INTERNATIONAL STANDARD

ISO 11929-7

First edition
2005-02-15

Determination of the detection limit and decision threshold for ionizing radiation measurements —

Part 7:
Fundamentals and general applications

Détermination de la limite de détection et du seuil de décision des mesurages de rayonnements ionisants —

Partie 7: Principes fondamentaux et leurs applications générales

Reference number
ISO 11929-7:2005(E)

ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.org
Web www.iso.org

Published in Switzerland

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 11929-7 was prepared by Technical Committee ISO/TC 85, *Nuclear energy*, Subcommittee SC 2, *Radiation protection*.

ISO 11929 consists of the following parts, under the general title *Determination of the detection limit and decision threshold for ionizing radiation measurements*:

— *Part 1: Fundamentals and application to counting measurements without the influence of sample treatment*

— *Part 2: Fundamentals and application to counting measurements with the influence of sample treatment*

— *Part 3: Fundamentals and application to counting measurements with high resolution gamma spectrometry without the influence of sample treatment*

— *Part 4: Fundamentals and application to measurements by use of linear-scale analogue ratemeters, without the influence of sample treatment*

— *Part 5: Fundamentals and applications to counting measurements on filters during accumulation of radioactive material*

— *Part 6: Fundamentals and applications to measurements by use of transient mode*

— *Part 7: Fundamentals and general applications*

— *Part 8: Fundamentals and applications to unfolding of spectrometric measurements without the influence of sample treatment*

Introduction

This part of ISO 11929 gives basic information on the statistical principles for the determination of the detection limit, of the decision threshold and of the limits of the confidence interval for general applications of nuclear radiation measurements.

ISO 11929-1 and ISO 11929-2, respectively, deal with integral counting measurements with or without consideration of the sample treatment. High-resolution spectrometric measurements are covered in ISO 11929-3. ISO 11929-4 deals with measurements using linear-scale analogue ratemeters, ISO 11929-5 with monitoring of the concentration of aerosols in exhaust gas, air or waste water, ISO 11929-6 with measurements by use of a transient measuring mode, and ISO 11929-8 with unfolding of spectrometric measurements.

Whereas the earlier parts 1 to 4 were elaborated for special measuring tasks in nuclear radiation measurements based on the principles defined by Altschuler and Pasternack [1], Nicholson [2], Currie [3], this restriction does not apply to this part, or to part 5, part 6 and part 8. The determination of the characteristic limits mentioned above is separated from the evaluation of the measurement. Consequently, this part of ISO 11929 is generally applicable and can be applied to any suitable procedure for the evaluation of a measurement. Since the uncertainty of measurement plays a fundamental role in this part of ISO 11929, evaluations of measurements and the determination of the uncertainties of measurement have to be performed according to the Guide for the Expression of Uncertainty in Measurement.

This part, as well as parts 5, 6 and 8, of ISO 11929 is based on methods of Bayesian statistics (see [4] to [6] in the Bibliography) in order to be able to account also for such uncertain quantities and influences which do not behave randomly in repeated or counting measurements.

For this purpose, Bayesian statistical methods are used to specify the following statistical values, called “characteristic limits”:

— the *decision threshold*, which allows a decision to be made for a measurement with a given probability of error as to whether the result of the measurement indicates the presence of the physical effect quantified by the measurand.

— the *detection limit*, which specifies the minimum true value of the measurand which can be detected with a given probability of error using the measuring procedure in question. This consequently allows a decision to be made as to whether or not a measuring method checked using this part of ISO 11929 satisfies certain requirements and is consequently suitable for the given purpose of measurement.

— the *limits of the confidence interval*, which define an interval which contains the true value of the measurand with a given probability, in the case that the result of the measurement exceeds the decision threshold.

Determination of the detection limit and decision threshold for ionizing radiation measurements —

Part 7: Fundamentals and general applications

1 Scope

This part of ISO 11929 specifies suitable statistical values which allow an assessment of the detection capabilities in ionizing radiation measurements and of the physical effect quantified by the measurand. For this purpose, Bayesian statistical methods are used to specify characteristic limits.

This part of ISO 11929 deals with fundamentals and general applications.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 11929-1:2000, *Determination of the detection limit and decision threshold for ionizing radiation measurements — Part 1: Fundamentals and application to counting measurements without the influence of sample treatment*

ISO 11929-2:2000, *Determination of the detection limit and decision threshold for ionizing radiation measurements — Part 2: Fundamentals and application to counting measurements with the influence of sample treatment*

BIPM/IEC/IFCC/ISO/IUPAC/IUPAP/OIML, *Guide to the Expression of Uncertainty in Measurement,* Geneva, 1993

BIPM/IEC/IFCC/ISO/IUPAC/IUPAP/OIML, *International Vocabulary of Basic and General Terms in Metrology,* 2nd edition, Geneva, 1993

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1
measuring method
any logical sequence of operations, described generically, used in the performance of measurements

NOTE Adapted from the International Vocabulary of Basic and General Terms in Metrology:1993.

3.2
measurand
particular quantity subject to measurement

[International Vocabulary of Basic and General Terms in Metrology:1993]

NOTE In this part of ISO 11929, a measurand is non-negative and quantifies a nuclear radiation effect. The effect is not present if the value of the measurand is zero. Examples for a measurand are the net count rate of a sample of radioactive material, the net activity of a sample of radioactive material given the activity of a blank sample, the increase of the specific activity or activity concentration of a gas flow, or the intensity of a line in a spectrum above the background in a spectrometric measurement.

3.3
uncertainty (of measurement)
parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand

[Guide for the Expression of Uncertainty in Measurement:1993]

NOTE The uncertainty of measurement defined in the Guide for the Expression of Uncertainty in Measurement comprises, in general, many components. Some of these components may be evaluated from the results of series of measurements or counting measurements and can be characterized by experimental standard deviations. The other components, which also can be characterized by standard deviations, are evaluated from assumed or known probability distributions based on experience and other information.

3.4
mathematical model of the evaluation
a set of mathematical relationships between all measured and other quantities involved in the evaluation of measurements

3.5
decision quantity
random variable for the decision whether or not the physical effect to be measured is present

3.6
decision threshold
fixed value of the decision quantity by which, when exceeded by the result of an actual measurement of a measurand quantifying a physical effect, one decides that the physical effect is present

NOTE The decision threshold is the critical value of a statistical test to decide between the hypothesis that the physical effect is not present and the alternative hypothesis that it is present. When the critical value is exceeded by the result of an actual measurement, this is taken to indicate that the hypothesis should be rejected. The statistical test will be designed such that the probability of wrongly rejecting the hypothesis (error of the first kind) is equal at most to a given value α.

3.7
detection limit
smallest true value of the measurand which is detectable by the measuring method

NOTE 1 The detection limit is the smallest true value of the measurand which is associated with the statistical test and hypotheses according to 3.6 by the following characteristics: if in reality the true value is equal to or exceeds the detection limit, the probability of wrongly not rejecting the hypothesis (error of the second kind) will be at most equal to a given value β.

NOTE 2 The difference between using the decision threshold and using the detection limit is that measured values are to be compared with the decision threshold, whereas the detection limit is to be compared with the guideline value.

3.8
confidence limits
values which define a confidence interval to be specified for the measurand in question which, if the result exceeds the decision threshold, includes the true value of the measurand with the given probability $(1 - \gamma)$

3.9
guideline value
value which corresponds to scientific, legal or other requirements for which the measuring procedure is intended to assess

EXAMPLE Activity, specific activity or activity concentration, surface activity, or dose rate.

4 Quantities and symbols

$\hat{\xi}$ Random variable as an estimator for a non-negative measurand quantifying the physical effect in question

ξ Value of the estimator $\hat{\xi}$ of the measurand; true value of the measurand

X Random variable as decision quantity; estimator of the measurand

x Primary result of a measurement of the measurand; obtained value of the decision quantity X; primary estimate of the measurand

$u(x)$ Standard uncertainty of the measurand associated with the primary result x of a measurement

$\tilde{u}(\xi)$ Standard uncertainty of the decision quantity X as a function of the true value ξ of the measurand

z Best estimate of the measurand

$u(z)$ Standard uncertainty associated with the best estimate z of the measurand

x^* Decision threshold for the measurand

ξ^* Detection limit for the measurand ξ_l, ξ_u, respectively, the lower and upper limit of the confidence interval for the measurand

α Probability of the error of the first kind; the probability of rejecting a hypothesis if it is true

β Probability of the error of the second kind; the probability of accepting a hypothesis if it is false

$1 - \gamma$ Probability attributed to the confidence interval of the measurand; probability that the true value of the measurand is included by the confidence interval

k_p Quantiles of the standardized normal distribution for the probability p (see Table 1); ($p = 1 - \alpha$), $(1 - \beta)$, $(1 - \gamma/2)$

E Operator for the formation of the expectation value of a random variable

Var Operator for the formation of the variance of a random variable

Φ Distribution function of the standardized normal (Gaussian) distribution

5 Statistical values and confidence interval

5.1 General aspects

For a particular task involving nuclear radiation measurements, first the particular physical effect which is the objective of the measurement has to be described. Then a non-negative measurand has to be defined which quantifies the physical effect and which assumes the value zero if the effect is not present in an actual case.

A random variable, called a decision quantity X has to be attributed to the measurand. It is also an estimator of the measurand. It is required that the expectation value $\mathrm{E}X$ of the decision quantity X equals the true value ξ of the measurand. A value x of the estimator X derived from measurements is a primary estimate of the measurand. The primary estimate x of the measurand, and its associated standard uncertainty $u(x)$, have to be calculated as the primary complete result of the measurement according to the Guide for the expression of uncertainty in measurement, by evaluation of measured quantities and of other information using a mathematical model of the evaluation which takes into account all relevant quantities. Generally, the fact that the measurand is non-negative will not be explicitly made use of. Therefore, x may become negative, in particular, if the true value of the measurand is close to zero.

NOTE The model of the evaluation of the measurement need not necessarily be given in the form of explicit mathematical formulas. It can also be represented by an algorithm or a computer code (see A.2).

For the determination of the decision threshold and the detection limit, the standard uncertainty of the decision quantity has to be calculated, if possible, as a function $\tilde{u}(\xi)$ of the true value ξ of the measurand. In the case that this is not possible, approximate solutions are described below.

ξ is the value of another, non-negative estimator $\hat{\xi}$ of the measurand. The estimator $\hat{\xi}$, in contrast to X, makes use of the knowledge that the measurand is non-negative. The limits of the confidence interval to be determined refer to this estimator $\hat{\xi}$ (compare 5.4). Besides the limits of the confidence interval, the expectation value $\mathrm{E}\hat{\xi}$ of this estimator as a best estimate z of the measurand, and the standard deviation $[\mathrm{Var}(\hat{\xi})]^{1/2}$ as the standard uncertainty $u(z)$ associated with the best estimate z of the measurand have to be calculated (see 6.3).

For the numerical calculation of the decision threshold and the detection limit, the function $\tilde{u}(\xi)$ is needed, which is the standard uncertainty of the decision quantity X as a function of the true value ξ of the measurand. The function $\tilde{u}(\xi)$ generally has to be determined by the user of this part of ISO 11929, in the course of the evaluation of the measurement according to the Guide for the Expression of Uncertainty in Measurement. For examples see Annex A. This function is often only slowly increasing. Therefore, it is justified in many cases to use the approximation $\tilde{u}(\xi) = u(x)$. This applies, in particular, if the primary estimate x of the measurand is not much larger than its standard uncertainty $u(x)$ associated with x. If the value x is calculated as the difference (net effect) of two approximately equal values y_1 and y_0 obtained from independent measurements, that is $x = y_1 - y_0$, one gets $\tilde{u}^2(\xi) = u^2(y_1) + u^2(y_0)$ with the standard uncertainties $u(y_1)$ and $u(y_0)$ associated with y_1 and y_0, respectively.

If only $\tilde{u}(0)$ and $u(x)$ are known, an approximation by linear interpolation is often sufficient for $x > 0$ according to:

$$\tilde{u}^2(\xi) = \tilde{u}^2(0) \cdot (1 - \xi / x) + u^2(x) \cdot \xi / x \tag{1}$$

NOTE In many practical cases, $\tilde{u}^2(\xi)$ is a slowly increasing linear function of ξ. This justifies the approximations above, in particular, the linear interpolation of $\tilde{u}^2(\xi)$ instead of $\tilde{u}(\xi)$ itself.

5.2 Decision threshold

The decision threshold x^* of a non-negative measurand quantifying the physical effect, according to 5.1, is a value of the decision quantity X which, when it is exceeded by a result x of a measurement, indicates that the physical effect is present. If $x \leqslant x^*$ one decides that the physical effect is not present. If this decision rule is observed, a wrong decision in favour of the presence of the physical effect occurs with a probability not greater than α (error of the first kind).

The decision threshold is given by:

$$x^{*} = k_{1-\alpha} \cdot \tilde{u}(0) \tag{2}$$

Values of the quantiles $k_{1-\alpha}$ of the standardized normal distribution are given in Table 1. It is $\Phi(k_{1-\alpha}) = 1 - \alpha$.

If the approximation $\tilde{u}(\xi) = u(x)$ is sufficient, one gets $x^{*} = k_{1-\alpha} \cdot u(x)$.

5.3 Detection limit

The detection limit ξ^{*}, which is the smallest true value of the measurand detectable with the measuring method, is so much larger than the decision threshold that the probability of an error of the second kind is not greater than β. The detection limit is given by:

$$\xi^{*} = x^{*} + k_{1-\beta} \cdot \tilde{u}(\xi^{*}) \tag{3}$$

Equation (3) is an implicit one. The detection limit can be calculated from it by iteration using, for example, the starting approximation $\xi^{*} = 2x^{*}$. The iteration converges in most cases. Equation (3) may have multiple solutions. In this case the detection limit is the smallest one. If Equation (3) has no solution, the measuring procedure is not suited for the measuring purpose.

If the approximation $\tilde{u}(\xi) = u(x)$ is sufficient, then $\xi^{*} = (k_{1-\alpha} + k_{1-\beta}) \cdot u(x)$ is valid. If $\tilde{u}(\xi)$ is not explicitly known for $\xi > 0$, one gets with $\tilde{u}(0)$ and with a result x of a measurement and its associated uncertainty $u(x)$, an approximation of ξ^{*} using the interpolation formula according to Equation (1):

$$\xi^{*} = a + \sqrt{a^{2} + (k_{1-\beta}^{2} - k_{1-\alpha}^{2}) \cdot \tilde{u}^{2}(0)} \quad \text{with} \quad a = k_{1-\alpha} \cdot \tilde{u}(0) + \frac{1}{2}(k_{1-\alpha}^{2} / x) \cdot [u^{2}(x) - \tilde{u}^{2}(0)] \tag{4}$$

For $\alpha = \beta$ one obtains $\xi^{*} = 2a$.

When using the approximation of Equation (4) to calculate the detection limit ξ^{*} and when type B uncertainties are not negligible, a measurement result $x > \approx 2x^{*}$ shall be chosen. If $x \gg 2x^{*}$ holds, one obtains an unreasonably high detection limit. In this case, the approximation yields only an upper limit of ξ^{*}. If type B uncertainties are negligible, Equations (3) and (4) converge to the same result for the detection limit.

Values of the quantiles $k_{1-\alpha}$, $k_{1-\beta}$, of the standardized normal distribution are given in Table 1. It is $\Phi(k_{1-\alpha}) = 1 - \alpha$ and $\Phi(k_{1-\beta}) = 1 - \beta$.

5.4 Confidence limits

For a result x of a measurement which exceeds the decision threshold x^{*}, the confidence interval includes the true value of the measurand with the given probability $(1 - \gamma)$. It is enclosed by the confidence limits ξ_{l} and ξ_{u} according to:

$$\xi_{l} = x - k_{p} \cdot u(x) \text{ with } p = \kappa \cdot (1 - \gamma / 2) \tag{5}$$

$$\xi_{u} = x + k_{p} \cdot u(x) \text{ with } q = 1 - (\kappa \cdot \gamma / 2) \tag{6}$$