TECHNICAL REPORT

ISO/TR 11146-3

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Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios —

Part 3: Intrinsic and geometrical laser beam classification, propagation and details of test methods

Lasers et équipements associés aux lasers — Méthodes d'essai des largeurs du faisceau, des angles de divergence et des facteurs de propagation du faisceau —

Partie 3: Classification intrinsèque et géométrique du faisceau laser, propagation et détails des méthodes d'essai

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Foreword

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International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

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ISO/TR 11146-3 was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 9, *Electro-optical systems*.

This first edition of ISO/TR 11146-3, together with ISO 11146-1, cancels and replaces ISO 11146:1999, which has been technically revised.

ISO 11146 consists of the following parts, under the general title *Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios*:

— *Part 1: Stigmatic and simple astigmatic beams*

— *Part 2: General astigmatic beams*

— *Part 3: Intrinsic and geometrical laser beam classification, propagation and details of test methods* (Technical Report)

Introduction

The propagation properties of every laser beam can be characterized within the method of second-order moments by ten independent parameters. However, most laser beams of practical interest need less parameters for a complete description due to their higher symmetry. These beams are stigmatic or simple astigmatic, e.g. due to the used resonator design.

The theoretical description of beam characterization and propagation as well as the classification of laser beams based on the second-order moments of the Wigner distribution is given in this part of ISO 11146.

The measurement procedures introduced in ISO 11146-1 and ISO 11146-2 are essentially based on (but not restricted to) the acquisition of power (energy) density distributions by means of matrix detectors, as for example CCD cameras. The accuracy of results based on these data depends strongly on proper data pre-processing, namely background subtraction and offset correction. The details of these procedures are given here.

In some situations accuracy obtainable with matrix detectors might not be satisfying or matrix detectors might simply be unavailable. In such cases, other, indirect methods for the determination of beam diameters or beam width are viable alternatives, as long as comparable results are achieved. Some alternative measurement methods are presented here.

Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios —

Part 3: Intrinsic and geometrical laser beam classification, propagation and details of test methods

1 Scope

This part of ISO 11146 specifies methods for measuring beam widths (diameter), divergence angles and beam propagation ratios of laser beams in support of ISO 11146-1. It provides the theoretical description of laser beam characterization based on the second-order moments of the Wigner distribution, including geometrical and intrinsic beam characterization, and offers important details for proper background subtraction methods recommendable for matrix detectors such as CCD cameras. It also presents alternative methods for the characterization of stigmatic or simple astigmatic beams that are applicable where matrix detectors are unavailable or deliver unsatisfying results.

2 Second-order laser beam characterization

2.1 General

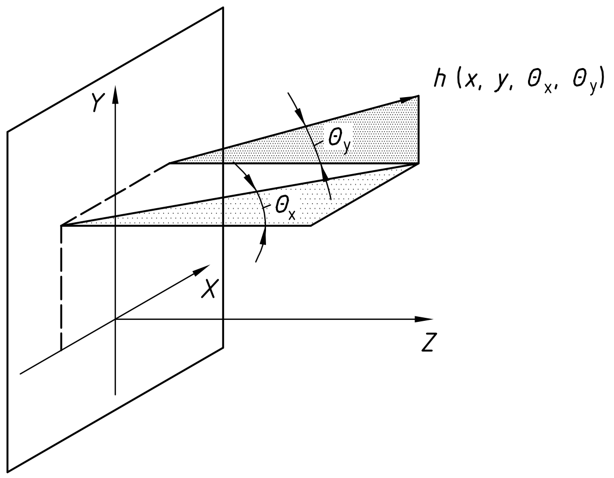
Almost any coherent or partially coherent laser beam can be characterized by a maximum of ten independent parameters, the so-called second-order moments of the Wigner distribution. Laser beams showing some kind of symmetry, stigmatism or simple astigmatism, need even fewer parameters. The knowledge of these parameters allows the prediction of beam properties behind arbitrary aberration-free optical systems.

Here and throughout this document the term “power density distribution $E(x,y,z)$” refers to continuous wave sources. It might be replaced by “energy density distribution $H(x,y,z)$” in the case of pulsed sources. Furthermore, a coordinate system is assumed where the z axis is almost parallel to the direction of beam propagation and the x and y axes are horizontal and vertical, respectively.

2.2 Wigner distribution

The Wigner distribution $h(x,y,\Theta_x,\Theta_y;z)$ is a general and complete description of narrow-band coherent and partially coherent laser beams in a measurement plane. Generally speaking, it gives the amount of beam power of a beam passing the measurement plane at the lateral position (x,y) with a horizontal paraxial angle of Θ_x and a vertical paraxial angle of Θ_y to the z axis, as shown in Figure 1.

NOTE The Wigner distribution is a function of the axial location z, i.e. the Wigner distribution of the same beam is different at different z locations. Hence, quantities derived from the Wigner distribution are in general also functions of z. Throughout this document this z dependence will be dropped. The Wigner distribution then refers to an arbitrarily chosen location z, the measurement plane.



x,y spatial coordinates

Θ_x, Θ_y corresponding angular coordinates

Figure 1 — Coordinates of Wigner distribution

The power density distribution $E(x,y)$ in a measurement plane is related to the Wigner distribution by

$$E(x,y) = \int_{-\infty}^{\infty} h(x,y,\Theta_x,\Theta_y)\,\mathrm{d}\Theta_x\mathrm{d}\Theta_y \tag{1}$$

NOTE The integration limits in the equation above are finite, representing the maximum angles of the rays contained in the beam, in paraxial; they are conventionally extended to infinity.

2.3 First- and second-order moments of Wigner distribution

The first-order moments of the Wigner distribution are defined as

$$\langle x\rangle = \frac{1}{P}\int h(x,y,\Theta_x,\Theta_y)\,x\,\mathrm{d}x\mathrm{d}y\mathrm{d}\Theta_x\mathrm{d}\Theta_y \tag{2}$$

$$\langle y\rangle = \frac{1}{P}\int h(x,y,\Theta_x,\Theta_y)\,y\,\mathrm{d}x\mathrm{d}y\mathrm{d}\Theta_x\mathrm{d}\Theta_y \tag{3}$$

$$\langle \Theta_x\rangle = \frac{1}{P}\int h(x,y,\Theta_x,\Theta_y)\,\Theta_x\,\mathrm{d}x\mathrm{d}y\mathrm{d}\Theta_x\mathrm{d}\Theta_y \tag{4}$$

$$\langle \Theta_y\rangle = \frac{1}{P}\int h(x,y,\Theta_x,\Theta_y)\,\Theta_y\,\mathrm{d}x\mathrm{d}y\mathrm{d}\Theta_x\mathrm{d}\Theta_y \tag{5}$$

where P is the beam power given by

$$P = \int h(x,y,\Theta_x,\Theta_y)\,\mathrm{d}x\mathrm{d}y\mathrm{d}\Theta_x\mathrm{d}\Theta_y \tag{6}$$

or, using Equation (1),

$$P = \int E(x,y)\,\mathrm{d}x\mathrm{d}y \tag{7}$$

The spatial moments $\langle x \rangle$ and $\langle y \rangle$ give the lateral position of the beam centroid in the measurement plane. The angular moments $\langle \Theta_x \rangle$ and $\langle \Theta_y \rangle$ specify the direction of propagation of the beam centroid.

The (centred) second-order moments are given by

$$\left\langle x^k y^\ell \Theta_x^m \Theta_y^n \right\rangle = \frac{1}{P} \int_{-\infty}^{\infty} h\left(x, y, \Theta_x, \Theta_y\right) \left(x - \langle x \rangle\right)^k \left(y - \langle y \rangle\right)^\ell \left(\Theta_x - \langle \Theta_x \rangle\right)^m \left(\Theta_y - \langle \Theta_y \rangle\right)^n \mathrm{d}x \mathrm{d}y \mathrm{d}\Theta_x \mathrm{d}\Theta_y \qquad (8)$$

where k, ℓ, m and n are non-negative integers and $k + \ell + m + n = 2$. Therefore, there are ten different second-order moments.

The three spatial second-order moments $\left\langle x^2 \right\rangle, \left\langle y^2 \right\rangle$ and $\left\langle x\, y \right\rangle$ are related to the lateral extent of the power density distribution in the measurement plane, the three angular moments $\left\langle \Theta_x^2 \right\rangle, \left\langle \Theta_y^2 \right\rangle$ and $\left\langle \Theta_x \Theta_y \right\rangle$ to the beam divergence, and the four mixed moments $\left\langle x\Theta_x \right\rangle, \left\langle x\Theta_y \right\rangle, \left\langle y\Theta_x \right\rangle$ and $\left\langle y\Theta_y \right\rangle$ to the phase properties in the measurement plane. More details on the relation between the ten second-order moments and the physical beam properties are discussed below.

The spatial first- and second-order moments can be directly obtained from the power density distribution $E(x,y)$. From Equation (1) it follows:

$$\left\langle x \right\rangle = \frac{1}{P} \int E\left(x, y\right) x \mathrm{d}x \mathrm{d}y \qquad (9)$$

$$\left\langle y \right\rangle = \frac{1}{P} \int E\left(x, y\right) y \mathrm{d}x \mathrm{d}y \qquad (10)$$

and

$$\left\langle x^2 \right\rangle = \frac{1}{P} \int E\left(x, y\right) \left(x - \langle x \rangle\right)^2 \mathrm{d}x \mathrm{d}y \qquad (11)$$

$$\left\langle xy \right\rangle = \frac{1}{P} \int E\left(x, y\right) \left(x - \langle x \rangle\right) \left(y - \langle y \rangle\right) \mathrm{d}x \mathrm{d}y \qquad (12)$$

$$\left\langle y^2 \right\rangle = \frac{1}{P} \int E\left(x, y\right) \left(y - \langle y \rangle\right)^2 \mathrm{d}x \mathrm{d}y \qquad (13)$$

The other second-order moments are obtained by measuring the spatial moments in other planes and using the propagation law of the second-order moments (see below).

NOTE The details of measuring all ten second-order moments are given in ISO 11146-2.

2.4 Beam matrix

The ten second-order moments are collected into the symmetric 4 × 4 beam matrix

$$\boldsymbol{P} = \begin{pmatrix} \boldsymbol{W} & \boldsymbol{M} \\ \boldsymbol{M}^{\mathrm{T}} & \boldsymbol{U} \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle xy \rangle & \langle x\Theta_x \rangle & \langle x\Theta_y \rangle \\ \langle xy \rangle & \langle y^2 \rangle & \langle y\Theta_x \rangle & \langle y\Theta_y \rangle \\ \langle x\Theta_x \rangle & \langle y\Theta_x \rangle & \langle \Theta_x^2 \rangle & \langle \Theta_x\Theta_y \rangle \\ \langle x\Theta_y \rangle & \langle y\Theta_y \rangle & \langle \Theta_x\Theta_y \rangle & \langle \Theta_x^2 \rangle \end{pmatrix} \tag{14}$$

with the symmetric submatrix of the spatial moments

$$\boldsymbol{W} = \begin{pmatrix} \langle x^2 \rangle & \langle xy \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix} \tag{15}$$

the symmetric submatrix of the angular moments

$$\boldsymbol{U} = \begin{pmatrix} \langle \Theta_x^2 \rangle & \langle \Theta_y\Theta_y \rangle \\ \langle \Theta_y\Theta_y \rangle & \langle \Theta_y^2 \rangle \end{pmatrix} \tag{16}$$

and the submatrix of the mixed moments

$$\boldsymbol{M} = \begin{pmatrix} \langle x\Theta_x \rangle & \langle x\Theta_y \rangle \\ \langle y\Theta_x \rangle & \langle y\Theta_y \rangle \end{pmatrix} \tag{17}$$

2.5 Propagation though aberration-free optical systems

Aberration-free optical systems are represented by 4 × 4 system matrices $\boldsymbol{S}$ known from geometrical optics. The propagation of the second-order moments through such a system is given by

$$\boldsymbol{P}_{\mathrm{out}} = \boldsymbol{S} \cdot \boldsymbol{P}_{\mathrm{in}} \cdot \boldsymbol{S}^{\mathrm{T}} \tag{18}$$

where $\boldsymbol{P}_{\mathrm{in}}$ and $\boldsymbol{P}_{\mathrm{out}}$ are the beam matrices in entry and exit plane of the optical system, respectively.

Examples for system matrices are given in Annex A.

2.6 Relation between second-order moments and physical beam quantities

The ten second-order moments are closely related to well known physical quantities of a beam.

The three spatial moments describe the lateral extent of the power density distribution of the beam in the measurement plane. The directions of minimum and maximum extent, called principal axes, are always orthogonal to each other. Any power density distribution is characterized by the extents along its principal axes and the orientation of those axes. The beam width along the direction of the principal axis that is closer to the x-axis of the laboratory system is given by

$$d_{\sigma x} = 2\sqrt{2}\left\{\left(\langle x^2 \rangle + \langle y^2 \rangle\right) + \gamma\left[\left(\langle x^2 \rangle - \langle y^2 \rangle\right)^2 + 4\left(\langle x\,y \rangle\right)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}} \tag{19}$$

and the beam width along the direction of that principal axis, which is closer to the y-axis by

$$d_{\sigma y} = 2\sqrt{2}\left\{\left(\langle x^2\rangle + \langle y^2\rangle\right) - \gamma\left[\left(\langle x^2\rangle - \langle y^2\rangle\right)^2 + 4\left(\langle x y\rangle\right)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}} \qquad (20)$$

where

$$\gamma = \operatorname{sgn}\left(\langle x^2\rangle - \langle y^2\rangle\right) = \frac{\langle x^2\rangle - \langle y^2\rangle}{\left|\langle x^2\rangle - \langle y^2\rangle\right|} \qquad (21)$$

If the principal axes make the angle + or – $\pi/4$ with x- or y-axis, when $\langle x^2\rangle = \langle y^2\rangle$, then $d_{\sigma x}$ is by convention the larger of the two beam widths, and

$$d_{\sigma x} = 2\sqrt{2}\left\{\left(\langle x^2\rangle + \langle y^2\rangle\right) + 2\left|\langle xy\rangle\right|\right\}^{\frac{1}{2}} \qquad (22)$$

$$d_{\sigma y} = 2\sqrt{2}\left\{\left(\langle x^2\rangle + \langle y^2\rangle\right) - 2\left|\langle xy\rangle\right|\right\}^{\frac{1}{2}} \qquad (23)$$

The azimuthal angle between that principal axis, which is closer to the x-axis, and the x-axis is obtained by

$$\varphi = \frac{1}{2}\arctan\left(2\langle xy\rangle / \left(\langle x^2\rangle - \langle y^2\rangle\right)\right) \qquad (24)$$

valid for $\langle x^2\rangle \neq \langle y^2\rangle$; for $\langle x^2\rangle = \langle y^2\rangle$, $\varphi(z)$ is obtained as

$$\varphi(z) = \operatorname{sgn}\left(\langle xy\rangle\right)\cdot\frac{\pi}{4} \qquad (25)$$

where

$$\operatorname{sgn}\left(\langle xy\rangle\right) = \frac{\langle xy\rangle}{\left|\langle xy\rangle\right|} \qquad (26)$$

See Figure 2.

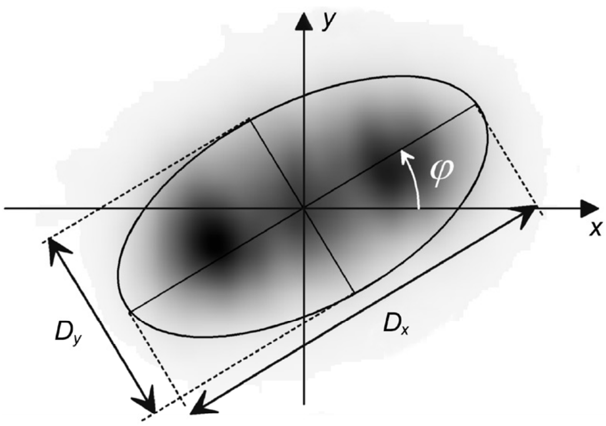


Figure 2 — Azimuthal angle and beam widths along principal axes of power density distribution

Very similar, the three angular moments describe the beam divergence characterized by the orthogonal directions of its maximum and minimum extent. These directions are called the principal axes of the beam divergence and may not coincide with the principal axes of the power density distribution in the measurement plane. The beam divergence along the direction of that principal axis, which is closer to the x-axis of the laboratory system is given by

$$\Theta_{\sigma x} = 2\sqrt{2}\left\{\left(\left\langle \Theta_x^2 \right\rangle + \left\langle \Theta_y^2 \right\rangle\right) + \tau\left[\left(\left\langle \Theta_x^2 \right\rangle - \left\langle \Theta_y^2 \right\rangle\right)^2 + 4\left(\left\langle \Theta_x \Theta_y \right\rangle\right)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}} \tag{27}$$

and the beam divergence along the direction of that principal axis, which is closer to the y-axis by

$$\Theta_{\sigma y} = 2\sqrt{2}\left\{\left(\left\langle \Theta_x^2 \right\rangle + \left\langle \Theta_y^2 \right\rangle\right) - \tau\left[\left(\left\langle \Theta_x^2 \right\rangle - \left\langle \Theta_y^2 \right\rangle\right)^2 + 4\left(\left\langle \Theta_x \Theta_y \right\rangle\right)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}} \tag{28}$$

where

$$\tau = \mathrm{sgn}\left(\left\langle \Theta_x^2 \right\rangle - \left\langle \Theta_y^2 \right\rangle\right) = \frac{\left\langle \Theta_x^2 \right\rangle - \left\langle \Theta_y^2 \right\rangle}{\left|\left\langle \Theta_x^2 \right\rangle - \left\langle \Theta_y^2 \right\rangle\right|} \tag{29}$$

If the principal axes of the beam divergence make the angle + or − π/4 with x- or y-axis, when $\langle \Theta_x \rangle = \langle \Theta_y \rangle$, then $\Theta_{\sigma x}$ is by convention the larger of the two beam divergences, and

$$\Theta_{\sigma x} = 2\sqrt{2}\left\{\left(\left\langle \Theta_x^2 \right\rangle + \left\langle \Theta_y^2 \right\rangle\right) + 2\left|\left\langle \Theta_x \Theta_y \right\rangle\right|\right\}^{\frac{1}{2}} \tag{30}$$

$$\Theta_{\sigma y} = 2\sqrt{2}\left\{\left(\left\langle \Theta_x^2 \right\rangle + \left\langle \Theta_y^2 \right\rangle\right) - 2\left|\left\langle \Theta_x \Theta_y \right\rangle\right|\right\}^{\frac{1}{2}} \tag{31}$$

The divergence azimuthal angle between that principal axis, which is closer to the x-axis, and the x-axis is obtained by

$$\varphi_{\Theta} = \frac{1}{2}\arctan\left(2\left\langle \Theta_x \Theta_y \right\rangle \Big/ \left(\left\langle \Theta_x^2 \right\rangle - \left\langle \Theta_y^2 \right\rangle\right)\right) \tag{32}$$

valid for $\left\langle \Theta_x^2 \right\rangle \neq \left\langle \Theta_y^2 \right\rangle$; for $\left\langle \Theta_x^2 \right\rangle = \left\langle \Theta_y^2 \right\rangle$, $\varphi_{\Theta}(z)$ is obtained as

$$\varphi_{\Theta}(z) = \mathrm{sgn}\left(\left\langle \Theta_x \Theta_y \right\rangle\right) \cdot \frac{\pi}{4} \tag{33}$$

where

$$\mathrm{sgn}\left(\left\langle \Theta_x \Theta_y \right\rangle\right) = \frac{\left\langle \Theta_x \Theta_y \right\rangle}{\left|\left\langle \Theta_x \Theta_y \right\rangle\right|} \tag{34}$$

The four mixed moments are related to the average phase properties of the beam in the measurement plane. The best-fitting phase paraboloid is characterized by the orthogonal directions of maximum and minimum curvature. These curvatures can take also negative or zero values, independently, along the principal axes. The directions of maximum and minimum curvature, called the principal axes of the phase paraboloid, may