Designation: F 1811 – 97 (Reapproved 2002)

Standard Practice for Estimating the Power Spectral Density Function and Related Finish Parameters from Surface Profile Data[1]

This standard is issued under the fixed designation F 1811; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice defines the methodology for calculating a set of commonly used statistical parameters and functions of surface roughness from a set of measured surface profile data. Its purposes are to provide fundamental procedures and notation for processing and presenting data, to alert the reader to related issues that may arise in user-specific applications, and to provide literature references where further details can be found.

1.2 The present practice is limited to the analysis of one-dimensional or profile data taken at uniform intervals along straight lines across the surface under test, although reference is made to the more general case of two-dimensional measurements made over a rectangular array of data points.

1.3 The data analysis procedures described in this practice are generic and are not limited to specific surfaces, surface-generation techniques, degrees of roughness, or measuring techniques. Examples of measuring techniques that can be used to generate profile data for analysis are mechanical profiling instruments using a rigid contacting probe, optical profiling instruments that sample over a line or an array over an area of the surface, optical interferometry, and scanning-microscopy techniques such as atomic-force microscopy. The distinctions between different measuring techniques enter the present practice through various parameters and functions that are defined in Sections 3 and 5, such as their sampling intervals, bandwidths, and measurement transfer functions.

1.4 The primary interest here is the characterization of random or periodic aspects of surface finish rather than isolated surface defects such as pits, protrusions, scratches or ridges. Although the methods of data analysis described here can be equally well applied to profile data of isolated surface features, the parameters and functions that are derived using the procedures described in this practice may have a different physical significance than those derived from random or periodic surfaces.

1.5 The statistical parameters and functions that are discussed in this practice are, in fact, mathematical abstractions that are generally defined in terms of an infinitely-long linear profile across the surface, or the "ensemble" average of an infinite number of finite-length profiles. In contrast, real profile data are available in the form of one or more sets of digitized height data measured at a finite number of discrete positions on the surface under test. This practice gives both the abstract definitions of the statistical quantities of interest, and numerical procedures for determining values of these abstract quantities from sets of measured data. In the notation of this practice these numerical procedures are called "estimators" and the results that they produce are called "estimates".

1.6 This practice gives "periodogram" estimators for determining the root-mean-square (rms) roughness, rms slope, and power spectral density (PSD) of the surface directly from profile height or slope measurements. The statistical literature uses a circumflex to distinguish an estimator or estimate from its abstract or ensemble-average value. For example, $\hat{A}$ denotes an estimate of the quality A. However, some word-processors cannot place a circumflex over consonants in text. Any symbolic or verbal device may be used instead.

1.7 The quality of estimators of surface statistics are, in turn, characterized by higher-order statistical properties that describe their "bias" and "fluctuation" properties with respect to their abstract or ensemble-average versions. This practice does not discuss the higher-order statistical properties of the estimators given here since their practical significance and use are application-specific and beyond the scope of this document. Details of these and related subjects can be found in References (**1–10**)[2] at the end of this practice.

1.8 Raw measured profile data generally contain trending components that are independent of the microtopography of the surface being measured. These components must be subtracted before the difference or residual errors are subjected to the statistical-estimation routines given here. These trending components originate from both extrinsic and intrinsic sources. Extrinsic trends arise from the rigid-body positioning of the part under test in the measuring apparatus. In optics these displacement and rotation contributions are called "piston" and "tilt" errors. In contrast, intrinsic trends arise from deliberate or accidental shape errors inherent in the surface under test, such

[1] This practice is under the jurisdiction of ASTM Committee F01 on Electronicsand is the direct responsibility of Subcommittee F01.06 on Silicon Materials and Process Control.

Current edition approved June 10, 1997. Published August 1997.

[2] The boldface numbers in parentheses refer to the list of references at the end of this practice.

as a circular or parabolic curvature. In the absence of a-priori information about the true surface shape, the intrinsic shape error is frequently limited to a quadratic (parabolic) curvature of the surface. Detrending of intrinsic and extrinsic trends is generally accomplished simultaneously by subtracting a detrending polynomial from the raw measured data, where the polynomial coefficients are determined by least-squares fitting to the measured data.

1.9 Although surfaces and surface measuring instruments exist in real or configuration space, they are most easily understood in frequency space, also known as Fourier transform, reciprocal or spatial-frequency space. This is because any practical measurement process can be considered to be a "linear system", meaning that the measured profile is the convolution of the true surface profile and the impulse response of the measuring system; and equivalently, the Fourier-amplitude spectrum of the measured profile is the product of that of the true profile and the frequency-dependent "transfer function" of the measurement system. This is expressed symbolically by the following equation:

$$A_{meas}(f_x) = A_{true}(f_x) \cdot T(f_x)$$

where:

A = the Fourier amplitudes,
$T(f_x)$ = instrument response function or the measurement transfer function, and
f_x = surface spatial frequency.

This factorization permits the surface and the measuring system to be discussed independently of each other in frequency space, and is an essential feature of any discussion of measurement systems.

1.10 Figure 1 sketches different forms of the measurement transfer function, $T(f_x)$:

1.10.1 Case (*a*) is a perfect measuring system, which has $T(f_x) = 1$ for all spatial frequencies, $0 \leq f_x \leq \infty$. This is unrealistic since no real measuring instrument is equally sensitive to all spatial frequencies. Case (*b*) is an ideal measuring system, which has $T(f_x) = 1$ for $LFL \leq f_x \leq HFL$ and $T(f_x) = 0$ otherwise, where *LFL* and *HFL* denote the low-frequency and high-frequency limits of the measurement. The range $LFL \leq f_x \leq HFL$ is called the bandpass or bandwidth of the measurement, and ratio *HFL/LFL* is called the dynamic range of the measurement. Case (*c*) represents a realistic measuring system, since it includes the fact that $T(f_x)$ need not be unity within the measurement bandpass or strictly zero outside the bandpass.

1.11 If the measurement transfer function is known to deviate significantly from unity within the measurement bandpass, the measured power spectral density (*PSD*) can be transformed into the form that would have been measured by an instrument with the ideal rectangular form through the process of digital "restoration." In its simplest form restoration involves dividing the measured *PSD* by the known form of $|T(f_x)|^2$ over the measurement bandpass. Restoration is particularly relevant to measuring instruments that involve optical microscopes since the transfer functions of microscope systems are not unity over their bandpass but tend to fall linearly between unity at $T(0) = 1$ and $T(HFL) = 0$. The need for, and methodology of digital restoration is instrument specific and this practice places no requirements on its use.

1.12 This practice requires that any data on surface finish parameters or functions generated by the procedures described herein be accompanied by an identifying description of measuring instrument used, estimates of its low- and high-frequency limits, *LFL* and *HFL*, and a statement of whether or not restoration techniques were used.

1.13 In order to make a quantitative comparison between profile data obtained from different measurement techniques, the statistical parameters and functions of interest must be compared over the same or comparable spatial-frequency regions. The most common quantities used to compare surfaces are their root-mean-square (rms) roughness values, which are the square roots of the areas under the *PSD* between specified surface-frequency limits. Surface statistics derived from measurements involving different spatial-frequency ranges cannot be compared quantitatively except in an approximate way. In some cases measurements with partially or even nonoverlapping bandwidths can be compared by using analytic models of the *PSD*s to extrapolate the *PSD*s outside their measurement bandwidth.

1.14 Examples of specific band-width limits can be drawn from the optical and semiconductor industries. In optics the so-called total integrated scatter or TIS measurement technique leads to rms roughness values involving an annulus in two-dimensional spatial frequencies space from 0.069 to 1.48 μm^{-1}; that is, a dynamic range of 1.48/0.069 = 21/1. In contrast, the range of spatial frequencies involved in optical and mechanical scanning techniques are generally much larger than this, frequently having a dynamic ranges of 512/1 or more. In the latter case the subrange of 0.0125 to 1 μm^{-1} has been used to discuss the rms surface roughness in the semiconductor industry. These numbers are provided to illustrate the magnitudes and ranges of *HFL* and *LFL* encountered in practice but do not constitute a recommendation of particular limits for the specification of surface finish parameters. Such selections are application dependent, and are to be made at the users' discretion.

1.15 The limits of integration involved in the determination of rms roughness and slope values from measured profile data are introduced by multiplying the measured *PSD* by a factor equal to zero for spatial frequencies outside the desired bandpass and unity within the desired bandpass, as shown in Case (*b*) in Fig. 1. This is called a top-hat or binary filter function. Before the ready availability of digital frequency-domain processing as employed in this practice, bandwidth limits were imposed by passing the profile data through analog or digital filters without explicitly transforming them into the frequency domain and multiplying by a top-hat function. The two processes are mathematically equivalent, providing the data filter has the desired frequency response. Real data filters, however, frequently have Gaussian or *RC* forms that only approximate the desired top-hat form that introduces some ambiguity in their interpretation. This practice recommends the determination of rms roughness and slope values using top-hat windowing of the measured *PSD* in the frequency domain.

1.16 The *PSD* and rms roughness are surface statistics of particular interest to the optics and semiconductor industries because of their direct relationship to the functional properties of such surfaces. In the case of rougher surfaces these are still valid and useful statistics, although the functional properties of such surfaces may depend on additional statistics as well. The ASME Standard on Surface Texture, B46.1, discusses additional surface statistics, terms, and measurement methods applicable to machined surfaces.

1.17 The units used in this practice are a self-consistent set of SI units that are appropriate for many measurements in the semiconductor and optics industry. This practice does not mandate the use of these units, but does require that results expressed in other units be referenced to SI units for ease of comparison.

1.18 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

E 284 Terminology Relating to Appearance of Materials[3]

E 1392 Practice for Angle Resolved Optical Scatter Measurements on Specular or Diffuse Surfaces[4]

F 1048 Test Method for Measuring the Effective Surface Roughness of Optical Components by Total Integrated Scattering[4]

2.2 *ANSI Standard:*

ANSI/ASME B46.1 Surface Texture (Surface Roughness, Waviness and Lay)[5]

3. Terminology

3.1 *Definitions: Introduction—This section provides the definitions of special terms used in this practice, and includes the mathematical definitions of different profile statistics in terms of continuous, infinitely-long profiles. The corresponding estimators of those statistics based on linear, sampled, finite-trace-length data are given in Section 5. Definitions of terms not included here will be found in Terminology E 284, Practice E 1392, Test Method F 1048 or ANSI/ASME B46.1.*

3.2 *aperture averaging, local averaging, data averaging*—As used here, aperture and local averaging mean that an estimate of the power spectral density function (*PSD*) is "smoothed" by replacing its value at a given spatial frequency by its average over a local frequency range using a particular weighting function. Data averaging means the numerical averaging of statistical estimates of the PSD, the mean-square surface roughness or the mean-square profile slope derived from different measurements, in order to obtain a single, composite result. For example, a rectangular or square array of measurements can be separated into a set of parallel profile measurements which can be analyzed separately and the results averaged.

3.2.1 *Discussion*—The averaged quantities must include the same range of surface spatial frequencies.

3.3 *bandwidth, bandwidth limits*—The range of surface spatial frequencies included in a measurement or specification. It is specified by a high-frequency limit (*HFL*) and a low-frequency limit (*LFL*).

3.3.1 *Discussion*—The bandwidth and the measurement transfer function over the bandwidth must be taken into account when measurements or statistical properties are compared. Different measuring instruments are generally sensitive to different ranges of surface spatial frequencies; that is, they have different bandwidth limits. Real bandwidth limits are necessarily finite since no measuring instrument is sensitive to infinitely-low or to infinitely-high surface spatial frequencies.

3.4 *bias error*—The average deviation between an estimate of a statistical quantity and its true value.

3.4.1 *Discussion*—The periodogram estimator of the power spectral density (*PSD*) given in this practice is a zero-bias or unbiased estimator of the *PSD*. On the other hand, local averaging of the periodogram can introduce bias errors in regions where the spectrum varies rapidly with frequency.

3.5 *deterministic profile, deterministic roughness*—A deterministic profile is a surface profile that is a known function of surface position, with no random dependencies on position.

3.5.1 *Discussion*—In contrast, a random profile is known only in terms of a probability distribution function.

3.6 *dynamic range*—The ratio of the high- to low-frequency limits of the bandwidth of a given measurement technique:

Dynamic range = HFL/LFL.

[3] *Annual Book of ASTM Standards*, Vol 06.01.

[4] *Annual Book of ASTM Standards*, Vol 10.05.

[5] Available from the American National Standards Institute, 25 W. 43rd St., 4th Floor, New York, NY 10036.

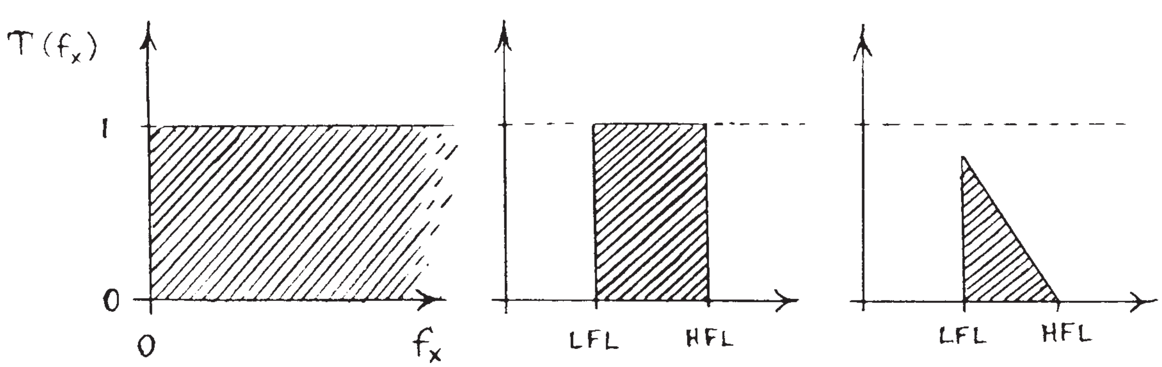

FIG. 1 Different Forms of the Measurement-Transfer or Instrumental-Response Function as a Function of Spatial Frequency, f_x.

3.6.1 *Discussion*—This is a useful single-number characteristic of a measuring apparatus. It completely describes the measurement effects on surfaces with power-law power spectra.

3.7 *detrended profile, $Z_d(x)$*—The raw or measured profile after removing instrumental and surface trends. The detrended profile is the input for the statistical estimation routines described in Section 5.

3.7.1 *Discussion*—If the parametric form of the trend is known, its least-squares-fitted form can be subtracted from the measured profile data. Otherwise a generic power-series form can be used. This practice describes the procedures for removing a zero-, first- or second-order polynomial in the trace distance. A zero-order polynomial removes piston; a first-order polynomial removes piston and tilt; and a second-order polynomial removes piston, tilt and quadratic curvature. In each case the detrended data set has zero mean. The coefficients of constant and linear terms correspond to the rigid-body orientation of the part being measured and need not be recorded. However, the coefficient of the quadratic term represents the intrinsic curvature of the surface being measured and should be recorded.

3.8 *ensemble, ensemble-average value*—An "ensemble" is an infinitely large collection (infinite ensemble) of quantities, the properties of which are governed by some statistical distribution law. For example, surface profiles, and rms roughness values. An "ensemble average value" is the value of a particular surface parameter or function averaged over the appropriate distribution functions. The ensemble average value of the quantity *A* is denoted by $<A>$.

3.8.1 *Discussion*—Estimates of ensemble-average quantities based on a finite collection of measurements (finite ensemble) can deviate from their infinite-ensemble values by fluctuation and bias errors.

3.9 *estimator, estimated value, or estimate*—An estimator is an algorithm or mathematical procedure for calculating an "estimate" the ensemble-average value of a roughness statistic from a finite set of measured profile data.

3.9.1 *Discussion*—In this practice a circumflex is used to distinguish estimators and estimates from the corresponding ensemble-average quantities (see also 1.6).

3.10 *fast fourier transform or FFT*—An algorithm for calculating the Fourier transform (discrete Fourier transform or *DFT*) of a set of numerical data. It is now ubiquitous and can be found in any computer data analysis package (see 5.4.2 for details).

3.10.1 *Discussion*—The discovery of the *FFT* is generally attributed to Cooley and Tukey, although it was used and reported in the earlier literature by a number of others, including Gauss, two centuries before.

3.11 *finish parameters and functions*—Numbers or functions that characterize surface height fluctuations. Their values and forms may vary depending on the bandwidth of surface frequencies that they contain, and the shapes of the transfer functions of the measurement instruments involved. These quantities are represented by their ensemble-average values derived from measurements using specific estimation routines.

3.11.1 *Discussion*—In general, the finish parameters and functions of an area are different from those of profiles taken across the surface. In the case of surfaces that are statistically isotropic, however, the area and profile statistics have a one-to-one relationship. Except for incidental remarks, this practice is concerned exclusively with the properties of surface profiles.

3.12 *fluctuation error*—A general term denoting the deviation of a quantity from its mean, average or detrended value. Fluctuation errors are usually measured in terms of their mean-square or rms values.

3.12.1 *Discussion*—For example, R_q is the rms fluctuation error in the surface height and Δ_q is the rms fluctuation error in the profile slope. In turn, the estimates of R_q and Δ_q have their own fluctuation errors. The magnitudes of these higher fluctuation errors not discussed in this practice.

3.13 *high-frequency limit, HFL, 1/micrometers*—The highest spatial frequency contained in a profile data set or specification. The *HFL* of a measurement is determined by the details of the measurement process, and its value in specifications is determined by the user.

3.13.1 *Discussion*—If the sampling interval in the measurement process is *D*, the extreme value of the *HFL* is given by the Nyquist criterion: HFL = ½D. However, other electrical, mechanical, or optical filtering mechanisms may further limit the *HFL*. Examples of such mechanisms are: the stylus tip radius, projected measurement pixel size, optical resolution, and electrical and digital filters, all of which contribute to the high-frequency roll-off of the instrument transfer function. If the Nyquist frequency is used to determine the *HFL*, care should be taken to determine that the true *HFL* is not reduced by these additional mechanisms.

3.14 *intrinsic surface or finish parameters*—Surface parameters such as the rms roughness or rms slope that contain all surface spatial frequencies from zero to infinity.

3.14.1 *Discussion*—Intrinsic parameters are statistical abstractions that cannot be measured or estimated directly since real measurements are sensitive to only limited ranges of surface spatial frequencies. They can, however, be inferred from real measurements by augmenting measurements with a-priori information about very low and very high spatial frequencies contained in physically-based models of the *PSD*s of the surfaces involved. All measured finish parameters are finite but their corresponding intrinsic values need not be. The important distinction between intrinsic and measured (bandwidth limited) finish parameters is not always made in the literature.

3.15 *impulse response*—The impulse response of a profile-measuring system is the measured shape of an impulse or infinitely-sharp ridge lying perpendicularly to the profile direction. In the case of a linear measuring system the impulse response is the Fourier transform of the system transfer function.

3.15.1 *Discussion*—The impulse response of a perfect measuring system would be an infinitely sharp spike or delta function. In contrast, the impulse response of real measuring systems has a finite width.

3.16 *isotropic surface, statistically-isotropic surface*—A surface whose intrinsic finish parameters and functions are independent of the rotational position of the surface about its surface normal.

3.16.1 *Discussion*—The rms roughness of profiles taken across an isotropically rough surface is independent of the profile directions, and equals the rms roughness of the surface area. The rms slope of an isotropically rough surface is also independent of the profile direction and equals $1/\sqrt{(2)}$ of the rms area gradient. The one-dimensional or profile power spectrum of an isotropic surface is also independent of the direction of the profile on the surface, and is related to the two-dimensional spectrum of the surface area by an integral transform. Examples of this are given in 3.37.

3.17 *linear systems, linear measurement system*—A signal-processing concept more precisely described as a linear, shift-invariant system. For the present purposes, a linear measurement of the surface profile is the true profile convolved with the impulse response of the measuring system, or equivalently, the Fourier amplitude spectrum of the measurement is the true amplitude spectrum times the measurement transfer function as indicated in 1.9.

3.17.1 *Discussion*—All practical measurement systems are taken to be linear over their operating ranges.

3.18 *low-frequency limit, LFL, 1/micrometers*—The lowest spatial frequency contained in a profile data set or specification.

3.18.1 *Discussion*—The minimum *LFL* in a profile measurement is the reciprocal of the length of the surface profile. The estimated value of the *PSD* at this value of the *LFL* is generally attenuated by the detrending process. To avoid this effect the lowest practical *LFL* is sometimes taken to be 3 to 5 times the reciprocal of the scan length. The *LFL* in surface specifications is determined by the user.

3.19 *mean-square profile roughness, R_q^2, nanometers squared*—The ensemble-average value of the square of the height of the detrended profile:

$$R_q^2 = \lim_{L \to \infty} \frac{1}{L} \int_{-L/2}^{+L/2} dx\, (Z_d(x))^2 = \int_0^{+\infty} df_x \; S_1(f_x)$$

where:

$Z_d(x)$ is the detrended surface profile, and

$S_1(f_x)$ is its power spectral density.

The optics literature uses the symbol σ for R_q.

3.19.1 *Discussion*—The intrinsic value of the mean-square roughness of an isotropically-rough surface area equals the mean-square roughness of any profile across it. The rms roughness, R_q, is distinct from the arithmetic-average roughness, R_a. The two are only related through a specific height-distribution function. For example, for a Gaussian height distribution,

$$R_a = \sqrt{(2/\pi)}\; R_q = 0.798\; R_q .$$

3.20 *mean-square profile slope, Δ_q^2, units of choice*—The average value of the square of the slope of the detrended profile:

$$\Delta_q^2 = \lim_{L \to \infty} \frac{1}{L} \int_{-L/2}^{+L/2} dx \left(\frac{dZ_d}{dx}\right)^2 = \int_0^{+\infty} df_x S_1(f_x) \cdot (2\pi f_x)^2$$

3.20.1 *Discussion*—This expression assumes that the average slope has been removed in the detrending process. The integrand in the frequency integral on the right can be viewed as the slope power spectral density. The mean-square surface slope of an isotropically-rough two-dimensional surface is half the mean-square gradient of the surface itself.

3.21 *measured profile parameters and functions*—Quantities derived from detrended profile data that include the bandwidth and transfer function effects of the particular measurement system used.

3.21.1 *Discussion*—Measured parameters and functions can be used for comparing surfaces quality providing the same measurement system is used in all cases. In order to compare quantitative measurements made by different measurement systems, or to estimate intrinsic surface properties, the system bandwidths and transfer functions must be taken into account. In the early literature, measurement systems were taken to be "perfect" in the sense of 1.10.1, and the effects of their bandwidths and transfer functions were ignored.

3.22 *Nyquist frequency, 1/micrometers*—The spatial frequency equal to the reciprocal of twice the sampling interval. See 3.13.1.

3.22.1 *Discussion*—The Nyquist frequency represents the highest undistorted frequency involved in a series of uniformly-spaced profile measurements. Higher-frequency components in the surface appear at lower-frequencies through the process of aliasing. Unless the effects of aliasing are removed by anti-aliasing mechanisms in the measurement process, they will corrupt the measured spectrum immediately below the Nyquist frequency. In that case the *HFL* should be taken to be a factor of 3 to 5 below the Nyquist frequency.

3.23 *periodic roughness, periodic random roughness*—Purely periodic roughness is deterministic. Periodic random roughness is modified version of purely periodic roughness that has a definite fundamental spatial frequency but random variations in its phase or amplitude.

3.23.1 *Discussion*—The power spectra of periodic and periodic random roughness appear as isolated peaks in the power spectral density function. This pattern is distinct from the broad variations appearing for purely random surfaces. Random surfaces can be viewed as periodic surfaces with a broad distribution of fundamental periods.

3.24 *periodogram estimator, periodogram estimates*—The periodogram is the particular estimator for the power spectral density discussed in this practice. It is proportional to the square magnitude of the discrete Fourier transform of the detrended data set. Periodogram estimates are estimates of particular finish parameters that are derived from the periodogram estimate of the power spectrum.

3.25 *power spectral density, PSD or power spectrum*—A statistical function that shows how the mean-square $(\mathrm{rms})^2$ of a given quantity is distributed among the various surface spatial frequencies inherent in the profile height.

3.25.1 *Discussion*—The two conventional measures of surface roughness, R_a and R_q do not carry any information about the transverse scale of the surface roughness. That is, they are independent of how much the surface profile is squeezed or stretched parallel to the surface plane. The *PSD* is the simplest statistic that carries that important additional information.

(1) *profile or one-dimensional PSD of the surface height, micrometers-cubed*—This quantity has the units of μm^3, and is a function of the spatial frequency, f_x, in units of inverse micrometers, μm^{-1}. It is defined as follows:

$$S_1(f_x) = \lim_{L \to \infty} \left\langle \frac{2}{L} \left| \int_{-L/2}^{+L/2} dx \; Z(x) e^{i2\pi f_x x} \right|^2 \right\rangle, \quad f_x > 0$$

3.25.1.1 *Discussion*—The subscript "*x*" on "f_x" corresponds to the direction of the profile on the surface and can be omitted if no confusion is involved. In this definition the spatial frequency, f_x, is always positive and greater than zero. The value at $f_x = 0$ corresponds to the average value of the profile height, which is zero for detrended profiles. The factor of 2 accounts for the equal contribution from negative frequencies and ensures that the area under the positive-frequency profile spectrum equals the rms-squared (mean-square) profile height.

3.25.1.2 *Discussion*—A mathematical variant of the periodogram estimator is the correlation method. This is a two-step process that requires the estimation of an intermediate function, the autocovariance function, which is then Fourier transformed to obtain the periodogram estimate of the power spectrum. This method is not discussed in this practice since it is indirect, and when properly applied gives identically the same results as the direct transform method recommended in this practice.

3.25.1.3 *Discussion*—The signal-processing literature contains many different estimators of the power spectrum in addition to the periodogram. In general, they differ from the periodogram in that they incorporate different types and degrees of a-priori physical or mathematical information about the original data set. The periodogram, in contrast, includes the maximum number of degrees of freedom and is always used for first-cut evaluation and analysis. Details of the correlation and other spectral estimation methods are discussed in the literature found in "References" at the end of this practice.

(2) *area or two-dimensional PSD of the surface height, micrometers-fourth power*—This quantity has the dimensions of μm^4, and is a function of the spatial frequencies in both the *x* and *y* directions on the surface, f_x and f_y, in units of inverse micrometers, μm^{-1}. It is defined as follows:

$$S_2(f_x, f_y) = \lim_{A \to \infty} \left\langle \frac{1}{A} \left| \int\int_A dx\, dy\; e^{i2\pi(f_x x + f_y y)} Z(x, y) \right|^2 \right\rangle,$$
$$-\infty < f_x, f_y < +\infty$$

3.25.1.4 *Discussion*—The spatial frequency ranges included in this definition cover the entire frequency plane and are not limited to positive frequencies only as in the case of the profile spectrum. In the case of an isotropically-rough surface the area spectrum is a function only of the magnitude of the two-dimensional frequency vector: $f = \sqrt{(f_x^2 + f_y^2)}$. The profile spectrum can be derived from the area spectrum, but the area spectrum cannot, in general, be derived from the profile spectrum. Uniaxial and isotropically-rough surfaces are exceptions.

3.25.1.5 *Discussion of units*—The surface height fluctuations of optical surfaces are usually measured in units of nanometers (1 nm = 10^{-3} µm), or the non-SI units of Ångstroms (1 Å = 10^{-4} µm). Values of the *PSD*s estimated using height data in these units can be converted to the recommended units by multiplying by the following conversion factors:

(1) To convert S_1 in units of nm ^{2}µm to units of μm^3 multiply it by 10^{-6},

(2) To convert S_1 in units of Å ^{2}µm to units of μm^3 multiply it by 10^{-8},

(3) To convert S_2 in units of nm ^{2}µm ^{2} to units of μm^4 multiply it by 10^{-6}, and

(4) To convert S_2 in units of Å ^{2}µm^2 to units of μm^4 multiply it by 10^{-8}.

If the sample interval is given in millimeters instead of micrometers, the conversion factors for S_1 should be multiplied by an additional factor of 10^3, and those for S_2 should be multiplied by an additional factor of 10^6.

3.26 *radius of curvature, $\hat{R}_x$, units of choice—The radius of a circle fitted to the measured surface profile.*

3.26.1 *Discussion*—When the radius us large relative to the profile length its magnitude is most easily determined from the quadratic term in the detrending polynomial. If the average surface profile is written as $Z(x) = a + bx + cx^2$, the estimate of the radius of curvature in the *x* direction is $\hat{R}_x = 1/(2c)$. If *Z* and *x* are expressed in micrometers, $\hat{R}_x$ will be in micrometers. Since the radii of curvature of nominally flat surfaces can be quite large, other reporting units, such as meters or kilometers, may be more appropriate.

3.27 *random roughness, random surface profile*—A surface height profile that involves parameters that are distributed according to statistical distribution laws rather than having fixed or deterministic values.

3.27.1 *Discussion*—For example, the profile $Z(x) = A \cos(2\pi f_x x + \phi)$ is deterministic if ϕ = const., but random if ϕ has a finite-width probability distribution function $P(\phi)$. Finish parameters and functions such as $Z(x)^2$, are then the values of those quantities averaged over $P(\phi)$.

3.28 *restoration*—The signal-processing procedure in which measurements are compensated for a non-unit measurement transfer function by passing them through a digital filter that restores the effective measurement function to unity over its bandpass.

3.28.1 *Discussion*—The measured profile can be restored and the statistics of the restored profile can then be estimated. The most common spatial- and frequency-domain filters used for this purpose are "inverse" and "Wiener" filters. This practice does not discuss the details of such restoration processes, which may be found in standard signal-processing texts such as those given in "References" at the end of this practice.

3.29 *RMS profile roughness, Rq, nanometers*—The square root of the mean-square profile roughness.

3.30 *RMS profile slope, Δq, units of convenience*—The square root of the mean-square profile slope.

3.30.1 *Discussion*—The slope is dimensionless, although the fundamental unit is the radian. In practice it may be convenient to express the rms slope of highly polished surfaces in microradians.