($Y_{\delta\,rel\,T} = Y_\delta / Y_{\delta T}$ accounts for the notch sensitivity of the material);

$Y_{R\,rel\,T}$ is the relative surface condition factor (see clause 11) ($Y_{R\,rel\,T} = Y_R / Y_{RT}$ accounts for the surface condition at the root fillet, related to the conditions at the test gear);

Y_X is the size factor for tooth root strength (see clause 12), which accounts for the influence of the module on the tooth root strength;

Y_{NT} is the life factor, which accounts for the influence of required numbers of cycles of operation (see clause 13).

6.3.2 Calculated safety factor

The calculated safety factor against tooth breakage shall be determined separately for pinion and wheel. On the basis of the allowable stress number (bending), determined for the standard test gear:

$$S_F = \frac{\sigma_{FE}\, Y_{NT}}{\sigma_{F0}}\, \frac{Y_{\delta\,rel\,T}\, Y_{R\,rel\,T}\, Y_X}{K_A\, K_v\, K_{F\beta}\, K_{F\alpha}} \quad (8)$$

NOTE This is the calculated safety factor with respect to the transmitted torque.

See ISO 10300-1 in reference to the safety factor and the risk (probability) of failure.

7 Tooth form, Y_{Fa}, and correction, Y_{Sa}, factors — Method B1

7.1 General

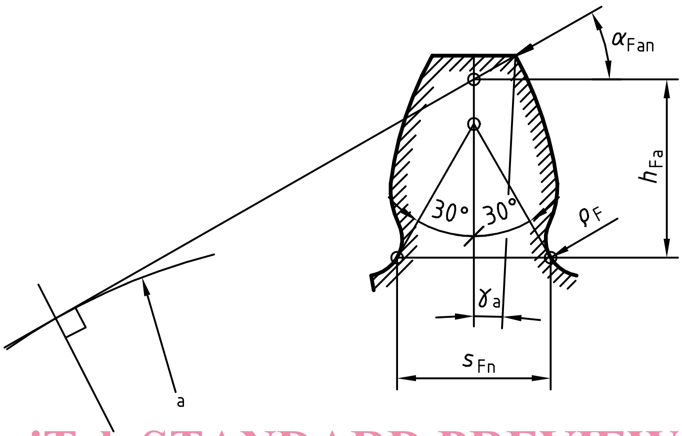
The tooth form factor, Y_{Fa}, accounts for the influence of the tooth form on the nominal bending stress in the case of load application at the tooth tip. It is determined separately for pinion and wheel.

NOTE In the case of gears with tip and root relief, the actual bending moment arm is slightly smaller, and the calculation is on the safe side.

Bevel gears generally have octoid teeth and a tip and root relief. However, deviations from an involute profile are small, especially in view of the tooth root chord and bending moment arm, and thus they may be neglected when calculating the tooth-form factor.

The distance between the contact points of the 30° tangents at the root fillets of the tooth profile of the virtual cylindrical gear is taken as a cross-section for calculation (see Figure 1).

In this part of ISO 10300, Y_{Fa} and Y_{Sa} are determined for the nominal gear without tolerances. The slight reduction in tooth thickness for backlash between teeth may be neglected for the load capacity calculation. The size reduction shall be taken into account when the outer tooth thickness allowance A_{sne} > 0,05 m_{mn}.



[a] Base circle of virtual cylindrical gear

Figure 1 — Tooth root chord, s_{Fn}, and bending moment arm for load application at the tooth tip, h_{Fa}, of the virtual cylindrical gear tooth profile

7.2 Y_{Fa} for generated gears

7.2.1 General

Equation (9) applies to virtual cylindrical gears in normal section with and without profile shift. The calculation is valid under the following premises:

a) The contact point of the 30° tangent lies on the fillet curve generated by the tool tip radius.

b) The tool is manufactured with a finite tip radius ($\rho_{a0} \neq 0$).

$$Y_{Fa} = \frac{6 \frac{h_{Fa}}{m_{mn}} \cos \alpha_{Fan}}{\left(\frac{s_{Fn}}{m_{mn}} \right)^2 \cos \alpha_n} \qquad (9)$$

See Figure 1 for the respective definitions; see ISO 6336-3 for an evaluation of the decisive normal tooth load and tooth form factor.

7.2.2 Auxiliary quantities

For the calculation of the tooth root chord, s_{Fn}, and bending moment arm, h_{Fa}, first the auxiliary quantities E, G, H and ϑ need to be determined:

$$E = \left(\frac{\pi}{4} - x_{sm} \right) m_{mn} - h_{a0} \tan \alpha_n - \frac{\rho_{a0} (1 - \sin \alpha_n) - s_{pr}}{\cos \alpha_n} \qquad (10)$$

$$G = \frac{\rho_{a0}}{m_{mn}} - \frac{h_{a0}}{m_{mn}} + x_{hm} \tag{11}$$

$$H = \frac{2}{z_{vn}} \left(\frac{\pi}{2} - \frac{E}{m_{mn}} \right) - \frac{\pi}{3} \tag{12}$$

$$\vartheta = \frac{2G}{z_{vn}} \tan\vartheta - H \tag{13}$$

For the solution of the transcendent Equation (13), $\vartheta = \pi/6$ may be inserted as the initial value. In most cases, the Equation already converges after a few iteration steps.

7.2.3 Tooth root chord, s_{Fn}

$$\frac{s_{Fn}}{m_{mn}} = z_{vn} \sin\left(\frac{\pi}{3} - \vartheta \right) + \sqrt{3} \left(\frac{G}{\cos\vartheta} - \frac{\rho_{a0}}{m_{mn}} \right) \tag{14}$$

7.2.4 Fillet radius, ρ_F, at contact point of 30° tangent

$$\frac{\rho_F}{m_{mn}} = \frac{\rho_{a0}}{m_{mn}} + \frac{2G^2}{\cos\vartheta \left(z_{vn} \cos^2\vartheta - 2G \right)} \tag{15}$$

7.2.5 Bending moment arm, h_{Fa}

$$\alpha_{an} = \arccos\left(\frac{d_{vbn}}{d_{van}} \right) \tag{16}$$

$$\gamma_a = \frac{1}{z_{vn}} \left[\frac{\pi}{2} + 2\,(x_{hm} \tan\alpha_n + x_{sm}) \right] + \operatorname{inv}\alpha_n - \operatorname{inv}\alpha_{an} \tag{17}$$

$$\alpha_{Fan} = \alpha_{an} - \gamma_a \tag{18}$$

$$\frac{h_{Fa}}{m_{mn}} = \frac{1}{2} \left[\left(\cos\gamma_a - \sin\gamma_a \tan\alpha_{Fan} \right) \frac{d_{van}}{m_{mn}} - z_{vn} \cos\left(\frac{\pi}{3} - \vartheta \right) - \frac{G}{\cos\vartheta} + \frac{\rho_{a0}}{m_{mn}} \right] \tag{19}$$

See annex A of ISO 10300-1:2001 for data of the virtual cylindrical gear in normal section. Dimensions at the basic rack profile of the tooth are shown in Figure 2 of this part of ISO 10300, while Y_{Fa} may be taken from Figure 3 for a basic rack profile of the tool with data $\alpha_n = 20°$, $h_{a0}/m_{mn} = 1{,}25$, and $\rho_{a0}/m_{mn} = 0{,}25$ for $x_{sm} = 0$. Diagrams for other basic rack profiles are given in ISO 6336-3.

See Figures 4 to 6 for the combined tooth form factor $Y_{FS} = Y_{Fa}\, Y_{Sa}$ for generated bevel gears.