Designation: C 680 – 03$^{\epsilon 1}$

Standard Practice for Estimate of the Heat Gain or Loss and the Surface Temperatures of Insulated Flat, Cylindrical, and Spherical Systems by Use of Computer Programs[1]

This standard is issued under the fixed designation C 680; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

ϵ^1 NOTE—Footnote 3 was editorially added in October 2003.

1. Scope

1.1 This practice provides the algorithms and calculation methodologies for predicting the heat loss or gain and surface temperatures of certain thermal insulation systems that can attain one dimensional, steady- or quasi-steady-state heat transfer conditions in field operations.

1.2 This practice is based on the assumption that the thermal insulation systems can be well defined in rectangular, cylindrical or spherical coordinate systems and that the insulation systems are composed of homogeneous, uniformly dimensioned materials that reduce heat flow between two different temperature conditions.

1.3 Qualified personnel familiar with insulation-systems design and analysis should resolve the applicability of the methodologies to real systems. The range and quality of the physical and thermal property data of the materials comprising the thermal insulation system limit the calculation accuracy.

1.4 The computer program that can be generated from the algorithms and computational methodologies defined in this practice is described in Section 7 of this practice. The computer program is intended for flat slab, pipe and hollow sphere insulation systems. An executable version of a program based on this standard may be obtained from ASTM.

1.5 The values stated in inch-pound units are to be regarded as the standard. The values given in parentheses are for information only.

1.6 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

C 168 Terminology Relating to Thermal Insulating Materials[2]
C 177 Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Guarded Hot Plate Apparatus[2]
C 335 Test Method for Steady-State Heat Transfer Properties of Horizontal Pipe Insulation[2]
C 518 Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Heat Flow Meter Apparatus[2]
C 585 Practice for Inner and Outer Diameters of Rigid Thermal Insulation for Nominal Sizes of Pipe and Tubing (NPS System)[2]
C 1055 Guide for Heated System Surface Conditions That Produce Contact Burn Injuries[2]
C 1057 Practice for Determination of Skin Contact Temperature from Heated Surfaces Using a Mathematical Model and Thermesthesiometer[2]

3. Terminology

3.1 *Definitions*—For definitions of terms used in this practice, refer to Terminology C 168.

3.1.1 *thermal insulation system*—for this practice, a thermal insulation system is a system comprised of a single layer or layers of homogeneous, uniformly dimensioned material(s) intended for reduction of heat transfer between two different temperature conditions. Heat transfer in the system is steady-state. Heat flow for a flat system is normal to the flat surface, and heat flow for cylindrical and spherical systems is radial.

3.2 *Symbols*—The following symbols are used in the development of the equations for this practice. Other symbols will be introduced and defined in the detailed description of the development.

where:
h = surface conductance, Btu/(h·ft^2·°F) (W/(m^2·K)) h_i at inside surface; h_o at outside surface

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[1] This practice is under the jurisdiction of ASTM Committee C16 on Thermal Insulation and is the direct responsibility of Subcommittee C16.30 on Thermal Measurements.

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[2] *Annual Book of ASTM Standards*, Vol 04.06.

k = apparent thermal conductivity, Btu·in./(h·ft^2·°F) (W/(m·K))
k_e = effective thermal conductivity over a prescribed temperature range, Btu·in./(h·ft^2·°F) (W/(m·K))
q = heat flux, Btu/(h·ft^2) (W/m^2)
q_p = time rate of heat flow per unit length of pipe, Btu/(h·ft) (W/m)
R = thermal resistance, °F·h·ft^2/Btu (K·m^2/W)
r = radius, in. (m); $r_{m+1} - r_m$ = thickness
t = local temperature, °F (K)
t_i = inner surface temperature of the insulation, °F (K)
t_1 = inner surface temperature of the system
t_o = temperature of ambient fluid and surroundings, °F (K)
x = distance, in. (m); $x_{m+1} - x_m$ = thickness

4. Summary of Practice

4.1 The procedures used in this practice are based on standard, steady-state, one dimensional, conduction heat transfer theory as outlined in textbooks and handbooks. Heat flux solutions are derived for temperature dependent thermal conductivity in a material. Algorithms and computational methodologies for predicting heat loss or gain of single or multi-layer thermal insulation systems are provided by this practice for implementation in a computer program. In addition, interested parties can develop computer programs from the computational procedures for specific applications and for one or more of the three coordinate systems considered in Section 6.

4.1.1 The computer program combines functions of data input, analysis and data output into an easy to use, interactive computer program. By making the program interactive, little operator training is needed to perform accurate calculations.

4.2 The operation of the computer program follows the procedure listed below:

4.2.1 *Data Input*—The computer requests and the operator inputs information that describes the system and operating environment. The data includes:

4.2.1.1 Analysis identification.

4.2.1.2 Date.

4.2.1.3 Ambient temperature.

4.2.1.4 Surface conductance or ambient wind speed, system surface emittance and system orientation.

4.2.1.5 *System Description*—Material and thickness for each layer (define sequence from inside out).

4.2.2 *Analysis*—Once input data is entered, the program calculates the surface conductances (if not entered directly) and layer thermal resistances. The program then uses this information to calculate the heat transfer and surface temperature. The program continues to repeat the analysis using the previous temperature data to update the estimates of layer thermal resistance until the temperatures at each surface repeat within specified tolerances between the previous and present temperatures at the various surface locations in the system.

4.2.3 Once convergence of the temperatures is reached, the program prints a table that presents the input data, calculated thermal resistance of the system, heat flux and the inner surface and external surface temperatures.

5. Significance and Use

5.1 Manufacturers of thermal insulation express the performance of their products in charts and tables showing heat gain or loss per unit surface area or unit length of pipe. This data is presented for typical insulation thicknesses, operating temperatures, surface orientations (facing up, down, horizontal, vertical), and in the case of pipes, different pipe sizes. The exterior surface temperature of the insulation is often shown to provide information on personnel protection or surface condensation. However, additional information on effects of wind velocity, jacket emittance, ambient conditions and other influential parameters may also be required to properly select an insulation system. Due to the infinite combinations of size, temperature, humidity, thickness, jacket properties, surface emittance, orientation, and ambient conditions, it is not practical to publish data for each possible case.

5.2 Users of thermal insulation faced with the problem of designing large thermal insulation systems encounter substantial engineering cost to obtain the required information. This cost can be substantially reduced by the use of accurate engineering data tables, or available computer analysis tools, or both. The use of this practice by both manufacturers and users of thermal insulation will provide standardized engineering data of sufficient accuracy for predicting thermal insulation system performance. However, it is important to note that the accuracy of results is extremely dependent on the accuracy of the input data. Certain applications may need specific data to produce meaningful results.

5.3 The use of analysis procedures described in this practice can also apply to designed or existing systems. In the rectangular coordinate system, Practice C 680 can be applied to heat flows normal to flat, horizontal or vertical surfaces for all types of enclosures, such as boilers, furnaces, refrigerated chambers and building envelopes. In the cylindrical coordinate system, Practice C 680 can be applied to radial heat flows for all types of piping circuits. In the spherical coordinate system, Practice C 680 can be applied to radial heat flows to or from stored fluids such as liquefied natural gas (LNG).

5.4 Practice C 680 is referenced for use with Guide C 1055 and Practice C 1057 for burn hazard evaluation for heated surfaces. Infrared inspection, in-situ heat flux measurements, or both are often used in conjunction with Practice C 680 to evaluate insulation system performance and durability of operating systems. This type of analysis is often made prior to system upgrades or replacements.

5.5 All porous and non-porous solids of natural or man-made origin have temperature dependent thermal conductivities. The change in thermal conductivity with temperature is different for different materials, and for operation at a relatively small temperature difference, an average thermal conductivity may suffice. Thermal insulating materials (k < 0.85 {Btu·in}/{h·ft^2·°F}) are porous solids where the heat transfer modes include conduction in series and parallel flow through the matrix of solid and gaseous portions, radiant heat exchange between the surfaces of the pores or interstices, as well as transmission through non-opaque surfaces, and to a lesser extent, convection within and between the gaseous portions. With the existence of radiation and convection modes of heat transfer, the measured value should be called apparent thermal conductivity as described in Terminology C 168. The main reason for this is that the premise for pure heat conduction is no longer valid, because the other modes of heat transfer obey different laws. Also, phase change of a gas, liquid, or solid within a solid matrix or phase change by other mechanisms will provide abrupt changes in the temperature dependence of thermal conductivity. For example, the condensation of the gaseous portions of thermal insulation in extremely cold conditions will have an extremely influential effect on the apparent thermal conductivity of the insulation. With all of this considered, the use of a single value of thermal conductivity at an arithmetic mean temperature will provide less accurate predictions, especially when bridging temperature regions where strong temperature dependence occurs.

5.6 The calculation of surface temperature and heat loss or gain of an insulated system is mathematically complex, and because of the iterative nature of the method, computers best handle the calculation. Computers are readily available to most producers and consumers of thermal insulation to permit the use of this practice.

5.7 Computer programs are described in this practice as a guide for calculation of the heat loss or gain and surface temperatures of insulation systems. The range of application of these programs and the reliability of the output is a primary function of the range and quality of the input data. The programs are intended for use with an "interactive" terminal. Under this system, intermediate output guides the user to make programming adjustments to the input parameters as necessary. The computer controls the terminal interactively with program-generated instructions and questions, which prompts user response. This facilitates problem solution and increases the probability of successful computer runs.

5.8 The user of this practice may wish to modify the data input and report sections of the computer programs presented in this practice to fit individual needs. Also, additional calculations may be desired to include other data such as system costs or economic thickness. No conflict exists with such modifications as long as the user verifies the modifications using a series of test cases that cover the range for which the new method is to be used. For each test case, the results for heat flow and surface temperature must be identical (within resolution of the method) to those obtained using the practice described herein.

5.9 This practice has been prepared to provide input and output data that conforms to the system of units commonly used by United States industry. Although modification of the input/output routines could provide an SI equivalent of the heat flow results, no such "metric" equivalent is available for some portions of this practice. To date, there is no accepted system of metric dimensions for pipe and insulation systems for cylindrical shapes. The dimensions used in Europe are the SI equivalents of American sizes (based on Practice C 585), and each has a different designation in each country. Therefore, no SI version of the practice has been prepared, because a standard SI equivalent of this practice would be complex. When an international standard for piping and insulation sizing occurs, this practice can be rewritten to meet those needs. In addition, it has been demonstrated that this practice can be used to calculate heat transfer for circumstances other than insulated systems; however, these calculations are beyond the scope of this practice.

6. Method of Calculation

6.1 *Approach*:

6.1.1 The calculation of heat gain or loss and surface temperature requires: (*1*) The thermal insulation be homogeneous as outlined by the definition of thermal conductivity in Terminology C 168; (*2*) the system operating temperature be known; (*3*) the insulation thickness be known; (*4*) the surface conductances of the system be known, reasonably estimated or estimated from algorithms defined in this practice based on sufficient information; and, (*5*) the thermal conductivity as a function of temperature for each system layer be known in detail.

6.1.2 The solution is a procedure calling for (*1*) estimation of the system temperature distribution; (*2*) calculation of the thermal resistances throughout the system based on that distribution; (*3*) calculation of heat flux; and (*4*) reestimation of the system temperature distribution. The iterative process continues until a calculated distribution is in reasonable agreement with the previous distribution. The layer thermal resistance is calculated each time with the effective thermal conductivity being obtained by integration of the thermal conductivity curve for the layer being considered. This practice uses the temperature dependence of the thermal conductivity of any insulation or multiple layer combination of insulations to calculate heat flow.

6.2 *Development of Equations*—The development of the mathematical equations is for conduction heat transfer through homogeneous solids having temperature dependent thermal conductivities. To proceed with the development, several precepts or guidelines must be cited:

6.2.1 *Steady-state Heat Transfer*—For all the equations it is assumed that the temperature at any point or position in the solid is invariant with time. Thus, heat is transferred solely by temperature difference from point to point in the solid.

6.2.2 *One-dimensional Heat Transfer*—For all equations it is assumed there is heat flow in only one dimension of the particular coordinate system being considered. Heat transfer in the other dimensions of the particular coordinate system is considered to be zero.

6.2.3 *Conduction Heat Transfer*—The premise here is that the heat flux normal to any surface is directly proportional to the temperature gradient in the direction of heat flow, or

$$q = -k\frac{dt}{dp} \quad (1)$$

where the thermal conductivity, k, is the proportionality constant, and p is the space variable through which heat is flowing. For steady-state conditions, one-dimensional heat flow, and temperature dependent thermal conductivity, the equation becomes

$$q = -k(t)\frac{dt}{dp} \quad (2)$$

where at all surfaces normal to the heat flux, the total heat flow through these surfaces is the same and changes in the thermal conductivity must dictate changes in the temperature gradient. This will ensure that the total heat passing through a given surface does not change from that surface to the next.

6.2.4 *Solutions from Temperature Boundary Conditions*—The temperature boundary conditions on a uniformly thick, homogeneous mth layer material are:

$$t = t_m \quad \text{at} \quad x = x_m \quad (r = r_m); \qquad (3)$$
$$t = t_{m+1} \quad \text{at} \quad x = x_{m+1} \quad (r = r_m)$$

For heat flow in the flat slab, let $p = x$ and integrate Eq 2:

$$q \int_{x_m}^{x_{m+1}} dx = - \int_{t_m}^{t_{m+1}} k(t)dt \qquad (4)$$

$$q = k_{e,m} \frac{t_m - t_{m+1}}{x_{m+1} - x_m}$$

For heat flow in the hollow cylinder, let $p = r$, $q = Q/(2\pi rl)$ and integrate Eq 2:

$$\frac{Q}{2\pi l} \int_{r_m}^{r_{m+1}} \frac{dr}{r} = - \int_{t_m}^{t_{m+1}} k(t)dt \qquad (5)$$

$$Q = k_{e,m} \frac{t_m - t_{m+1}}{\ln(r_{m+1}/r_m)} 2\pi l$$

Divide both sides by $2\pi rl$

$$q = k_{e,m} \frac{t_m - t_{m+1}}{r \ln(r_{m+1}/r_m)}$$

For radial heat flow in the hollow sphere, let $p = r$, $q = Q/(4\pi r^2)$ and integrate Eq 2:

$$\frac{Q}{4\pi} \int_{r_m}^{r_{m+1}} \frac{dr}{r^2} = \int_{t_m}^{t_{m+1}} k(t)dt \qquad (6)$$

$$Q = k_{e,m} \frac{t_m - t_{m+1}}{\frac{1}{r_m} - \frac{1}{r_{m+1}}} 4\pi$$

Divide both sides by $4\pi r^2$ and multiply both sides by $r_m r_{m+1}/r_m r_{m+1}$

$$q = k_{e,m} \frac{r_m r_{m+1}}{r^2} \frac{t_m - t_{m+1}}{r_{m+1} - r_m}$$

Note that the effective thermal conductivity over the temperature range is:

$$k_{e,m} = \frac{\int_{t_m}^{t_{m+1}} k(t)dt}{t_{m+1} - t_m} \qquad (7)$$

6.3 *Case 1, Flat Slab Systems*:

6.3.1 From Eq 4, the temperature difference across the *m*th layer material is:

$$t_m - t_{m+1} = qR_m \qquad (8)$$

$$\text{where} \quad R_m = \frac{(x_{m+1} - x_m)}{k_{e,m}}$$

Note that R_m is defined as the thermal resistance of the *m*th layer of material. Also, for a thermal insulation system of *n* layers, $m = 1,2...n$, it is assumed that perfect contact exists between layers. This is essential so that continuity of temperature between layers can be assumed.

6.3.2 Heat is transferred between the inside and outside surfaces of the system and ambient fluids and surrounding surfaces by the relationships:

$$q = h_i(t_i - t_1) \qquad (9)$$
$$q = h_o(t_{n+1} - t_o)$$

where h_i and h_o are the inside and outside surface conductances. Methods for estimating these conductances are found in 6.7. Eq 9 can be rewritten as:

$$t_i - t_1 = qR_i \qquad (10)$$
$$t_{n+1} - t_o = qR_o$$
$$\text{where} \quad R_i = \frac{1}{h_i}, \quad R_o = \frac{1}{h_o}$$

For the computer programs, the inside surface conductance, h_i, can be assumed to be very large such that $R_i = 0$, and $t_1 = t_i$ is the given surface temperature.

6.3.3 Adding Eq 8 and Eq 10 yields the following equation:

$$t_i - t_o = q(R_1 + R_2 + ... + R_n + R_i + R_o) \qquad (11)$$

From the previous equation a value for q can be calculated from estimated values of the resistances, R. Then, by rewriting Eq 8 to the following:

$$t_{m+1} = t_m - qR_m \qquad (12)$$
$$t_1 = t_i - qR_i, \quad \text{for } R_i > 0$$

The temperature at the interface(s) and the outside surface can be calculated starting with $m = 1$. Next, from the calculated temperatures, values of $k_{e,m}$ (Eq 7) and R_m (Eq 8) can be calculated as well as R_o and R_i. Then, by substituting the calculated R-values back into Eq 11, a new value for q can be calculated. Finally, desired (correct) values can be obtained by repeating this calculation methodology until all values agree with previous values.

6.4 *Case 2, Cylindrical (Pipe) Systems*:

6.4.1 From Eq 5, the heat flux through any layer of material is referenced to the outer radius by the relationship:

$$q_n = q_m \frac{r}{r_{n+1}} = k_{e,m} \frac{t_m - t_{m+1}}{r_{n+1} \ln(r_{m+1}/r_m)} \qquad (13)$$

and, the temperature difference can be defined by Eq 8, where:

$$R_m = \frac{r_{n+1} \ln(r_{m+1}/r_m)}{k_{e,m}} \qquad (14)$$

Utilizing the methodology presented in case 1 (6.3), the heat flux, q_n, and the surface temperature, t_{n+1}, can be found by successive iterations. However, one should note that the definition of R_m found in Eq 14 must be substituted for the one presented in Eq 8.

6.4.2 For radial heat transfer in pipes, it is customary to define the heat flux in terms of the pipe length:

$$q_p = 2\pi r_{n+1} q_n \qquad (15)$$

where q_p is the time rate of heat flow per unit length of pipe. If one chooses not to do this, then heat flux based on the interior radius must be reported to avoid the influence of outer-diameter differences.

6.5 *Case 3, Spherical Systems*:

6.5.1 From Eq 6, the flux through any layer of material is referenced to the outer radius by the relationship:

$$q_n = q_m \frac{r^2}{r_{n+1}^2} = k_{e,m} \frac{r_m r_{m+1}(t_m - t_{m+1})}{r_{n+1}^2 (r_{m+1} - r_m)} \quad (16)$$

The temperature difference can be defined by Eq 8, where:

$$R_m = \frac{r_{n+1}^2 (r_{m+1} - r_m)}{k_{e,m} r_m r_{m+1}} \quad (17)$$

Again, utilizing the methodology presented in case 1 (6.3), the heat flux, q_n, and the surface temperature, t_{n+1}, can be found by successive iterations. However, one should note that the definition of R_m found in Eq 17 must be substituted for the one presented in Eq 8.

6.6 *Calculation of Effective Thermal Conductivity*:

6.6.1 In the calculational methodologies of 6.3, 6.4, and 6.5, it is necessary to evaluate $k_{e,m}$ as a function of the two surface temperatures of each layer comprising the thermal insulating system. This is accomplished by use of Eq 7 where *k(t)* is defined as a polynomial function or a piecewise continuous function comprised of individual, integrable functions over specific temperature ranges. It is important to note that temperature can either be in °F (°C) or absolute temperature, because the thermal conductivity versus temperature relationship is regression dependent. It is assumed for the programs in this practice that the user regresses the *k* versus *t* functions using °F.

6.6.1.1 When *k(t)* is defined as a polynomial function, such as $k(t) = a + bt + ct^2 + dt^3$, the expression for the effective thermal conductivity is:

$$k_{e,m} = \frac{\int_{t_m}^{t_{m+1}} (a + bt + ct^2 + dt^3)dt}{(t_{m+1} - t_m)} \quad (18)$$

$$k_{e,m} = \frac{a(t_{m+1} - t_m) + \frac{b}{2}(t_{m+1}^2 - t_m^2) + \frac{c}{3}(t_{m+1}^3 - t_m^3) + \frac{d}{4}(t_{m+1}^4 - t_m^4)}{(t_{m+1} - t_m)}$$

$$k_{e,m} = a + \frac{b}{2}(t_m + t_{m+1}) + \frac{c}{3}(t_m^2 + t_m t_{m+1} + t_{m+1}^2) + \frac{d}{4}(t_m^3 + t_m^2 t_{m+1} + t_m t_{m+1}^2 + t_{m+1}^3)$$

It should be noted here that for the linear case, $c = d = 0$, and for the quadratic case, $d = 0$.

6.6.1.2 When *k(t)* is defined as an exponential function, such as $k(t) = e^{a+bt}$, the expression for the effective thermal conductivity is:

$$k_{e,m} = \frac{\int_{t_m}^{t_{m+1}} e^{a+bt}dt}{(t_{m+1} - t_m)} \quad (19)$$

$$k_{e,m} = \frac{\frac{1}{b}(e^{a+bt_{m+1}} - e^{a+bt_m})}{(t_{m+1} - t_m)}$$

$$k_{e,m} = \frac{(e^{a+bt_{m+1}} - e^{a+bt_m})}{b(t_{m+1} - t_m)}$$

6.6.1.3 The piece-wise continuous function may be defined as:

$$\begin{aligned} k(t) &= k_1(t) \qquad t_{bl} \le t \le t_l \\ &= k_2(t) \qquad t_l \le t \le t_u \qquad t_{bl} \le t_m \text{ and } t_{m+1} \le t_{bu} \\ &= k_3(t) \qquad t_u \le t \le t_{bu} \end{aligned} \quad (20)$$

where t_{bl} and t_{bu} are the experimental lower and upper boundaries for the function. Also, each function is integrable, and $k_1(t_l) = k_2(t_l)$ and $k_2(t_u) = k_3(t_u)$. In terms of the effective thermal conductivity, some items must be considered before performing the integration in Eq 8. First, it is necessary to determine if t_{m+1} is greater than or equal to t_m. Next, it is necessary to determine which temperature range t_m and t_{m+1} fit into. Once these two parameters are decided, the effective thermal conductivity can be determined using simple calculus. For example, if $t_{bl} \le t_m \le t_l$ and $t_u \le t_{m+1} \le t_{bu}$ then the effective thermal conductivity would be:

$$k_{e,m} = \frac{\int_{t_m}^{T_l} k_1(t)dt + \int_{T_l}^{T_u} k_2(t) + \int_{T_u}^{t_{m+1}} k_3(t)}{(t_{m+1} - t_m)} \quad (21)$$

It should be noted that other piece-wise functions exist, but for brevity, the previous is the only function presented.

6.6.2 It should also be noted that when the relationship of *k* with *t* is more complex and does not lend itself to simple mathematical treatment, a numerical method might be used. It is in these cases that the power of the computer is particularly useful. There are a wide variety of numerical techniques available. The most suitable will depend of the particular situation, and the details of the factors affecting the choice are beyond the scope of this practice.

6.7 *Surface Conductances*:

6.7.1 The surface conductance, *h*, as defined in Terminology C 168, assumes that the principal surface is at a uniform temperature and that the ambient fluid and other visible surfaces are at a different uniform temperature. The conductance includes the combined effects of radiant, convective, and conductive heat transfer. The conductance is defined by:

$$h = h_r + h_c \quad (22)$$

where h_r is the component due to radiation and h_c is the component due to convection and conduction. In subsequent sections, algorithms for these components will be presented.

6.7.1.1 The algorithms presented in this practice for calculating surface conductances are used in the computer program; however, surface conductances may be estimated from published values or separately calculated from algorithms other than the ones presented in this practice. One special note, care must be exercised at low or high surface temperatures to ensure reasonable values.

6.7.2 *Radiant Heat Transfer Conductance*—The radiation conductance is simply based on radiant heat transfer and is calculated from the Stefan-Bolzmann Law divided by the average difference between the surface temperature and the air temperature. In other words:

$$h_r = \frac{\sigma\epsilon\,(T_s^4 - T_o^4)}{T_s - T_o} \quad \text{or} \quad (23)$$

$$h_r = \sigma\epsilon \cdot (T_s^3 + T_s^2 T_o + T_s T_o^2 + T_o^3) \quad \text{or}$$

$$h_r = \sigma\epsilon \cdot 4T_m^3 \left[1 + \left(\frac{T_s - T_o}{T_s + T_o}\right)^2\right]$$

where:

ϵ = effective surface emittance between outside surface and the ambient surroundings, dimensionless,
σ = Stefan-Boltzman constant, 0.1714×10^{-8} Btu/(h·ft^2·°R^4) (5.6697×10^{-8} W/(m^2·K^4)),
T_s = absolute surface temperature, °R (K),
T_o = absolute surroundings (ambient air if assumed the same) temperature, °R (K), and
T_m = $(T_s + T_o)/2$

6.7.3 *Convective Heat Transfer Conductance*—Certain conditions need to be identified for proper calculation of this component. The conditions are: (*a*) Surface geometry—plane, cylinder or sphere; (*b*) Surface orientation—from vertical to horizontal including flow dependency; (*c*) Nature of heat transfer in fluid—from free (natural) convection to forced convection with variation in the direction and magnitude of fluid flow; (*d*) Condition of the surface—from smooth to various degrees of roughness (primarily a concern for forced convection).

6.7.3.1 Modern correlation of the surface conductances are presented in terms of dimensionless groups, which are defined for fluids in contact with solid surfaces. These groups are:

$$\text{Nusselt,}\quad \overline{Nu}_L = \frac{\bar{h}_c L}{k_f} \quad \text{or} \quad \overline{Nu}_D = \frac{\bar{h}_c D}{k_f} \tag{24}$$

$$\text{Rayleigh,}\quad Ra_L = \frac{g \cdot \beta \cdot \rho \cdot c_p(\Delta T)L^3}{\nu \cdot k_f} \quad \text{or} \quad Ra_D = \frac{g \cdot \beta \cdot \rho \cdot c_p(\Delta T)D^3}{\nu \cdot k_f} \tag{25}$$

$$\text{Reynolds,}\quad Re_L = \frac{VL}{\nu} \quad \text{or} \quad Re_D = \frac{VD}{\nu} \tag{26}$$

$$\text{Prandtl,}\quad Pr = \frac{\nu \cdot \rho \cdot c_p}{k_f} \tag{27}$$

where:
L = characteristic dimension for horizontal and vertical flat surfaces, and vertical cylinders,
D = characteristic dimension for horizontal cylinders and spheres,
c_p = specific heat of ambient fluid, Btu/(lb·°R) (J/(kg·K)),
$\bar{h}_c$ = average convection conductance, Btu/(h·ft^2·°F) (W/(m^2·K)),
k_f = thermal conductivity of ambient fluid, Btu/(h·ft·°F) (W/(m·K)),
V = free stream velocity of ambient fluid, ft/h (m/s),
ν = kinematic viscosity of ambient fluid, ft^2/h (m^2/s),
g = acceleration due to gravity, ft/h^2 (m/s^2),
β = volumetric thermal expansion coefficient of ambient fluid, °R^{-1}(K^{-1}),
ρ = density of ambient fluid, lb/ft^3 (kg/m^3), and
ΔT = absolute value of temperature difference between surface and ambient fluid, °R (K).

It needs to be noted here that (except for spheres–forced convection) the above fluid properties must be calculated at the film temperature, T_f, which is the average of surface and ambient fluid temperatures. For this practice, it is assumed that the ambient fluid is dry air at atmospheric pressure. The properties of air can be found in references such as *Tables of Thermodynamic and Transport Properties of Air*. in *NBS Circular 564*, U.S. Dept. of Commerce, by Hilsenrath, et al. This reference contains equations for some of the properties and polynomial fits for others, and the equations are summarized in Table A1.1.

6.7.3.2 When a heated surface is exposed to flowing fluid, the convective heat transfer will be a combination of forced and free convection. For this mixed convection condition, Churchill **(26)** recommends the following equation. For each geometric shape and surface orientation the overall average Nusselt number is to be computed from the average Nusselt number for forced convection and the average Nusselt number for natural convection. The film conductance, h, is then computed from Eq 24. The relationship is:

$$(\overline{Nu} - \delta)^j = (\overline{Nu}_f - \delta)^j + (\overline{Nu}_n - \delta)^j \tag{28}$$

where the exponent, j, and the constant, δ, are defined based on the geometry and orientation.

6.7.4 *Convection Conductances for Flat Surfaces*:

6.7.4.1 From *Heat Transfer* by Churchill and Ozoe as cited in *Fundamentals of Heat and Mass Transfer* (p. 354) by Incropera and Dewitt, the relation for forced convection by laminar flow over an isothermal flat surface is:

$$\overline{Nu}_{f,L} = \frac{0.6774\, Re_L^{1/2} Pr^{1/3}}{[1 + (0.0468 / Pr)^{2/3}]^{1/4}} \qquad Re_L < 5 \times 10^5 \tag{29}$$

For forced convection by turbulent flow over an isothermal flat surface, Incropera and Dewitt (p. 356) suggest the following:

$$\overline{Nu}_{f,L} = (0.037\, Re_L^{4/5} - 871)\, Pr^{1/3} \qquad 5 \times 10^5 < Re_L < 10^8 \tag{30}$$

It should be noted that the upper bound for Re_L is an approximate value, and the user of the above equation must be aware of this.

6.7.4.2 In "Correlating Equations for Laminar and Turbulent Free Convection from a Vertical Plate" by Churchill and Chu, as cited by Incropera and Dewitt (p. 493), it is suggested for natural convection on isothermal, vertical flat surfaces that:

$$\overline{Nu}_{n,L} = \left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{[1 + (0.492 / Pr)^{9/16}]^{8/27}}\right\}^2 \qquad \text{All } Ra_L \tag{31}$$

For slightly better accuracy in the laminar range, it is suggested by the same source (p. 493) that:

$$\overline{Nu}_{n,L} = 0.68 + \frac{0.670\, Ra_L^{1/4}}{[1 + (0.492 / Pr)^{9/16}]^{4/9}} \qquad Ra_L < 10^9 \tag{32}$$

In the case of vertical surfaces the characteristic dimension is the vertical height. To compute the overall Nusselt number (Eq 28), set $j = 3$ and $\delta = 0$. Also, it is important to note that the free convection correlations apply to vertical cylinders in most cases.

6.7.4.3 For natural convection on horizontal flat surfaces, Incropera and Dewitt (p. 498) cite *Heat Transmission* by McAdams, "Natural Convection Mass Transfer Adjacent to Horizontal Plates" by Goldstein, Sparrow and Jones, and "Natural Convection Adjacent to Horizontal Surfaces of Various Platforms" for the following correlations:

$$\text{Heat flow up:}\quad \overline{Nu}_{n,L} = 0.54\, Ra_L^{1/4} \qquad 10^4 < Ra_L < 10^7 \tag{33}$$

$$\overline{Nu}_{n,L} = 0.15\, Ra_L^{1/3} \qquad 10^7 < Ra_L < 10^{11}$$