



# SLOVENSKI STANDARD

## SIST EN 61710:2014

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### Model eksponentnega pravila - Preskusi ujemanja in metode vrednotenja (IEC 61710:2013)

Power law model - Goodness-of-fit tests and estimation methods

Modèle de loi en puissance - Essai d'adéquation et méthodes d'estimation des paramètres

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**Power law model -  
Goodness-of-fit tests and estimation methods  
(IEC 61710:2013)**

Modèle de loi en puissance -  
Essais d'adéquation et méthodes  
d'estimation des paramètres  
(CEI 61710:2013)

Potenzgesetz-Modell -  
Anpassungstests und Schätzverfahren  
(IEC 61710:2013)

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## Foreword

The text of document 56/1500/FDIS, future edition 2 of IEC 61710, prepared by IEC/TC 56 "Dependability" was submitted to the IEC-CENELEC parallel vote and approved by CENELEC as EN 61710:2013.

The following dates are fixed:

- latest date by which the document has to be implemented at national level by publication of an identical national standard or by endorsement (dop) 2014-03-26
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NOTE When an international publication has been modified by common modifications, indicated by (mod), the relevant EN/HD applies.

<u>Publication</u>	<u>Year</u>	<u>Title</u>	<u>EN/HD</u>	<u>Year</u>
IEC 60050-191	1990	International Electrotechnical Vocabulary (IEV) - Chapter 191: Dependability and quality of service	-	-

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Edition 2.0 2013-05

# INTERNATIONAL STANDARD

# NORME INTERNATIONALE



Power law model – Goodness-of-fit tests and estimation methods

Modèle de loi en puissance – Essais d'adéquation et méthodes d'estimation  
des paramètres

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## INTERNATIONAL ELECTROTECHNICAL COMMISSION

**POWER LAW MODEL –  
GOODNESS-OF-FIT TESTS  
AND ESTIMATION METHODS**

## FOREWORD

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International Standard IEC 61710 has been prepared by IEC technical committee 56: Dependability.

This second edition cancels and replaces the first edition, published in 2000, and constitutes a technical revision.

The main changes with respect to the previous edition are listed below:

- the inclusion of an additional Annex C on Bayesian estimation for the power law model.

The text of this standard is based on the following documents:

FDIS	Report on voting
56/1500/FDIS	56/1508/RVD

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC web site under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

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- withdrawn,
- replaced by a revised edition, or
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## INTRODUCTION

This International Standard describes the power law model and gives step-by-step directions for its use. There are various models for describing the reliability of repairable items, the power law model being one of the most widely used. This standard provides procedures to estimate the parameters of the power law model and to test the goodness-of-fit of the power law model to data, to provide confidence intervals for the failure intensity and prediction intervals for the length of time to future failures. An input is required consisting of a data set of times at which relevant failures occurred, or were observed, for a repairable item or a set of copies of the same item, and the time at which observation of the item was terminated, if different from the time of final failure. All output results correspond to the item type under consideration.

Some of the procedures can require computer programs, but these are not unduly complex. This standard presents algorithms from which computer programs should be easy to construct.

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## POWER LAW MODEL – GOODNESS-OF-FIT TESTS AND ESTIMATION METHODS

### 1 Scope

This International Standard specifies procedures to estimate the parameters of the power law model, to provide confidence intervals for the failure intensity, to provide prediction intervals for the times to future failures, and to test the goodness-of-fit of the power law model to data from repairable items. It is assumed that the time to failure data have been collected from an item, or some identical items operating under the same conditions (e.g. environment and load).

### 2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-191:1990, *International Electrotechnical Vocabulary (IEV) – Chapter 191: Dependability and quality of service*

### 3 Terms and definitions

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For the purposes of this document, the terms and definitions of IEC 60050-191 apply.

### 4 Symbols and abbreviations

The following symbols and abbreviations apply:

$\beta$	shape parameter of the power law model
$\hat{\beta}$	estimated shape parameter of the power law model
$\beta_{LB}, \beta_{UB}$	lower, upper confidence limits for $\beta$
$C^2$	Cramer-von-Mises goodness-of-fit test statistic
$C_{1-\gamma}^2(M)$	critical value for the Cramer-von-Mises goodness-of-fit test statistic at $\gamma$ level of significance
$\chi^2$	Chi-square goodness-of-fit test statistic
$\chi_{\gamma}^2(v)$	$\gamma$ th fractile of the $\chi^2$ distribution with $v$ degrees of freedom
$d$	number of intervals for groups of failures
$E[N(t)]$	expected accumulated number of failures up to time $t$
$E[t_j]$	expected accumulated time to $j$ th failure

$\hat{E}[N[t(i)]]$	estimated expected accumulated number of failures up to $t(i)$
$\hat{E}[t_j]$	estimated expected accumulated time to $j$ th failure
$F_\gamma(\nu_1, \nu_2)$	$\gamma$ th fractile for the $F$ distribution with $(\nu_1, \nu_2)$ degrees of freedom
$i$	general purpose indicator
$j$	general purpose indicator
$k$	number of items
$L, U$	multipliers used in calculation of confidence intervals for failure intensity
$\lambda$	scale parameter of the power law model
$\hat{\lambda}$	estimated scale parameter of the power law model
$M$	parameter for Cramer-von-Mises statistical test
$N$	number of relevant failures
$N_j$	number of failures for $j$ th item
$N(t)$	accumulated number of failures up to time $t$
$N[t(i)]$	accumulated number of failures up to time $t(i)$
$R$	difference between the order number of future (predicted) failure and order number of last (observed) failure
$T$	accumulated relevant time
$T^*$	total accumulated relevant time for time terminated test
$T_j$	total accumulated relevant time for $j$ th item
$T_{RL}, T_{RU}$	lower, upper prediction limits for the length of time to the $R$ th future failure
$\hat{T}_{N+1}$	estimated median time to $(N+1)$ th failure
$t_i$	accumulated relevant time to the $i$ th failure
$t_{ij}$	$i$ th failure time for $j$ th item
$t_N$	total accumulated relevant time for failure terminated test
$t_{Nj}$	total accumulated relevant time to $N$ th failure of $j$ th item
$t(i-1), t(i)$	endpoints of $i$ th interval of time for grouped failures
$z(t)$	failure intensity at time $t$
$\hat{z}(t)$	estimated failure intensity at time $t$
$z_{LB}, z_{UB}$	lower, upper confidence limits for failure intensity

## 5 Power law model

The statistical procedures for the power law model use the relevant failure and time data from the test or field studies. The basic equations for the power law model are given in this clause. Background information on the model is given in Annex A and examples of its application are given in Annex B.

The expected accumulated number of failures up to test time  $t$  is given by:

$$E[N(t)] = \lambda t^\beta \quad \text{with } \lambda > 0, \beta > 0, t > 0$$

where

$\lambda$  is the scale parameter;

$\beta$  is the shape parameter ( $0 < \beta < 1$  corresponds to a decreasing failure intensity;  $\beta = 1$  corresponds to a constant failure intensity;  $\beta > 1$  corresponds to an increasing failure intensity).

The failure intensity at time  $t$  is given by:

$$z(t) = \frac{d}{dt} E[N(t)] = \lambda \beta t^{\beta-1} \text{ with } t > 0$$

Thus the parameters  $\lambda$  and  $\beta$  both affect the failure intensity in a given time.

Methods are given in 7.2 for maximum likelihood estimation of the parameters of  $\lambda$  and  $\beta$ . Subclause 7.3 gives goodness-of-fit tests for the model and 7.4 and 7.5 give confidence interval procedures. Subclause 7.6 gives prediction interval procedures and 7.7 gives tests for the equality of the shape parameters. The model is simple to evaluate. However when  $\beta < 1$ , theoretically  $z(0) = \infty$  (i.e.  $z(t)$  tends to infinity as  $t$  tends to zero) and  $z(\infty) = 0$  (i.e.  $z(t)$  tends to zero as  $t$  tends to infinity); but this theoretical limitation does not generally affect its practical use.

## 6 Data requirements

### 6.1 General

#### 6.1.1 Case 1 – Time data for every relevant failure for one or more copies from the same population

The normal evaluation methods assume the observed times to be exact times of failure of a single repairable item or a set of copies of the same repairable item. The figures below illustrate how the failure times are calculated for three general cases.

##### 6.1.2 Case 1a) – One repairable item

For one repairable item observed from time 0 to time  $T$ , the relevant failure time,  $t_i$ , is the elapsed operating time (that is, excluding repair and other down times) until the occurrence of the  $i$ -th failure as shown in Figure 1.

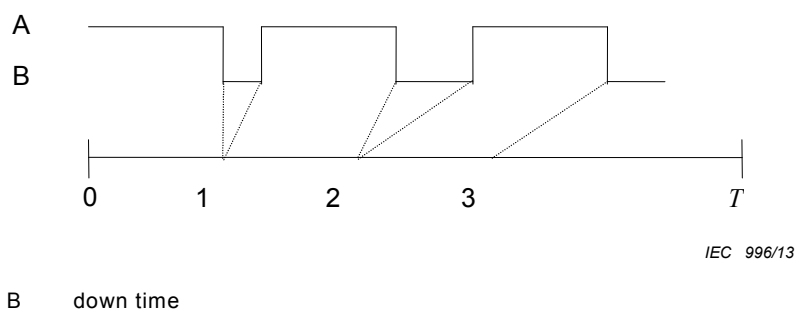


Figure 1 – One repairable item

Time terminated data are observed to  $T^*$ , which is not a failure time, and failure terminated data are observed to  $t_N$ , which is the time of the  $N$ th failure. Time terminated and failure terminated data use slightly different formulae.